

PURE MATHEMATICS

UNIT P1(IAL)

2020 — 2025

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EXAMINENT
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1 - (WMA11/P1(IAL)_Winter_2020_Q5) - *Algebra And Functions*

(a) Find, using algebra, all solutions of

$$20x^3 - 50x^2 - 30x = 0 \quad (3)$$

(b) Hence find all real solutions of

$$20(y + 3)^{\frac{3}{2}} - 50(y + 3) - 30(y + 3)^{\frac{1}{2}} = 0 \quad (4)$$

2 - (WMA11/P1(IAL)_Winter_2020_Q8) - *Algebra And Functions*

The straight line l has equation $y = k(2x - 1)$, where k is a constant.

The curve C has equation $y = x^2 + 2x + 11$

Find the set of values of k for which l does not cross or touch C . (6)

3 - (WMA11/P1(IAL)_Winter_2020_Q10) - *Algebra And Functions*

The curve C_1 has equation $y = f(x)$, where

$$f(x) = (4x - 3)(x - 5)^2$$

(a) Sketch C_1 showing the coordinates of any point where the curve touches or crosses the coordinate axes. (3)

(b) Hence or otherwise

(i) find the values of x for which $f\left(\frac{1}{4}x\right) = 0$

(ii) find the value of the constant p such that the curve with equation $y = f(x) + p$ passes through the origin. (2)

A second curve C_2 has equation $y = g(x)$, where $g(x) = f(x + 1)$

(c) (i) Find, in simplest form, $g(x)$. You may leave your answer in a factorised form.

(ii) Hence, or otherwise, find the y intercept of curve C_2 (3)

4 - (WMA11/P1(IAL)_Summer_2021_Q5) - Algebra And Functions

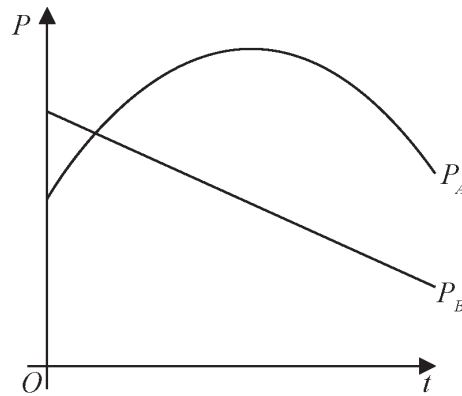


Figure 2

The share value of two companies, company *A* and company *B*, has been monitored over a 15-year period.

The share value P_A of **company A**, in millions of pounds, is modelled by the equation

$$P_A = 53 - 0.4(t - 8)^2 \quad t \geq 0$$

where t is the number of years after monitoring began.

The share value P_B of **company B**, in millions of pounds, is modelled by the equation

$$P_B = -1.6t + 44.2 \quad t \geq 0$$

where t is the number of years after monitoring began.

Figure 2 shows a graph of both models.

Use the equations of one or both models to answer parts (a) to (d).

- (a) Find the difference between the share value of **company A** and the share value of **company B** at the point monitoring began. (2)
- (b) State the maximum share value of **company A** during the 15-year period. (1)
- (c) Find, using algebra and showing your working, the times during this 15-year period when the share value of **company A** was greater than the share value of **company B**. (4)
- (d) Explain why the model for **company A** should not be used to predict its share value when $t = 20$ (1)

5 - (WMA11/P1(IAL)_Summer_2021_Q9) - Algebra And Functions

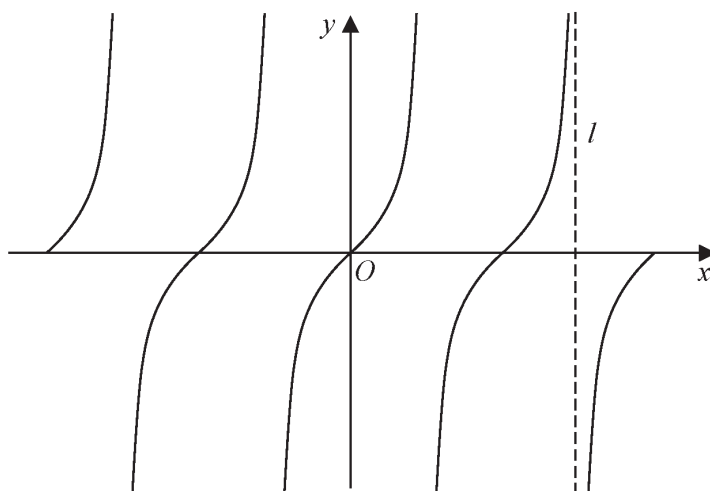
**Figure 4**

Figure 4 shows a sketch of the curve with equation

$$y = \tan x \quad -2\pi \leq x \leq 2\pi$$

The line l , shown in Figure 4, is an asymptote to $y = \tan x$

(a) State an equation for l .

(1)

A copy of Figure 4, labelled Diagram 1, is shown on the next page.

