

PURE MATHEMATICS

UNIT P3(IAL)

2020 — 2025

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1 - (WMA11/P3(IAL)_Summer_2020_Q3) - Algebra And Functions

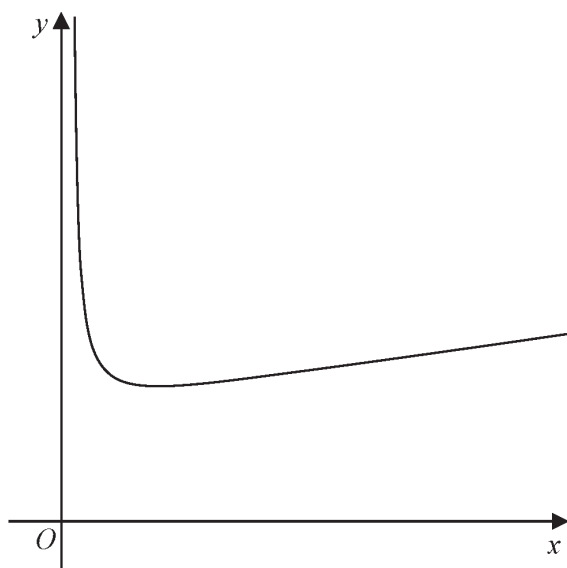


Figure 1

Figure 1 shows a sketch of a curve with equation $y = f(x)$ where

$$f(x) = \frac{2x + 3}{\sqrt{4x - 1}} \quad x > \frac{1}{4}$$

(a) Find, in simplest form, $f'(x)$.

(4)

(b) Hence find the range of f .

(3)

2 - (WMA11/P3(IAL)_Summer_2020_Q4) - Algebra And Functions

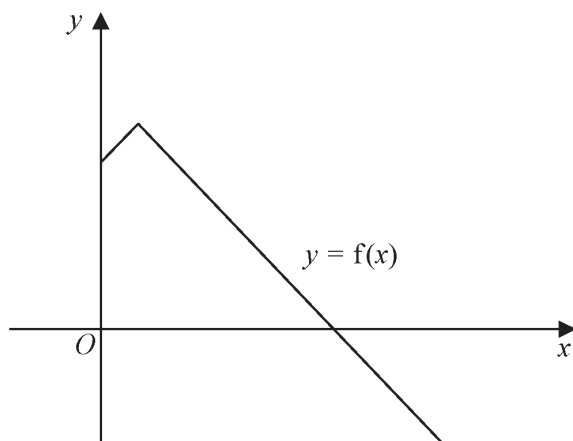
**Figure 2**

Figure 2 shows a sketch of part of the graph with equation $y = f(x)$ where

$$f(x) = 21 - 2|2 - x| \quad x \geq 0$$

(a) Find $ff(6)$ (2)

(b) Solve the equation $f(x) = 5x$ (2)

Given that the equation $f(x) = k$, where k is a constant, has exactly two roots,

(c) state the set of possible values of k . (2)

The graph with equation $y = f(x)$ is transformed onto the graph with equation $y = af(x - b)$

The vertex of the graph with equation $y = af(x - b)$ is $(6, 3)$.

Given that a and b are constants,

(d) find the value of a and the value of b . (2)

3 - (WMA11/P3(IAL)_Winter_2020_Q1) - Algebra And Functions

A population of a rare species of toad is being studied.

The number of toads, N , in the population, t years after the start of the study, is modelled by the equation

$$N = \frac{900e^{0.12t}}{2e^{0.12t} + 1} \quad t \geq 0, t \in \mathbb{R}$$

According to this model,

- (a) calculate the number of toads in the population at the start of the study, (1)
- (b) find the value of t when there are 420 toads in the population, giving your answer to 2 decimal places. (4)
- (c) Explain why, according to this model, the number of toads in the population can never reach 500 (1)

4 - (WMA11/P3(IAL)_Winter_2020_Q2) - Algebra And Functions

The function f and the function g are defined by

$$f(x) = \frac{12}{x+1} \quad x > 0, x \in \mathbb{R}$$

$$g(x) = \frac{5}{2} \ln x \quad x > 0, x \in \mathbb{R}$$

- (a) Find, in simplest form, the value of $fg(e^2)$ (2)
- (b) Find f^{-1} (3)
- (c) Hence, or otherwise, find all real solutions of the equation $f^{-1}(x) = f(x)$ (3)

5 - (WMA11/P3(IAL)_Winter_2020_Q4) - Algebra And Functions, Trigonometry

(i)
$$f(x) = \frac{(2x+5)^2}{x-3} \quad x \neq 3$$

(a) Find $f'(x)$ in the form $\frac{P(x)}{Q(x)}$ where $P(x)$ and $Q(x)$ are fully factorised quadratic expressions.

(b) Hence find the range of values of x for which $f(x)$ is increasing.

