

IB Diploma

MATHEMATICS AI

HL P1
2021 — 2025

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1 - (MATHAI/11_HL_Summer_2021_Q17) - Algebra-functions - Roots

[Maximum mark: 7]

The graph of the function $f(x) = \ln x$ is translated by $\begin{pmatrix} a \\ b \end{pmatrix}$ so that it then passes through the points $(0, 1)$ and $(e^3, 1 + \ln 2)$.

Find the value of a and the value of b .

2 - (MATHAI/12_HL_Summer_2021_Q2) - Algebra-functions - Roots

[Maximum mark: 7]

A function is defined by $f(x) = 2 - \frac{12}{x+5}$ for $-7 \leq x \leq 7$, $x \neq -5$.

(a) Find the range of f . [3]

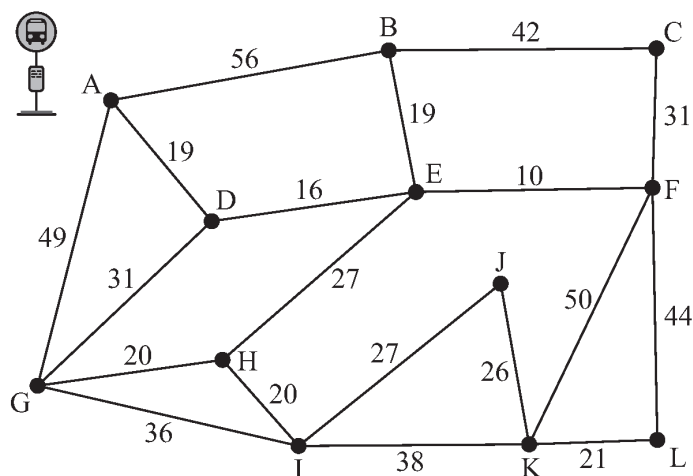
(b) Find an expression for the inverse function $f^{-1}(x)$. The domain is not required. [3]

(c) Write down the range of $f^{-1}(x)$. [1]

3 - (MATHAI/12_HL_Summer_2021_Q11) - Algebra-functions - Roots

[Maximum mark: 7]

The diagram below shows a network of roads in a small village with the weights indicating the distance of each road, in metres, and junctions indicated with letters.



Musab is required to deliver leaflets to every house on each road. He wishes to minimize his total distance.

(a) Musab starts and finishes from the village bus-stop at A. Determine the total distance Musab will need to walk. [5]

Instead of having to catch the bus to the village, Musab's sister offers to drop him off at any junction and pick him up at any other junction of his choice.

(b) Explain which junctions Musab should choose as his starting and finishing points. [2]

4 - (MATHAI/10_HL_Winter_2021_Q3) - Algebra-functions - Roots

[Maximum mark: 7]

Let the function $h(x)$ represent the height in centimetres of a cylindrical tin can with diameter x cm.

$$h(x) = \frac{640}{x^2} + 0.5 \text{ for } 4 \leq x \leq 14.$$

- (a) Find the range of h . [3]

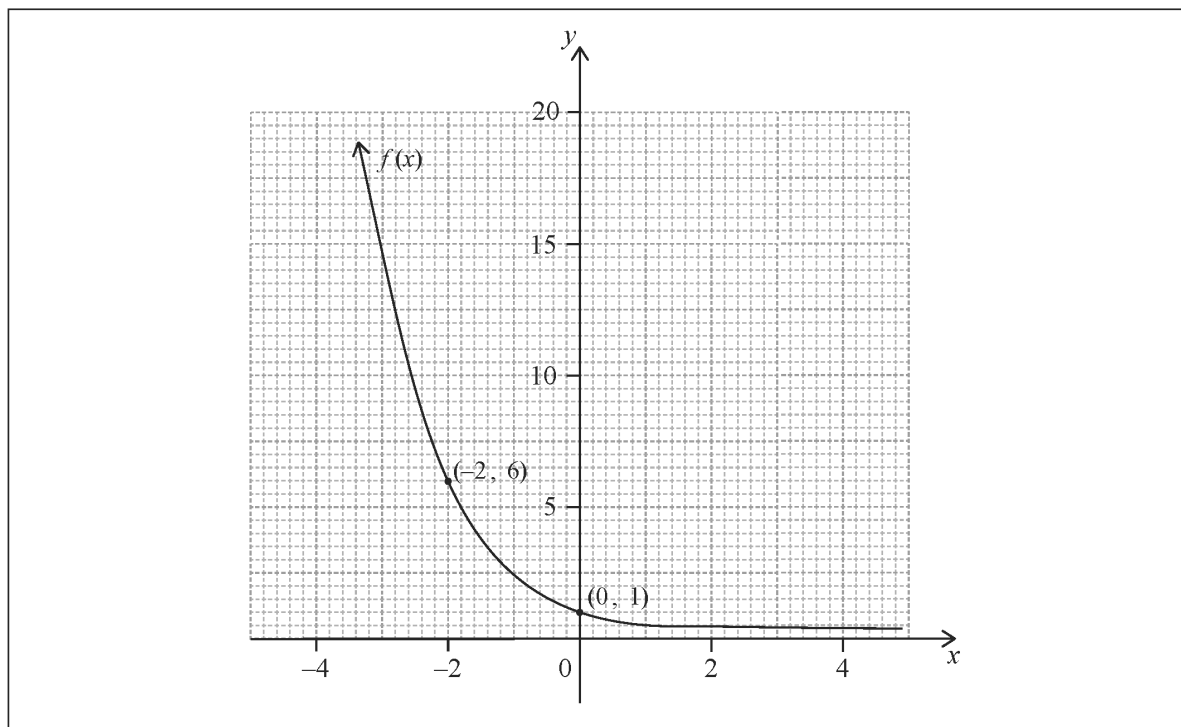
The function h^{-1} is the inverse function of h .

