

PURE MATHEMATICS

UNIT P4(IAL)

2020 — 2025

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1 - (WMA11/P4(IAL)_Winter_2021_Q4) - Algebra And Functions

The curve C is defined by the parametric equations

$$x = \frac{1}{t} + 2 \quad y = \frac{1-2t}{3+t} \quad t > 0$$

(a) Show that the equation of C can be written in the form $y = g(x)$ where g is the function

$$g(x) = \frac{ax + b}{cx + d} \quad x > k$$

where a, b, c, d and k are integers to be found.

(5)

(b) Hence, or otherwise, state the range of g .

(2)

2 - (WMA11/P4(IAL)_Summer_2024_Q5) - Coordinate Geometry In The (x, Y) Plane, Integration, Algebra And Functions

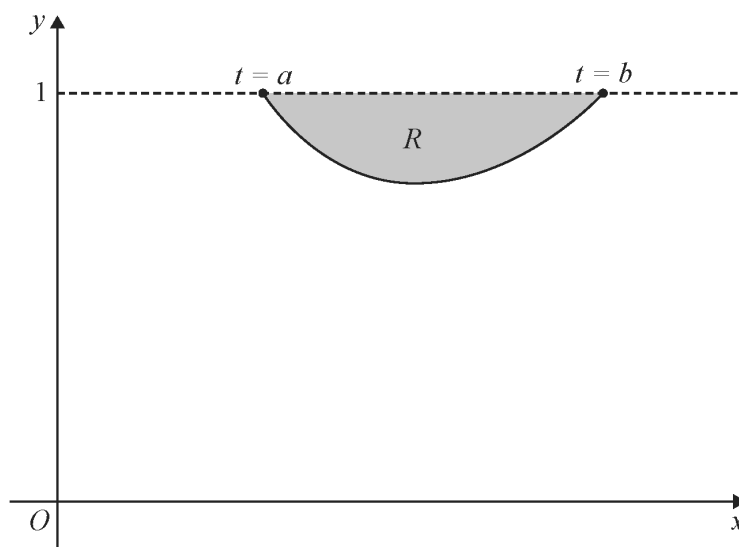


Figure 2

Figure 2 shows a sketch of the curve defined by the parametric equations

$$x = t^2 + 2t \quad y = \frac{2}{t(3-t)} \quad a \leq t \leq b$$

where a and b are constants.

The ends of the curve lie on the line with equation $y = 1$

(a) Find the value of a and the value of b

(2)

The region R , shown shaded in Figure 2, is bounded by the curve and the line with equation $y = 1$

(b) Show that the area of region R is given by

$$M - k \int_a^b \frac{t+1}{t(3-t)} dt$$

where M and k are constants to be found.

(5)

(c) (i) Write $\frac{t+1}{t(3-t)}$ in partial fractions.

(ii) Use algebraic integration to find the exact area of R , giving your answer in simplest form.

(6)

3 - (WMA11/P4(IAL)_Summer_2024_Q6) - Vectors, Algebra And Functions

With respect to a fixed origin O , the line l_1 is given by the equation

$$\mathbf{r} = \mathbf{i} + 2\mathbf{j} + 5\mathbf{k} + \lambda(8\mathbf{i} - \mathbf{j} + 4\mathbf{k})$$

where λ is a scalar parameter.

The point A lies on l_1

Given that $|\vec{OA}| = 5\sqrt{10}$

(a) show that at A the parameter λ satisfies

$$81\lambda^2 + 52\lambda - 220 = 0 \quad (3)$$

Hence

(b) (i) show that one possible position vector for A is $-15\mathbf{i} + 4\mathbf{j} - 3\mathbf{k}$

(ii) find the other possible position vector for A . (3)

The line l_2 is parallel to l_1 and passes through O .

