

PURE MATHEMATICS

UNIT P2(IAL)

2020 — 2025

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1 - (WMA11/P2(IAL)_Summer_2020_Q3) - Algebra And Functions, Differentiation

$$f(x) = ax^3 - x^2 + bx + 4$$

where a and b are constants.

When $f(x)$ is divided by $(x + 4)$, the remainder is -108

(a) Use the remainder theorem to show that

$$16a + b = 24$$

(2)

Given also that $(2x - 1)$ is a factor of $f(x)$,

(b) find the value of a and the value of b .

(3)

(c) Find $f'(x)$.

(1)

(d) Hence find the exact coordinates of the stationary points of the curve with equation $y = f(x)$.

(4)

2 - (WMA11/P2(IAL)_Winter_2020_Q3) - Algebra And Functions, Trigonometry

$$f(x) = 6x^3 + 17x^2 + 4x - 12$$

(a) Use the factor theorem to show that $(2x + 3)$ is a factor of $f(x)$.

(2)

(b) Hence, using algebra, write $f(x)$ as a product of three linear factors.

(4)

(c) Solve, for $\frac{\pi}{2} < \theta < \pi$, the equation

$$6 \tan^3 \theta + 17 \tan^2 \theta + 4 \tan \theta - 12 = 0$$

giving your answers to 3 significant figures.

(2)

3 - (WMA11/P2(IAL)_Winter_2021_Q1) - Algebra And Functions

$$f(x) = x^4 + ax^3 - 3x^2 + bx + 5$$

where a and b are constants.

When $f(x)$ is divided by $(x + 1)$, the remainder is 4

(a) Show that $a + b = -1$

(2)

When $f(x)$ is divided by $(x - 2)$, the remainder is -23

(b) Find the value of a and the value of b .

(4)

4 - (WMA11/P2(IAL)_Summer_2022_Q7) - Algebra And Functions, Integration

$$f(x) = Ax^3 + 6x^2 - 4x + B$$

where A and B are constants.

Given that

- $(x + 2)$ is a factor of $f(x)$

- $\int_3^5 f(x) dx = 176$

find the value of A and the value of B .

(7)

5 - (WMA11/P2(IAL)_Winter_2022_Q5) - Algebra And Functions

$$f(x) = 3x^3 + Ax^2 + Bx - 10$$

where A and B are integers.

Given that

- when $f(x)$ is divided by $(x - 1)$ the remainder is k
- when $f(x)$ is divided by $(x + 1)$ the remainder is $-10k$
- k is a constant

(a) show that

$$11A + 9B = 83$$

(3)

Given also that $(3x - 2)$ is a factor of $f(x)$,

(b) find the value of A and the value of B .

(3)

(c) Hence find the quadratic expression $g(x)$ such that

$$f(x) = (3x - 2)g(x)$$

(2)

6 - (WMA11/P2(IAL)_Summer_2023_Q2) - Algebra And Functions

$$f(x) = 4x^3 - 8x^2 + 5x + a$$

where a is a constant.

Given that $(2x - 3)$ is a factor of $f(x)$,

(a) use the factor theorem to show that $a = -3$

(2)

(b) Hence show that the equation $f(x) = 0$ has only one real root.

(4)

7 - (WMA11/P2(IAL)_Winter_2023_Q5) - Algebra And Functions

$$f(x) = x^3 + (p + 3)x^2 - x + q$$

where p and q are constants and $p > 0$

Given that $(x - 3)$ is a factor of $f(x)$

(a) show that

$$9p + q = -51 \quad (2)$$

Given also that when $f(x)$ is divided by $(x + p)$ the remainder is 9

(b) show that

$$3p^2 + p + q - 9 = 0 \quad (2)$$

(c) Hence find the value of p and the value of q . (3)

(d) Hence find a quadratic expression $g(x)$ such that

$$f(x) = (x - 3)g(x) \quad (2)$$

8 - (WMA11/P2(IAL)_Summer_2024_Q4) - Algebra And Functions

$$f(x) = (x - 2)(2x^2 + 5x + k) + 21$$

where k is a constant.

(a) State the remainder when $f(x)$ is divided by $(x - 2)$ (1)

Given that $(2x - 1)$ is a factor of $f(x)$

(b) show that $k = 11$ (2)

(c) Hence

(i) fully factorise $f(x)$,

(ii) find the number of real solutions of the equation

$$f(x) = 0$$

giving a reason for your answer.

(5)

9 - (WMA11/P2(IAL)_Winter_2024_Q1) - Algebra And Functions

$$f(x) = ax^3 + 3x^2 - 8x + 2 \quad \text{where } a \text{ is a constant}$$

Given that when $f(x)$ is divided by $(x - 2)$ the remainder is 3, find the value of a .