(6)

(ii)

$$g(x) = x\sqrt{\sin 4x} \quad 0 \leq x < \frac{\pi}{4}$$

The curve with equation $y = g(x)$ has a maximum at the point M .

Show that the x coordinate of M satisfies the equation

$$\tan 4x + kx = 0$$

where k is a constant to be found.

(5)

6 - (WMA11/P3(IAL)_Winter_2020_Q6) - Algebra And Functions

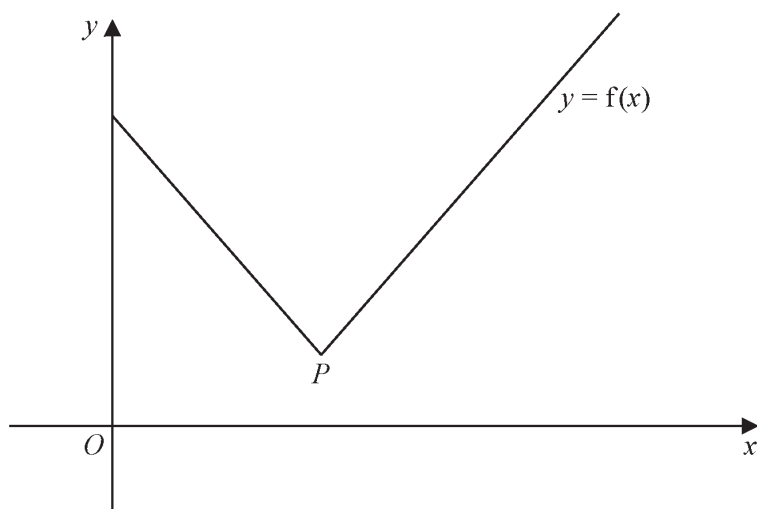


Figure 2

Figure 2 shows part of the graph with equation $y = f(x)$, where

$$f(x) = 2|2x - 5| + 3 \quad x \geq 0$$

The vertex of the graph is at point P as shown.

(a) State the coordinates of P . (2)

(b) Solve the equation $f(x) = 3x - 2$ (4)

Given that the equation

$$f(x) = kx + 2$$

where k is a constant, has exactly two roots,

(c) find the range of values of k . (3)

7 - (WMA11/P3(IAL)_Winter_2020_Q9) - Trigonometry, Algebra And Functions

$$f(\theta) = 5 \cos \theta - 4 \sin \theta \quad \theta \in \mathbb{R}$$

- (a) Express $f(\theta)$ in the form $R \cos(\theta + \alpha)$, where R and α are constants, $R > 0$ and $0 < \alpha < \frac{\pi}{2}$. Give the exact value of R and give the value of α , in radians, to 3 decimal places.

(3)

The curve with equation $y = \cos \theta$ is transformed onto the curve with equation $y = f(\theta)$ by a sequence of two transformations.

Given that the first transformation is a stretch and the second a translation,

- (b) (i) describe fully the transformation that is a stretch,
(ii) describe fully the transformation that is a translation.

(2)

Given

$$g(\theta) = \frac{90}{4 + (f(\theta))^2} \quad \theta \in \mathbb{R}$$

- (c) find the range of g .

(2)

8 - (WMA11/P3(IAL)_Summer_2021_Q4) - Algebra And Functions

The functions f and g are defined by

$$f(x) = \frac{4x + 6}{x - 5} \quad x \in \mathbb{R}, x \neq 5$$

$$g(x) = 5 - 2x^2 \quad x \in \mathbb{R}, x \leq 0$$

- (a) Solve the equation

$$fg(x) = 3$$

(4)

- (b) Find f^{-1}

(3)

- (c) Sketch and label, on the same axes, the curve with equation $y = g(x)$ and the curve with equation $y = g^{-1}(x)$. Show on your sketch the coordinates of the points where each curve meets or cuts the coordinate axes.

(3)

9 - (WMA11/P3(IAL)_Summer_2021_Q6) - Algebra And Functions

Given that k is a positive constant,

(a) on separate diagrams, sketch the graph with equation

(i) $y = k - 2|x|$

(ii) $y = \left| 2x - \frac{k}{3} \right|$

Show on each sketch the coordinates, in terms of k , of each point where the graph meets or cuts the axes.

(4)

(b) Hence find, in terms of k , the values of x for which

$$\left| 2x - \frac{k}{3} \right| = k - 2|x|$$

giving your answers in simplest form.