- (b) (i) Find $h^{-1}(10)$.
 (ii) In the context of the question, interpret your answer to part (b)(i).
 (iii) Write down the range of h^{-1} . [4]

5 - (MATHAI/10_HL_Winter_2021_Q10) - Algebra-functions - Roots

[Maximum mark: 4]

The graph of $y = f(x)$ is given on the following set of axes. The graph passes through the points $(-2, 6)$ and $(0, 1)$, and has a horizontal asymptote at $y = 0$.



Let $g(x) = 2f(x - 2) + 4$.

- (a) Find $g(0)$. [2]
 (b) On the same set of axes draw the graph of $y = g(x)$, showing any intercepts and asymptotes. [2]

6 - (MATHAI/10_HL_Winter_2021_Q12) - Algebra-functions - Roots

[Maximum mark: 5]

The following table shows the time, in days, from December 1st and the percentage of Christmas trees in stock at a shop on the beginning of that day.

Days since December 1st (d)	1	3	6	9	12	15	18
Percentage of Christmas trees left in stock (x)	100	51	29	21	18	16	14

The following table shows the natural logarithm of both d and x on these days to 2 decimal places.

$\ln(d)$	0	1.10	1.79	2.20	2.48	2.71	2.89
$\ln(x)$	4.61	3.93	3.37	3.04	2.89	2.77	2.64

- (a) Use the data in the second table to find the value of m and the value of b for the regression line, $\ln x = m(\ln d) + b$. [2]
- (b) Assuming that the model found in part (a) remains valid, estimate the percentage of trees in stock when $d = 25$. [3]

7 - (MATHAI/11_HL_Summer_2022_Q2) - Algebra-functions - Roots

[Maximum mark: 5]

The ticket prices for a concert are shown in the following table.

Ticket Type	Price (in Australian dollars, \$)
Adult	15
Child	10
Student	12

- A total of 600 tickets were sold.
- The total amount of money from ticket sales was \$7816.
- There were twice as many adult tickets sold as child tickets.

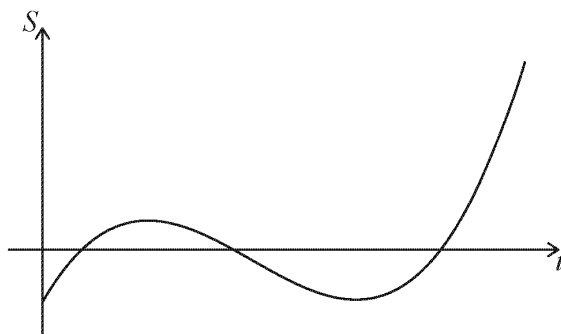
Let the number of adult tickets sold be x , the number of child tickets sold be y , and the number of student tickets sold be z .

- (a) Write down three equations that express the information given above. [3]
- (b) Find the number of each type of ticket sold. [2]

8 - (MATHAI/12_HL_Summer_2022_Q5) - Algebra-functions - Roots

[Maximum mark: 8]

The graph below shows the average savings, S thousand dollars, of a group of university graduates as a function of t , the number of years after graduating from university.



- (a) Write down one feature of this graph which suggests a cubic function might be appropriate to model this scenario.

[1]

The equation of the model can be expressed in the form $S = at^3 + bt^2 + ct + d$, where a , b , c and d are real constants.

The graph of the model must pass through the following four points.

t	0	1	2	3
S	-5	3	-1	-5

- (b) (i) Write down the value of d .
- (ii) Write down three simultaneous equations for a , b and c .
- (iii) Hence, or otherwise, find the values of a , b and c .

[4]

A negative value of S indicates that a graduate is expected to be in debt.