(3)

10 - (WMA11/P2(IAL)_Summer_2025_Q4) - Algebra And Functions

The function $f(x)$ is defined by

$$f(x) = ax^3 + bx^2 + 5x - 3$$

where a and b are constants.

Given that $(x + 3)$ is a factor of $f(x)$,

(a) show that

$$b = 3a + 2$$

(2)

Given further that when $f(x)$ is divided by $(2x - 1)$ the remainder is $\frac{7}{4}$

(b) find the value of a and the value of b .

(4)

(c) Using algebra, find the quotient and the remainder when $f(x)$ is divided by $(x - 2)$

(3)

ANSWERS

1 - (WMA11/P2(IAL)_Summer_2020_Q3) - Algebra And Functions, Differentiation

(a)	$a(-4)^3 - (-4)^2 + b(-4) + 4 = -108$ Attempts to set $f(-4) = -108$ to obtain an equation in a and b. Score when you see “- 4” embedded in the equation or 2 correct terms (excluding the “+ 4”) on lhs. May be implied by e.g. $-64a - 16 - 4b + 4 = -108$ Condone minor slips on the lhs e.g. one sign error between terms but must use - 108	M1
	As an alternative for the first mark we will condone an attempt at long division. This requires a complete method to divide $(ax^3 - x^2 + bx + 4)$ by $(x + 4)$ to obtain a remainder in terms of a and b which is then equated to -108 For reference, the quotient is $ax^2 - (1+4a)x + 16a + b + 4$ and the remainder is $-4b - 64a - 12$	
	$-64a - 16 - 4b + 4 = -108$ $\Rightarrow 16a + b = 24 *$	A1*
	Correct equation obtained with no errors and at least one line of intermediate working if starting with e.g. $a(-4)^3 - (-4)^2 + b(-4) + 4 = -108$	(2)
(b)	$a\left(\frac{1}{2}\right)^3 - \left(\frac{1}{2}\right)^2 + b\left(\frac{1}{2}\right) + 4 = 0$ Attempts to set $f\left(\frac{1}{2}\right) = 0$ to obtain an equation in a and b. Condone slips. Score when you see “$\frac{1}{2}$” embedded in the equation or 2 correct terms (excluding the “+ 4”) on lhs. May be implied by e.g. $\frac{a}{8} - \frac{1}{4} + \frac{b}{2} + 4 = 0$ The “= 0” may be implied when they attempt to solve simultaneously below	M1
	An alternative for the first mark is to attempt long division. This requires a complete method to divide $(ax^3 - x^2 + bx + 4)$ by $(2x - 1)$ to obtain a remainder in a and b which is then equated to 0 For reference, the quotient is $\frac{a}{2}x^2 + \left(\frac{a}{4} - \frac{1}{2}\right)x + \left(\frac{b}{2} - \frac{1}{4} + \frac{a}{8}\right)$ and the remainder is $\frac{15}{4} + \frac{b}{2} + \frac{a}{8}$	
	$16a + b = 24, a + 4b = -30$ $\Rightarrow a = \dots, b = \dots$	M1
	$a = 2, b = -8$	A1
		(3)
(c)	$f(x) = 2x^3 - x^2 - 8x + 4$ $\Rightarrow f'(x) = 6x^2 - 2x - 8$	B1ft
		(1)

(d)	$6x^2 - 2x - 8 = 0$ $\Rightarrow (3x - 4)(x + 1) = 0$ $\Rightarrow x = \dots$	Sets their $f'(x) = 0$ (may be implied) and solves a 3 term quadratic. Apply general guidance if necessary. You may need to check if a calculator has been used.	M1
	$x = \frac{4}{3}, -1 \Rightarrow y = \dots$	Uses at least one of their x values to find a value for y using their $f(x)$ <u>where x is from an attempt to solve $f'(x) = 0$</u> . You may need to check their y values if working is not shown.	M1
	$\left(\frac{4}{3}, -\frac{100}{27}\right)$ or $(-1, 9)$ Or e.g. $x = \frac{4}{3}, y = -\frac{100}{27}$ and $x = -1, y = 9$ One correct point. The fractional coordinates must be exact but allow 1.3 with a dot over the 3 and 3.703 with dots over the 7 and 3. Note that it is not necessary for the points to be written as coordinates as long as the pairing is clear. Depends on having scored both previous M marks.		A1
	$\left(\frac{4}{3}, -\frac{100}{27}\right)$ and $(-1, 9)$ Or e.g. $x = \frac{4}{3}, y = -\frac{100}{27}$ and $x = -1, y = 9$ Both correct points. The fractional coordinates must be exact but allow 1.3 with a dot over the 3 and 3.703 with dots over the 7 and 3. Note that it is not necessary for the points to be written as coordinates as long as the pairing is clear. Depends on having scored both previous M marks.		A1
	Fully correct answers with no working scores 4/4 following a <u>correct</u> part (c) i.e. $\Rightarrow f'(x) = 6x^2 - 2x - 8$		
			(4)
			Total 10