(b) (i) On Diagram 1, sketch the curve with equation

$$y = \frac{1}{x} + 1 \quad -2\pi \leq x \leq 2\pi$$

stating the equation of the horizontal asymptote of this curve.

(ii) Hence, **giving a reason**, state the number of solutions of the equation

$$\tan x = \frac{1}{x} + 1$$

in the region $-2\pi \leq x \leq 2\pi$

(4)

(c) State the number of solutions of the equation $\tan x = \frac{1}{x} + 1$ in the region

(i) $0 \leq x \leq 40\pi$

(ii) $-10\pi \leq x \leq \frac{5}{2}\pi$

(2)

6 - (WMA11/P1(IAL)_Winter_2021_Q2) - Algebra And Functions

A tree was planted.

Exactly 3 years after it was planted, the height of the tree was 2 m.

Exactly 5 years after it was planted, the height of the tree was 2.4 m.

Given that the height, H metres, of the tree, t years after it was planted, can be modelled by the equation

$$H^3 = pt^2 + q$$

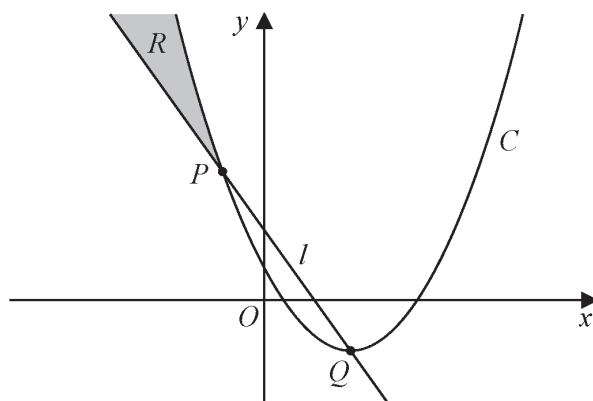
where p and q are constants,

- (a) find, to 3 significant figures where necessary, the value of p and the value of q . (4)

Exactly T years after the tree was planted, its height was 5 m.

- (b) Find the value of T according to the model, giving your answer to one decimal place. (2)

7 - (WMA11/P1(IAL)_Winter_2021_Q4) - Algebra And Functions

**Figure 2**

The points P and Q , as shown in Figure 2, have coordinates $(-2, 13)$ and $(4, -5)$ respectively.

The straight line l passes through P and Q .

- (a) Find an equation for l , writing your answer in the form $y = mx + c$, where m and c are integers to be found.
- (3)**

The quadratic curve C passes through P and has a minimum point at Q .

- (b) Find an equation for C .
- (3)**

The region R , shown shaded in Figure 2, lies in the second quadrant and is bounded by C and l only.

- (c) Use inequalities to define region R .
- (2)**

8 - (WMA11/P1(IAL)_Winter_2021_Q6) - Algebra And Functions

(a) Sketch the curve with equation

$$y = -\frac{k}{x} \quad k > 0 \quad x \neq 0 \quad (2)$$

(b) On a separate diagram, sketch the curve with equation

$$y = -\frac{k}{x} + k \quad k > 0 \quad x \neq 0$$

stating the coordinates of the point of intersection with the x -axis and, in terms of k , the equation of the horizontal asymptote.

(3)

(c) Find the range of possible values of k for which the curve with equation

$$y = -\frac{k}{x} + k \quad k > 0 \quad x \neq 0$$

does not touch or intersect the line with equation $y = 3x + 4$

(5)

ANSWERS

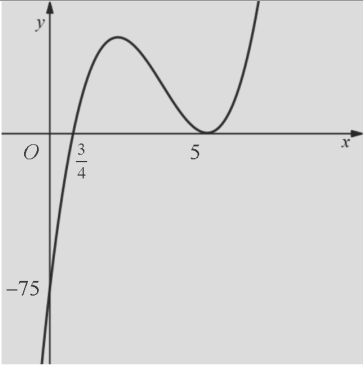
1 - (WMA11/P1(IAL)_Winter_2020_Q5) - Algebra And Functions

(a)	$20x^3 - 50x^2 - 30x = 0 \Rightarrow 10x(2x^2 - 5x - 3) = 0$ $\Rightarrow 10x(2x+1)(x-3) = 0$ $\Rightarrow x = 0, -\frac{1}{2}, 3$	M1 A1, A1 (3)
(b)	Sets or implies $(y+3)^{\frac{1}{2}} = 0$ or $-\frac{1}{2}$ or 3 Full method to find y $y = 6$ $y = -3, 6$	B1ft M1 A1ft, A1 (4) (7 marks)