(4)

ANSWERS

1 - (WMA11/P3(IAL)_Summer_2020_Q3) - Algebra And Functions

(a)	$\frac{dy}{dx} = \frac{(4x-1)^{\frac{1}{2}} \times 2 - (2x+3) \times 2(4x-1)^{-\frac{1}{2}}}{(4x-1)}$ $\frac{(4x-1)^{\frac{1}{2}} \times 2 - (2x+3) \times 2(4x-1)^{-\frac{1}{2}}}{(4x-1)} \times \frac{(4x-1)^{\frac{1}{2}}}{(4x-1)^{\frac{1}{2}}} = \frac{4x-8}{(4x-1)^{\frac{3}{2}}}$	M1 A1 dM1 A1 (4)
(b)	Turning point where $\frac{dy}{dx} = 0 \Rightarrow x = 2$ Find value of f at $x = 2 \Rightarrow f(x) = \sqrt{7}$ Hence range is $f \geq \sqrt{7}$	M1 dM1 A1 (3) (7 marks)

2 - (WMA11/P3(IAL)_Summer_2020_Q4) - Algebra And Functions

(a)	$f(6) = f(13) = -1$	M1 A1 (2)
(b)	Attempts $21 + 2(2-x) = 5x \Rightarrow x = \dots$ or $21 - 2(x-2) = 5x \Rightarrow x = \dots$ $x = \frac{25}{7}$ only	M1 A1 (2)
(c)	Either $k < 21$ or $k \geq 17$ $17 \leq k < 21$	M1 A1 (2)
(d)	$a = \frac{1}{7} \quad b = 4$	B1 B1 (2) (8 marks)

3 - (WMA11/P3(IAL)_Winter_2020_Q1) - Algebra And Functions

(a)	$P_0 = 300$	B1 (1)
(b)	$420 = \frac{900e^{0.12t}}{2e^{0.12t} + 1} \Rightarrow 60e^{0.12t} = 420$ Correct use of lns $\Rightarrow t = \frac{\ln 7}{0.12} = 16.22$	M1 A1 dM1 A1 (4)
(c)	States that maximum number (upper limit) is 450 so cannot reach 500	B1 (1) 6 marks

4 - (WMA11/P3(IAL)_Winter_2020_Q2) - Algebra And Functions

(a)	$fg(e^2) = f\left(\frac{5}{2}\ln e^2\right) = \frac{12}{\frac{5}{2}\ln e^2 + 1} = 2$	M1, A1 (2)
(b)	$f(x) = \frac{12}{x+1}$ $f^{-1}(x) = \frac{12}{x} - 1$ $0 < x < 12$	M1 A1 B1 (3)
(c)	$\frac{12}{x+1} = \frac{12}{x} - 1 \Rightarrow 12x = 12(x+1) - x(x+1)$ $\Rightarrow x^2 + x - 12 = 0 \Rightarrow x = \dots$ $x = 3$ only	Must be 3TQ dM1 A1 (3)
8 marks		
(c) Alts	Solves $f^{-1}(x) = x \Rightarrow \frac{12}{x} - 1 = x$ leading to quadratic equation, or solves $f(x) = x \Rightarrow \frac{12}{x+1} = x$ leading to quadratic equation $\Rightarrow x^2 + x - 12 = 0 \Rightarrow x = \dots$ $x = 3$ only	M1 dM1 A1 (3)

5 - (WMA11/P3(IAL)_Winter_2020_Q4) - Algebra And Functions, Trigonometry

(i) (a)	$f'(x) = \frac{4(x-3)(2x+5) - (2x+5)^2}{(x-3)^2}$ or $\frac{(x-3)(8x+20) - (4x^2 + 20x + 25)}{(x-3)^2}$ $= \frac{(2x+5)(2x-17)}{(x-3)^2}$	M1 A1 M1 A1
(b)	Attempts both critical values or finds one "correct" end $x < -2.5, x > 8.5$ (accept $x \leq -2.5, x \geq 8.5$)	M1 A1 (6)
(ii)	Attempts the chain rule on $(\sin 4x)^{\frac{1}{2}} \rightarrow A(\sin 4x)^{-\frac{1}{2}} \times \cos 4x$ $g(x) = x(\sin 4x)^{\frac{1}{2}} \Rightarrow g'(x) = (\sin 4x)^{\frac{1}{2}} + x \times \frac{1}{2}(\sin 4x)^{-\frac{1}{2}} 4 \cos 4x$ Sets $g'(x) = 0 \rightarrow (\sin 4x)^{\frac{1}{2}} + x \times \frac{2 \cos 4x}{(\sin 4x)^{\frac{1}{2}}} = 0$ and $\times \frac{(\sin 4x)^{\frac{1}{2}}}{\cos 4x}$ oe $\rightarrow \tan 4x + 2x = 0$	M1 M1 A1 M1 A1 (5)
11 marks		

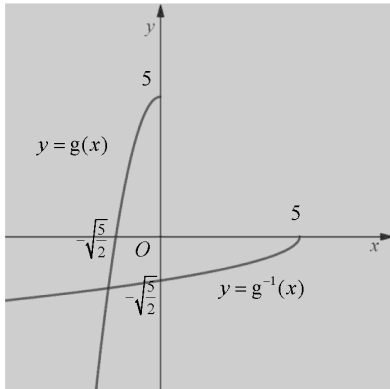
6 - (WMA11/P3(IAL)_Winter_2020_Q6) - Algebra And Functions