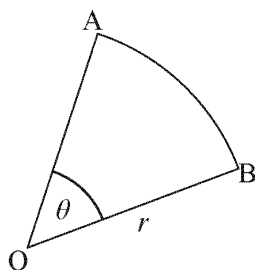
- (c) Use the model to determine the total length of time, in years, for which a graduate is expected to be in debt after graduating from university.

[3]

9 - (MATHAI/12_HL_Summer_2022_Q8) - Radian, Algebra-functions - Roots

[Maximum mark: 8]

The diagram shows a sector, OAB, of a circle with centre O and radius r , such that $\hat{AOB} = \theta$.



Sam measured the value of r to be 2 cm and the value of θ to be 30° .

- (a) Use Sam's measurements to calculate the area of the sector. Give your answer to four significant figures. [2]

It is found that Sam's measurements are accurate to only one significant figure.

- (b) Find the upper bound and lower bound of the area of the sector. [3]
- (c) Find, with justification, the largest possible percentage error if the answer to part (a) is recorded as the area of the sector. [3]

10 - (MATHAI/12_HL_Summer_2022_Q10) - Algebra-functions - Roots, Exponentials - Logarithms

[Maximum mark: 5]

The function $f(x) = \ln\left(\frac{1}{x-2}\right)$ is defined for $x > 2$, $x \in \mathbb{R}$.

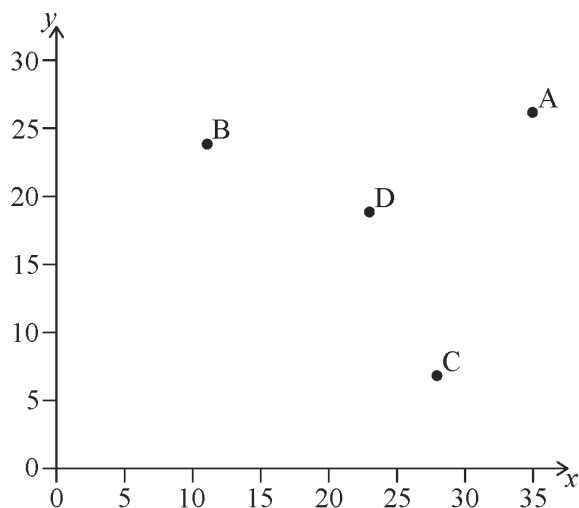
- (a) Find an expression for $f^{-1}(x)$. You are not required to state a domain. [3]
- (b) Solve $f(x) = f^{-1}(x)$. [2]

11 - (MATHAI/11_HL_Summer_2023_Q3) - Algebra-functions - Roots

[Maximum mark: 5]

Three towns have positions $A(35, 26)$, $B(11, 24)$, and $C(28, 7)$ according to the coordinate system shown where distances are measured in miles.

Dominique's farm is located at the position $D(24, 19)$.



- (a) Find AD . [2]

On a particular day, the mean temperatures recorded in each of towns A , B and C are 34°C , 29°C and 30°C respectively.

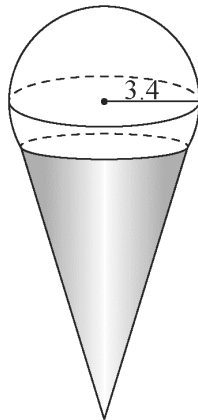
- (b) Use nearest neighbour interpolation to estimate the temperature at Dominique's farm on that particular day. [3]

12 - (MATHAI/12_HL_Summer_2023_Q6) - Algebra-functions - Roots

[Maximum mark: 5]

Ruhi buys a scoop of ice cream in the shape of a sphere with a radius of 3.4 cm. The ice cream is served in a cone, and it may be assumed that $\frac{1}{5}$ of the volume of the ice cream is inside the cone. This is shown in the following diagram.

diagram not to scale



- (a) Calculate the volume of ice cream that is not inside the cone.

[3]

The cone has a slant height of 11 cm and a radius of 3 cm.

The outside of the cone is covered with chocolate.

- (b) Calculate the surface area of the cone that is covered with chocolate. Give your answer correct to the nearest cm^2 .

[2]

13 - (MATHAI/10_HL_Winter_2023_Q1) - Algebra-functions - Roots

[Maximum mark: 7]

The growth of a particular type of seashell is being studied by Manon. At the end of each month Manon records the increase in the width of a seashell since the end of the previous month.

She models the monthly increase in the width of the seashell by a geometric sequence with common ratio 0.8. In the first month, the width of the seashell increases by 4 mm.

- (a) Find by how much the width of the seashell will increase during the third month, according to her model.

[2]

- (b) Find the total increase in the width of the seashell, predicted by Manon's model, during the first year.