Given that

- $\vec{OA} = -15\mathbf{i} + 4\mathbf{j} - 3\mathbf{k}$
- point B lies on l_2 where $|\vec{OB}| = 4\sqrt{10}$

(c) find the area of triangle OAB , giving your answer to one decimal place. (4)

4 - (WMA11/P4(IAL)_Summer_2024_Q9) - Integration, Trigonometry, Algebra And Functions

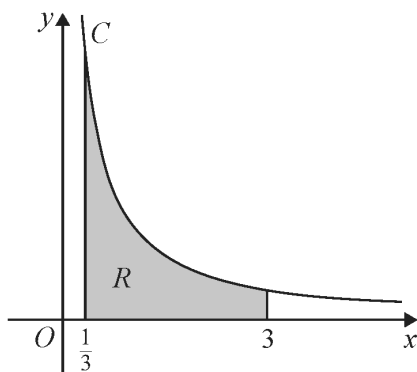


Figure 3

The curve C , shown in Figure 3, has equation

$$y = \frac{x^{-\frac{1}{4}}}{\sqrt{1+x} (\arctan \sqrt{x})}$$

The region R , shown shaded in Figure 3, is bounded by C , the line with equation $x = 3$, the x -axis and the line with equation $x = \frac{1}{3}$

The region R is rotated through 360° about the x -axis to form a solid.

Using the substitution $\tan u = \sqrt{x}$

(a) show that the volume V of the solid formed is given by

$$k \int_a^b \frac{1}{u^2} du$$

where k , a and b are constants to be found.

(6)

(b) Hence, using algebraic integration, find the value of V in simplest form.

(3)

5 - (WMA11/P4(IAL)_Summer_2025_Q4) - Integration, Algebra And Functions

$$f(x) = \frac{5 + 17x - 10x^2}{x(1-x)(2x+1)} \quad x > 1$$

(a) Write $f(x)$ in the form

$$\frac{A}{x} + \frac{B}{1-x} + \frac{C}{2x+1}$$

where A , B and C are constants to be found.

(3)

(b) Hence, use algebraic integration to find the exact value of

$$\int_2^4 f(x) \, dx$$

Write your answer in the form $p \ln 2 + q \ln \left(\frac{3}{5} \right)$, where p and q are integers to be found.

(5)

6 - (WMA11/P4(IAL)_Summer_2025_Q5) - Differentiation, Algebra And Functions

The curve C has parametric equations

$$x = \frac{3+2t}{1-t} \quad y = 1-t^2 \quad t \neq 1$$

The point P , where $t = 2$, lies on C .

(a) Use parametric differentiation to find the equation of the normal to C at P . Give your answer in the form $ax + by + c = 0$ where a , b and c are integers to be found.

(5)

(b) Show that a Cartesian equation for C can be expressed in the form

$$y = \frac{px + q}{(x + r)^2} \quad x \neq k$$

where p , q , r and k are integers to be found.

(4)

7 - (WMA11/P4(IAL)_Winter_2025_Q5) - Integration, Algebra And Functions, Exponentials And Logarithms

(i) Find

$$\int x^2 e^{4x} dx$$

writing the answer in simplest form.

(4)

(ii) Use partial fractions and algebraic integration to show that

$$\int_4^7 \frac{2x+11}{(2x+1)(2-x)} dx = \ln k$$

where k is a fully simplified rational constant to be found.

(6)

8 - (WMA11/P4(IAL)_Winter_2025_Q9) - Differentiation, Coordinate Geometry In The (x, Y) Plane, Algebra And Functions