(a)	$(2.5, 3)$ oe	B1 B1 (2)
(b)	Attempts one solution usually $4x - 10 + 3 = 3x - 2 \Rightarrow x = 5$ Attempts both solutions $-4x + 10 + 3 = 3x - 2 \Rightarrow x = \frac{15}{7}$	M1 A1 dM1 A1 (4)
(c)	Attempts to solve $y = kx + 2$ with $x = 2.5, y = 3$ or states that $k < 4$ $k \dots \frac{2}{5}$ States $\frac{2}{5} < k < 4$	M1 A1 A1 (3) 9 marks

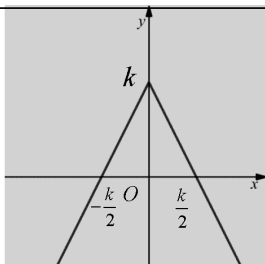
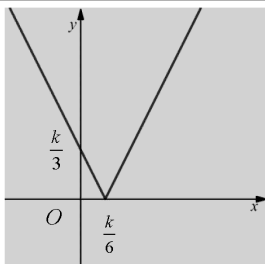
7 - (WMA11/P3(IAL)_Winter_2020_Q9) - Trigonometry, Algebra And Functions

(a)	$R = \sqrt{41}$ $\tan \alpha = \frac{4}{5} \Rightarrow \alpha = \text{awrt } 0.675$	B1 M1A1 (3)
(b)	(i) Describes stretch: stretch in the y direction by " $\sqrt{41}$ " (ii) Describes translation: E.g. translate by $\begin{pmatrix} -\arctan \frac{4}{5} \\ 0 \end{pmatrix}$	B1 ft B1 ft (2)
(c)	Attempts either $g(\theta) = \frac{90}{4 + (\sqrt{41})^2}$ OR $g(\theta) = \frac{90}{4}$ Range $2 \leq g(\theta) \leq 22.5$	M1 A1 (2) 7 marks

8 - (WMA11/P3(IAL)_Summer_2021_Q4) - Algebra And Functions

(a)	$fg(x) = \frac{4(5 - 2x^2) + 6}{5 - 2x^2 - 5}$	M1	
	$fg(x) = 3 \Rightarrow \frac{26 - 8x^2}{-2x^2} = 3 \Rightarrow x = -\sqrt{13}$	dM1 A1 A1	
		(4)	
(b)	$y = \frac{4x + 6}{x - 5} \Rightarrow yx - 5y = 4x + 6 \Rightarrow yx - 4x = 5y + 6$	M1	
	$\Rightarrow x = \frac{5y + 6}{y - 4}$	A1	
	$\Rightarrow f^{-1}(x) = \frac{5x + 6}{x - 4} \quad x \in \mathbb{R}, x \neq 4$	A1	
		(3)	
(c)		Shape and position for $y = g(x)$. A parabola starting on the y -axis that extends down in quadrant 2 to at least the x -axis. Condone slips of the pen e.g. if the curve heads back towards the y -axis in quadrant 3 slightly.	B1
		Shape and position for $y = g(x)$ as above that crosses the x -axis and $y = g^{-1}(x)$ a parabola starting on the x -axis with graphs crossing as shown in quadrant 3. Condone slips of the pen e.g. if $g^{-1}(x)$ heads back towards the x -axis in quadrant 3 slightly.	B1
		Correct intercepts. This mark is for the 4 correct intercepts as shown. Allow as values or coordinates and allow coordinates the wrong way round (e.g. (0, 5) for (5, 0)) as long as they are in the correct places. The negative intercepts must be exact as shown or equivalent. The graphs must cross the axes at these points and you can ignore any other intercepts.	B1
		Note that labels for $g(x)$ and $g^{-1}(x)$ are not required	
		(3)	
		(10 marks)	

9 - (WMA11/P3(IAL)_Summer_2021_Q6) - Algebra And Functions

(a)(i)		Shape	M1
		Correct intercepts.	A1
(ii)		Shape	M1
		Correct intercepts.	A1
			(4)
(b)	Attempts to solve either $k + 2x = -2x + \frac{k}{3}$ or $k - 2x = 2x - \frac{k}{3}$		M1
	Achieves either $x = -\frac{1}{6}k$ or $x = \frac{1}{3}k$ oe		A1
	Attempts to solve both $k + 2x = -2x + \frac{k}{3}$ and $k - 2x = 2x - \frac{k}{3}$		M1
	Achieves either $x = -\frac{1}{6}k$ and $x = \frac{1}{3}k$		A1
			(4)
			(8 marks)

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