2 - (WMA11/P2(IAL)_Winter_2020_Q3) - Algebra And Functions, Trigonometry

(a)	Attempts $f\left(-\frac{3}{2}\right) = 6\left(-\frac{3}{2}\right)^3 + 17\left(-\frac{3}{2}\right)^2 + 4\left(-\frac{3}{2}\right) - 12$ $= 0 \Rightarrow (2x + 3) \text{ is a factor} \quad *$	M1 A1*	(2)
(b)	$6x^3 + 17x^2 + 4x - 12 = (2x + 3)(3x^2 + 4x - 4)$ $= (2x + 3)(3x - 2)(x + 2)$	M1 A1 dM1 A1	(4)
(c)	Solves $\tan \theta = -\frac{3}{2}$ or “-2” or “ $\frac{2}{3}$ ” $\theta = \text{awrt } 2.03, 2.16$	M1 A1	(2)
			(8 marks)

3 - (WMA11/P2(IAL)_Winter_2021_Q1) - Algebra And Functions

(a)	$f(-1) = (-1)^4 + a(-1)^3 - 3(-1)^2 + b(-1) + 5 = 4$	M1
	$1 - a - 3 - b + 5 = 4 \Rightarrow a + b = -1$ *	A1*
		(2)
(b)	$f(2) = (2)^4 + a(2)^3 - 3(2)^2 + b(2) + 5 = -23$	M1
	$\Rightarrow 8a + 2b = -32$ oe (eg $4a + b = -16$)	A1
	$b = -1 - a \Rightarrow 4a - 1 - a = -16 \Rightarrow a = \dots$	dM1
	$a = -5, b = 4$	A1
		(4)
		(6 marks)

4 - (WMA11/P2(IAL)_Summer_2022_Q7) - Algebra And Functions, Integration

	$(x+2)$ a factor $\Rightarrow f(-2) = 0 \Rightarrow -8A + 24 + 8 + B = 0$	M1A1
	$\int f(x)dx = \frac{A}{4}x^4 + 2x^3 - 2x^2 + Bx$	M1 A1
	$\int_3^5 f(x)dx = 176 \Rightarrow \left[\frac{A}{4}x^4 + 2x^3 - 2x^2 + Bx \right]_3^5 = 176$ $\Rightarrow \left(\frac{A}{4}5^4 + 2(5^3) - 2(5^2) + 5B \right) - \left(\frac{A}{4}3^4 + 2(3^3) - 2(3^2) + 3B \right) = 176$	dM1
	$\left. \begin{array}{l} 8A - B = 32 \\ 136A + 2B = 12 \end{array} \right\} \Rightarrow A = \dots, B = \dots$	dM1
	$A = \frac{1}{2}, B = -28$	A1
		(7)
		(7 marks)

5 - (WMA11/P2(IAL)_Winter_2022_Q5) - Algebra And Functions

(a)	$3(1)^3 + A(1)^2 + B(1) - 10 = k$ or $3(-1)^3 + A(-1)^2 + B(-1) - 10 = -10k$	Attempts $f(\pm 1) = k$ or $f(\pm 1) = -10k$	M1
	$A + B - 7 = k, A - B - 13 = -10k$ $\Rightarrow -10A - 10B + 70 = A - B - 13$	Uses $f(\pm 1) = k$ and $f(\mp 1) = -10k$ to eliminate k and obtains an equation in A and B only	M1
	$\Rightarrow 11A + 9B = 83$ *	Correct proof with no errors	A1*
			(3)
(b)	$3\left(\frac{2}{3}\right)^3 + A\left(\frac{2}{3}\right)^2 + B\left(\frac{2}{3}\right) - 10 = 0$	Attempts $f\left(\frac{2}{3}\right) = 0$	M1
	$11A + 9B = 83, 12A + 18B = 246$ $\Rightarrow A = \dots, B = \dots$	Solves $11A + 9B = 83$ simultaneously with <i>their</i> equation in A and B	M1
	$A = -8, B = 19$	Correct values	A1
			(3)
(c)	$f(x) = (3x - 2)(x^2 + \dots x + \dots)$	Uses any appropriate method e.g. long division/inspection to obtain $x^2 + px + q$ where p and q are non-zero.	M1
	$g(x) = x^2 - 2x + 5$	Correct expression	A1
			(2)
			Total 8