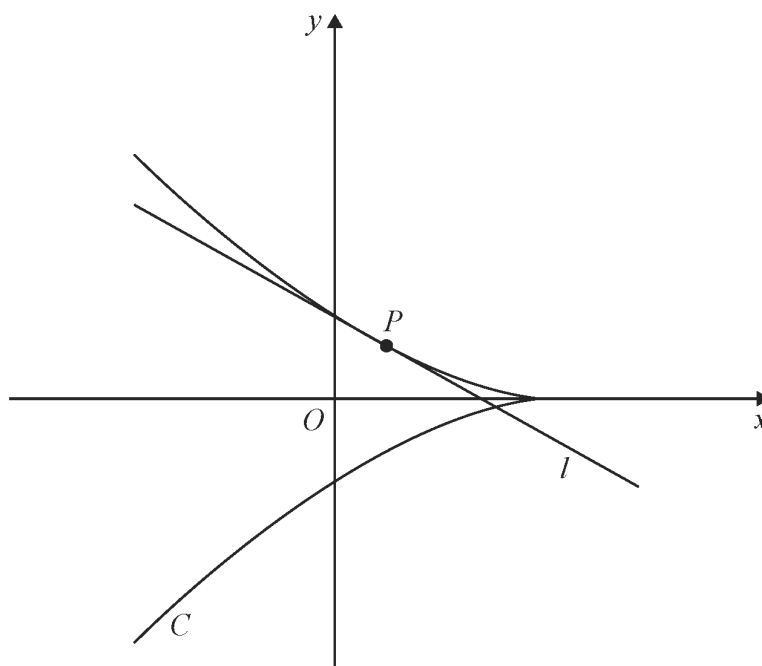


Figure 2

**In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.**

Figure 2 shows a sketch of the curve C with parametric equations

$$x = 2 \cos 2t \quad y = \sin^3 t \quad -\frac{\pi}{2} < t < \frac{\pi}{2}$$

where t is a parameter.

The point P lies on C where $t = \frac{\pi}{6}$

The line l , shown in Figure 2, is the tangent to C at P .

(a) Use parametric differentiation to show that

(i) $\frac{dy}{dx} = k \sin t$ where k is a constant to be found

(ii) an equation for l is $3x + 16y - 5 = 0$

(6)

The line l intersects the curve C again at the point Q .

(b) Using algebra and showing detailed reasoning, find the exact coordinates of Q .

(6)

ANSWERS

1 - (WMA11/P4(IAL)_Winter_2021_Q4) - Algebra And Functions

(a)	$k = 2 \text{ or } x > 2$	B1
	$t = \frac{1}{x-2} \Rightarrow y = \frac{1 - \frac{2}{x-2}}{3 + \frac{1}{x-2}}$	M1 A1
	$\frac{1 - \frac{2}{x-2}}{3 + \frac{1}{x-2}} = \frac{x-2-2}{\dots} \text{ or } \frac{\dots}{3(x-2)+1}$	A1 (M1 on EPEN)
	$y = \frac{x-4}{3x-5}$	A1
		(5)
(b)	$-2 < g < \frac{1}{3}$	M1 A1
		(2)
		(7 marks)

2 - (WMA11/P4(IAL)_Summer_2024_Q5) - Coordinate Geometry In The (x, Y) Plane, Integration, Algebra And Functions