2 - (WMA11/P1(IAL)_Winter_2020_Q8) - Algebra And Functions

	Equates $y = k(2x-1)$ and $y = x^2 + 2x + 11 \Rightarrow k(2x-1) = x^2 + 2x + 11$ $\Rightarrow x^2 + (2-2k)x + 11+k = 0$ Attempts "$b^2 - 4ac$"... $0 \Rightarrow (2-2k)^2 - 4(11+k) \leq 0$ and proceeds to critical values Critical values of $(k =) -5, -2$ No roots so $b^2 - 4ac < 0$ so choose inside region $-2 < k < 5$	M1 A1 M1 A1 M1 A1 (6) (6 marks)
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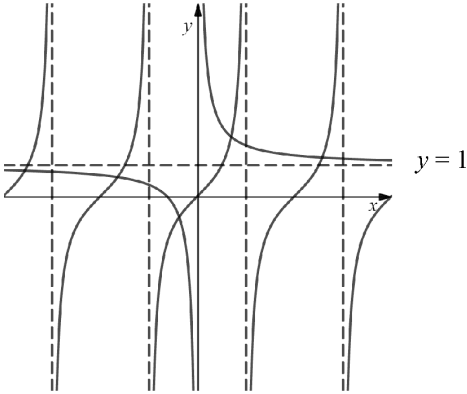
3 - (WMA11/P1(IAL)_Winter_2020_Q10) - Algebra And Functions

(a)		Shape for +ve x^3 Cuts x-axis at $\left(\frac{3}{4}, 0\right)$ and meets at $(5, 0)$ Crosses y-axis at $(0, -75)$	B1 B1 B1 (3)
(b)(i)	$(x =) 3, 20$		B1ft
(ii)	$(p =) 75$		B1ft
(c) (i)	$(g(x) =) (4(x+1)-3)(x+1-5)^2 = (4x+1)(x-4)^2$		M1 A1
(ii)	16		B1
			(3) (8 marks)

4 - (WMA11/P1(IAL)_Summer_2021_Q5) - Algebra And Functions

(a)	$P_B - P_A = 44.2 - (53 - 0.4 \times 8^2) = \dots$ awrt (£) 16.8 million	M1 A1
		(2)
(b)	(£) 53 (million)	B1
		(1)
(c)	$-1.6t + 44.2 = 53 - 0.4(t - 8)^2$ $\Rightarrow 0.4t^2 - 8t + \frac{84}{5} = 0 \Rightarrow t = \dots$ $t = 10 - \sqrt{58} = \text{awrt } 2.38 \text{ (years)}$ $"2.38" < t (\leq 15)$	M1 M1 A1 A1ft
		(4)
	“The share value would be negative” / “the model is known to hold for 15 years only (and 20 years is more than 15).”	B1
		(1)
		(8 marks)

5 - (WMA11/P1(IAL)_Summer_2021_Q9) - Algebra And Functions

(a)	$x = \frac{3\pi}{2}$ oe	B1
		(1)
(b)(i)		B1B1
(ii)	5 (solutions) Number of solutions are the number of points of intersections between the graphs	B1 B1
		(4)
(c)		
(i)	(Number of solutions) = 40	B1ft
(ii)	(Number of solutions) = 14	B1
		(2)
		(7 marks)

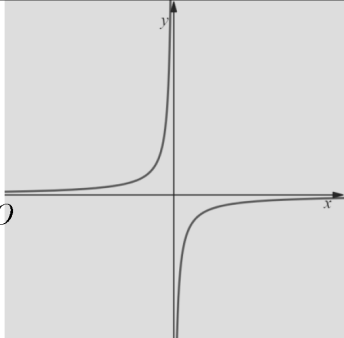
6 - (WMA11/P1(IAL)_Winter_2021_Q2) - Algebra And Functions

(a)	Attempts to use the given model at least once. Eg $2^3 = p \times 3^2 + q$ Two correct simplified equations $9p + q = 8$ $25p + q = 13.8(24)$ Solves simultaneously to get at least one of p or q $p = 0.364, q = 4.72(4)$	M1 A1 dM1 A1 (4)
(b)	Attempts to find T when $H = 5$ Eg. Calculates $\sqrt{\frac{125 - "q"}{"p"}}$ $(T =) 18.2$	M1 A1 (2)
		(6 marks)

7 - (WMA11/P1(IAL)_Winter_2021_Q4) - Algebra And Functions

(a)	Gradient $PQ = -3$ Attempts to find equation of l Eg. $y - 13 = -3(x + 2)$ $y = -3x + 7$	B1 M1 A1 (3)
(b)	Attempts to use minimum is $(4, -5)$ Eg $y = \dots(x - 4)^2 - 5$ Attempts to use $(-2, 13)$ with $y = a(x - 4)^2 - 5 \Rightarrow a = \dots$ $\Rightarrow y = \frac{1}{2}(x - 4)^2 - 5$ or $y = \frac{1}{2}x^2 - 4x + 3$ oe	M1 dM1 A1 (3)
(c)	Two of $y > -3x + 7$, $y < \frac{1}{2}(x - 4)^2 - 5$ $x < -2$ All three of $y > -3x + 7$, $y < \frac{1}{2}(x - 4)^2 - 5$ $x < -2$	M1 A1 (2) (8 marks)

8 - (WMA11/P1(IAL)_Winter_2021_Q6) - Algebra And Functions

(a)	 Negative reciprocal shape Fully correct	M1 A1 (2)
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