6 - (WMA11/P2(IAL)_Summer_2023_Q2) - Algebra And Functions

a	$\left(f\left(\frac{3}{2}\right) = 4\left(\frac{3}{2}\right)^3 - 8\left(\frac{3}{2}\right)^2 + 5\left(\frac{3}{2}\right) + a = 0 \Rightarrow a = \dots\right.$	M1
	$\Rightarrow \frac{27}{2} - 18 + \frac{15}{2} + a = 0 \Rightarrow a = -3$ *	A1*
		(2)
b	Example where $2x - 3$ is a linear factor: $ \begin{array}{r} 2x^2 - x + 1 \\ 2x - 3 \overline{) 4x^3 - 8x^2 + 5x - 3} \\ \underline{4x^3 - 6x^2} \\ -2x^2 + 5x \\ \underline{-2x^2 + 3x} \\ +2x - 3 \\ \underline{+2x - 3} \\ 0 \end{array} $	M1A1
	$(-1)^2 - 4 \times 2 \times 1 = -7 < 0 \Rightarrow$ no real roots so $x = \frac{3}{2}$ is the only one real root *	dM1A1*
		(4)
		(6 marks)

7 - (WMA11/P2(IAL)_Winter_2023_Q5) - Algebra And Functions

(a)	$3^3 + 3^2(p+3) - 3 + q = 0$	M1
	eg $\Rightarrow 27 + 9p + 27 - 3 + q = 0 \Rightarrow 9p + q = -51^*$	A1*
		(2)
(b)	$(-p)^3 + (p+3)(-p)^2 - (-p) + q = 9$	M1
	eg $-p^3 + p^3 + 3p^2 + p + q = 9 \Rightarrow 3p^2 + p + q - 9 = 0^*$	A1*
		(2)
(c)	$3p^2 + p + q - 9 = 0$ $\Rightarrow 3p^2 + p - 51 - 9p - 9 = 0$ $\Rightarrow 3p^2 - 8p - 60 = 0$	M1
	$p = 6$	A1
	$q = -51 - 9p = -105$	A1
		(3)
(d)	$f(x) = x^3 + 9x^2 - x - 105$ $f(x) = (x-3)(\dots x^2 + \dots x + \dots)$	M1
	$g(x) = x^2 + 12x + 35$	A1
		(2)
		Total 9

8 - (WMA11/P2(IAL)_Summer_2024_Q4) - Algebra And Functions

(a)	21	B1	(1)
(b)	Sets $f\left(\pm\frac{1}{2}\right) = 0 \rightarrow$ equation in k	M1	
	Eg. $\left(\frac{1}{2} - 2\right)\left(2 \times \frac{1}{4} + 5 \times \frac{1}{2} + k\right) + 21 = 0$ $\Rightarrow 3 + k = 14 \Rightarrow k = 11^*$	A1*	(2)
(c)	(i) $f(x) = (x-2)(2x^2 + 5x + 11) + 21 = 2x^3 + x^2 + x - 1$ $= (2x-1)(x^2 + x + 1)$	M1	
	(ii) Attempts to find the number of roots of their $(x^2 + x + 1)$	dM1, A1	
	• States $(x^2 + x + 1)$ has no roots with reason	M1	
	• Concludes $f(x) = 0$ has 1 root (at $x = \frac{1}{2}$)	A1	
			(5)
			(8 marks)

9 - (WMA11/P2(IAL)_Winter_2024_Q1) - Algebra And Functions

	$f(x) = ax^3 + 3x^2 - 8x + 2$	
	Sets $f(2) = 3 \Rightarrow 8a + 12 - 16 + 2 = 3$	M1
	$\Rightarrow 8a = 5 \Rightarrow a = \frac{5}{8}$	M1A1
		(3)
		(3 marks)

10 - (WMA11/P2(IAL)_Summer_2025_Q4) - Algebra And Functions

(a)	$f(x) = ax^3 + bx^2 + 5x - 3$ Sets $f(-3) = 0 \rightarrow a \times (-3)^3 + b \times (-3)^2 + 5 \times (-3) - 3 = 0$ $\Rightarrow -27a + 9b = 18 \Rightarrow b = 3a + 2^*$	M1 A1* (2)
(b)	Sets $f\left(\frac{1}{2}\right) = \frac{7}{4} \rightarrow a \times \left(\frac{1}{2}\right)^3 + b \times \left(\frac{1}{2}\right)^2 + 5 \times \left(\frac{1}{2}\right) - 3 = \frac{7}{4}$ $a + 2b = 18$ Solves $b = 3a + 2$ with their $a + 2b = 18 \Rightarrow a = \dots, b = \dots$ $a = 2, b = 8$	M1 A1 M1 A1 (4)
(c)	e.g. $\frac{2x^3 + 8x^2 + 5x - 3}{x - 2} = (\dots x^2 + \dots x + \dots) + \text{Remainder}$ (Quotient =) $2x^2 + 12x + 29$ Remainder = 55	M1 A1 B1 (3) (9 marks)

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