(a)	$y = \frac{2}{t(3-t)} = 1 \Rightarrow 2 = 3t - t^2 \Rightarrow t = \dots$	M1
	$(t^2 - 3t + 2 = (t-1)(t-2) = 0 \Rightarrow) a = 1, b = 2$	A1
		(2)
(b)	Area under curve $= \int_{t=1}^{t=2} y \, dx = \int_1^2 y \frac{dx}{dt} dt = \int_1^2 \frac{2}{t(3-t)} \times (2t+2) dt$	M1 A1
	$t = 1 \Rightarrow x = 3, \quad t = 2 \Rightarrow x = 8$	B1
	So area of R $= 1 \times (8-3) - \int_1^2 \frac{2}{t(3-t)} \times (2t+2) dt$	M1
	$= 5 - 4 \int_1^2 \frac{t+1}{t(3-t)} dt$	A1
		(5)
(c)(i)	$\frac{t+1}{t(3-t)} = \frac{A}{t} + \frac{B}{3-t}$	M1
	$\Rightarrow t+1 = A(3-t) + Bt$	
	E.g. $t = 0 \Rightarrow 1 = 3A \Rightarrow A = \frac{1}{3}; \quad t = 3 \Rightarrow 4 = 3B \Rightarrow B = \frac{4}{3}$	M1
	$\frac{t+1}{t(3-t)} = \frac{1}{3t} + \frac{4}{3(3-t)}$	A1
	$\int \frac{t+1}{t(3-t)} dt = \int \frac{1}{3t} + \frac{4}{3(3-t)} dt = \frac{1}{3} \ln t - \frac{4}{3} \ln(3-t)$	M1
	Area $= 5 - 4 \left[\frac{1}{3} \ln t - \frac{4}{3} \ln(3-t) \right]_1^2 = 5 - 4 \left(\frac{1}{3} \ln 2 - \frac{4}{3} \ln 1 - \frac{1}{3} \ln 1 + \frac{4}{3} \ln 2 \right)$	dM1
	$= 5 - 4 \left(\frac{1}{3} \ln 2 + \frac{4}{3} \ln 2 \right)$	
	$= 5 - \frac{20}{3} \ln 2$	A1
		(6)
(13 marks)		

3 - (WMA11/P4(IAL)_Summer_2024_Q6) - Vectors, Algebra And Functions

(a)	$ \overrightarrow{OA} ^2 = (1+8\lambda)^2 + (2-\lambda)^2 + (5+4\lambda)^2$	M1
	$ \overrightarrow{OA} = 5\sqrt{10} \Rightarrow (1+8\lambda)^2 + (2-\lambda)^2 + (5+4\lambda)^2 = 250$	M1
	$\Rightarrow 64\lambda^2 + 16\lambda + 1 + \lambda^2 - 4\lambda + 4 + 16\lambda^2 + 40\lambda + 25 = 250$ $\Rightarrow 81\lambda^2 + 52\lambda - 220 = 0^*$	A1*
		(3)
(b)	$81\lambda^2 + 52\lambda - 220 = 0 \Rightarrow (81\lambda - 110)(\lambda + 2) = 0 \Rightarrow \lambda = \dots \left(-2, \frac{110}{81}\right)$ $\Rightarrow \overrightarrow{OA} = \begin{pmatrix} 1+8 \times \text{"their } \lambda \text{"} \\ 2 - \text{"their } \lambda \text{"} \\ 5+4 \times \text{"their } \lambda \text{"} \end{pmatrix}$	M1
	E.g $\Rightarrow \overrightarrow{OA} = \begin{pmatrix} 1 \\ 2 \\ 5 \end{pmatrix} - 2 \begin{pmatrix} 8 \\ -1 \\ 4 \end{pmatrix} = \begin{pmatrix} -15 \\ 4 \\ -3 \end{pmatrix}^*$; or $\overrightarrow{OA} = \begin{pmatrix} 1+8 \times \frac{110}{81} \\ 2 - \frac{110}{81} \\ 5+4 \times \frac{110}{81} \end{pmatrix} = \begin{pmatrix} \frac{961}{81} \\ \frac{52}{81} \\ \frac{845}{81} \end{pmatrix}$	A1*; A1
		(3)
(c)	$\cos \theta = \pm \frac{\begin{pmatrix} -15 \\ 4 \\ -3 \end{pmatrix} \cdot \begin{pmatrix} 8 \\ -1 \\ 4 \end{pmatrix}}{\sqrt{15^2 + 4^2 + 3^2} \sqrt{8^2 + 1^2 + 4^2}} = \dots$	M1
	$= \pm \frac{-136}{45\sqrt{10}} \left(= \pm 0.9557\dots\right)$ or e.g. $\theta = \text{awrt } 0.299(\text{rad}) / 17.1^\circ$	A1
	$\text{Area } OAB = \frac{1}{2} 5\sqrt{10} \times 4\sqrt{10} \sin \theta = \dots$	M1
	$= \text{awrt } 29.4$	A1
		(4)
(10 marks)		

4 - (WMA11/P4(IAL)_Summer_2024_Q9) - Integration, Trigonometry, Algebra And Functions

(a)	Volume, $V = \pi \int_{\frac{1}{3}}^3 y^2 dx = \pi \int_{\frac{1}{3}}^3 \frac{x^{-\frac{1}{2}}}{(1+x)(\arctan(\sqrt{x}))^2} dx$	B1
	$\tan u = \sqrt{x} \Rightarrow \sec^2 u \frac{du}{dx} = \frac{1}{2} x^{-\frac{1}{2}}$ oe	M1 A1
	$\Rightarrow V = (\pi) \int \frac{2 \times \frac{1}{2} x^{-\frac{1}{2}}}{(\arctan(\sqrt{x}))^2 (1+x)} dx = (\pi) \int \frac{2}{u^2 (1 + \tan^2 u)} \sec^2 u du$	dM1
	$= \pi \int \frac{2}{u^2 \cancel{\sec^2 u}} \cancel{\sec^2 u} du = 2\pi \int \frac{1}{u^2} du$	A1
	$\left. \begin{array}{l} x=3 \Rightarrow u = \arctan \sqrt{3} = \frac{\pi}{3} \\ x=\frac{1}{3} \Rightarrow u = \arctan \frac{1}{\sqrt{3}} = \frac{\pi}{6} \end{array} \right\} \Rightarrow V = 2\pi \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{1}{u^2} du$	B1
		(6)
(b)	$V = (2\pi) \left[-\frac{1}{u} \right]_{\frac{\pi}{6}}^{\frac{\pi}{3}}$	M1
	$= (2\pi) \left[-\frac{1}{\pi/3} - \left(-\frac{1}{\pi/6} \right) \right]$	dM1
	Or $= (2\pi) \left[-\frac{1}{\arctan \sqrt{x}} \right]_{\frac{1}{3}}^3 = (2\pi) \left(-\frac{1}{\arctan \sqrt{3}} - \left(-\frac{1}{\arctan \sqrt{1/3}} \right) \right)$	
	$= 2\pi \left(-\frac{3}{\pi} + \frac{6}{\pi} \right) = 6$	A1
		(3)
(9 marks)		

5 - (WMA11/P4(IAL)_Summer_2025_Q4) - Integration, Algebra And Functions

(a)	e.g. $5 + 17x - 10x^2 = A(2x + 1)(1 - x) + Bx(2x + 1) + Cx(1 - x)$ $\Rightarrow A = \dots, B = \dots, C = \dots$	M1
	$A = 5, B = 4, C = 8 \Rightarrow f(x) = \frac{5}{x} + \frac{4}{(1-x)} + \frac{8}{(2x+1)}$	A1A1
		(3)
(b)	$\int \frac{5}{x} + \frac{4}{(1-x)} + \frac{8}{(2x+1)} dx = 5 \ln x - 4 \ln 1-x + 4 \ln 2x+1 (+c)$ $\left[5 \ln x - 4 \ln 1-x + 4 \ln 2x+1 \right]_2^4$ $= (5 \ln 4 - 4 \ln 3 + 4 \ln 9) - (5 \ln 2 - 4 \ln 1 + 4 \ln 5) = 5 \ln 2 + 4 \ln \left(\frac{3}{5} \right)$	M1A1ftA1
		dM1A1
		(5)
		(8 marks)

6 - (WMA11/P4(IAL)_Summer_2025_Q5) - Differentiation, Algebra And Functions

(a)	$\frac{dx}{dt} = \frac{2(1-t) - (-1)(3+2t)}{(1-t)^2} = \frac{5}{(1-t)^2}$ and $\frac{dy}{dt} = -2t$ At $t = 2 \Rightarrow \frac{dy}{dx} = \frac{(-1)^2}{5} \times (-2(2)) = -\frac{4}{5}$ $(-7, -3)$ $y + 3 = \frac{5}{4}(x + 7)$ $5x - 4y + 23 = 0$	M1A1
		B1 M1 A1
		(5)
(b)	$x = \frac{3+2t}{1-t} \rightarrow t = \frac{x-3}{x+2}$ $y = 1 - \left(\frac{x-3}{x+2} \right)^2 = \frac{(x+2)^2 - (x-3)^2}{(x+2)^2} = \frac{10x-5}{(x+2)^2}$ $x \neq -2$	M1
		dM1A1
		B1
		(4)
		(9 marks)

7 - (WMA11/P4(IAL)_Winter_2025_Q5) - Integration, Algebra And Functions, Exponentials And Logarithms

(i)	$\int x^2 e^{4x} dx = \frac{1}{4} x^2 e^{4x} - \int \frac{1}{2} x e^{4x} dx$	M1 A1
	$= \frac{1}{4} x^2 e^{4x} - \frac{1}{8} x e^{4x} + \int \frac{1}{8} e^{4x} dx$	dM1
	$= \frac{1}{4} x^2 e^{4x} - \frac{1}{8} x e^{4x} + \frac{1}{32} e^{4x} + c$	A1
		(4)
(ii)	$\frac{2x+11}{(2x+1)(2-x)} \equiv \frac{A}{(2x+1)} + \frac{B}{(2-x)} \Rightarrow A = \dots, B = \dots$	M1
	$= \frac{4}{(2x+1)} + \frac{3}{(2-x)}$	A1
	$\int \frac{2x+11}{(2x+1)(2-x)} dx = 2 \ln 2x+1 - 3 \ln 2-x $	dM1 A1ft
	$\int_4^7 \frac{2x+11}{(2x+1)(2-x)} dx = (2 \ln 15 - 3 \ln 5) - (2 \ln 9 - 3 \ln 2)$	ddM1
	$= \ln \frac{8}{45}$	A1
		(6)
		(10 marks)

8 - (WMA11/P4(IAL)_Winter_2025_Q9) - Differentiation, Coordinate Geometry In The (x, Y) Plane, Algebra And Functions

(a)(i)	$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{3 \sin^2 t \cos t}{-4 \sin 2t}$	M1A1
	$\frac{3 \sin^2 t \cos t}{-4 \sin 2t} = \frac{3 \sin^2 t \cos t}{-8 \sin t \cos t} = -\frac{3}{8} \sin t$ or e.g. $\frac{3 \sin^2 t \cos t}{-4 \sin 2t} = \frac{\frac{3}{2} \sin 2t \sin t}{-4 \sin 2t} = -\frac{3}{8} \sin t$	A1
(ii)	$t = \frac{\pi}{6} \Rightarrow x = 1, y = \frac{1}{8}$	B1
	$m = -\frac{3}{8} \sin\left(\frac{\pi}{6}\right) = -\frac{3}{16} \Rightarrow y - \frac{1}{8} = -\frac{3}{16}(x - 1)$	M1
	$3x + 16y - 5 = 0 \quad *$	A1*
		(6)
(b)	$3(2 \cos 2t) + 16(\sin^3 t) - 5 = 0$	M1
	$6(1 - 2 \sin^2 t) + 16 \sin^3 t - 5 = 0$	dM1
	$16 \sin^3 t - 12 \sin^2 t + 1 = 0$	A1
	$(2 \sin t - 1)^2 (4 \sin t + 1) = 0 \Rightarrow \sin t = -\frac{1}{4}$	ddM1
	$\sin t = -\frac{1}{4} \Rightarrow x = 2 \cos 2t = \dots \text{ and } y = \sin^3 t = \dots$	dddM1
	$Q\left(\frac{7}{4}, -\frac{1}{64}\right)$	A1
		(6)
		(12 marks)

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