[2]

Manon's seashell had a width of 25 mm at the beginning of the first month.

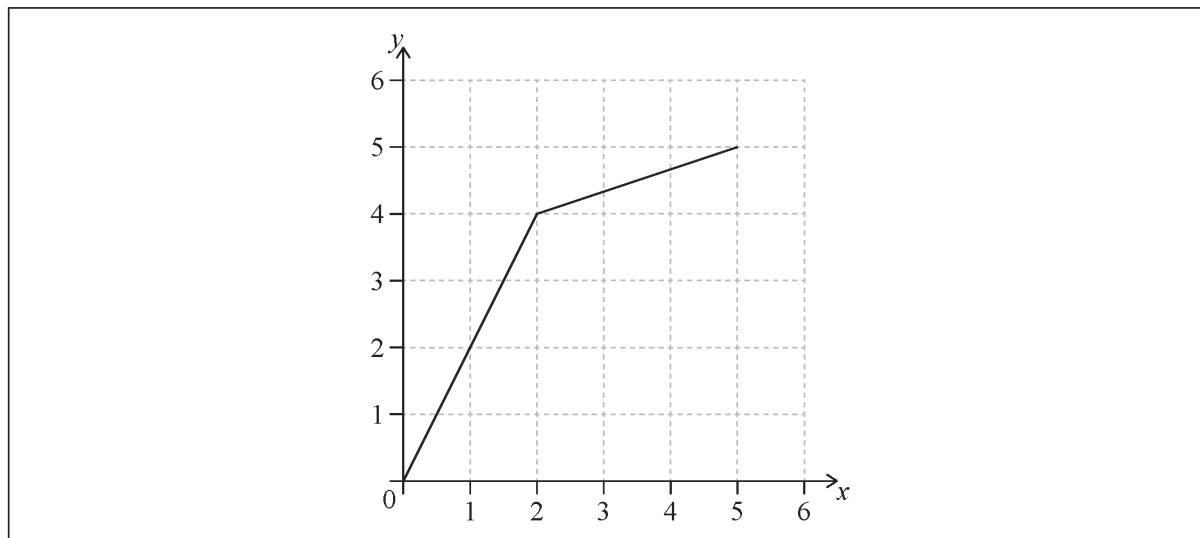
- (c) Find the maximum possible width of the seashell, predicted by Manon's model.

[3]

14 - (MATHAI/10_HL_Winter_2023_Q2) - Algebra-functions - Roots

[Maximum mark: 7]

The graph of the function f is given in the following diagram.



(a) Write down $f(2)$. [1]

(b) On the axes, sketch $y = f^{-1}(x)$. [2]

The function g is defined as $g(x) = 3x - 1$.

(c) Find an expression for $g^{-1}(x)$. [2]

(d) Find a value of x where $f^{-1}(x) = g^{-1}(x)$. [2]

15 - (MATHAI/10_HL_Winter_2023_Q3) - Algebra-functions - Roots

[Maximum mark: 6]

The Great Pyramid of Giza is the oldest of the Seven Wonders of the Ancient World. When it was built, 4500 years ago, the measurements of the pyramid were in Royal Egyptian Cubits (REC).



[Source: Nina Aldin Thune. https://en.wikipedia.org/wiki/Great_Pyramid_of_Giza#/media/File:Kheops-Pyramid.jpg. Licensed under CC BY 2.5 <https://creativecommons.org/licenses/by/2.5/#>. Image adapted.]

Viktor reads online that 1 REC is equal to 0.52 metres, rounded to two decimal places.

- (a) Write down the upper and lower bounds of 1 REC in metres. [2]

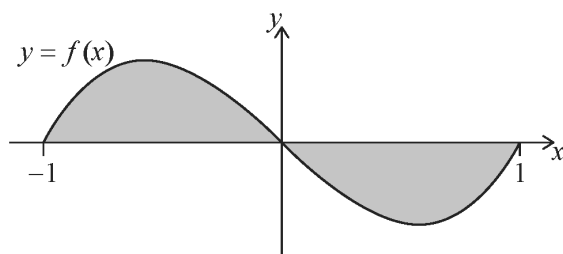
The Great Pyramid of Giza has a square base with side lengths of 440 REC and a height of 280 REC. Viktor assumes that these two measurements are exact and that the Great Pyramid can be modelled as a square-based pyramid with smooth faces.

- (b) Find the minimum possible volume of the pyramid in cubic metres. [4]

16 - (MATHAI/10_HL_Winter_2023_Q11) - Algebra-functions - Roots, Integration

[Maximum mark: 7]

Consider the function $f(x) = x^3 - x$, for $-1 \leq x \leq 1$. The shaded region, R , is bounded by the graph of $y = f(x)$ and the x -axis.



(a) (i) Write down an integral that represents the area of R .

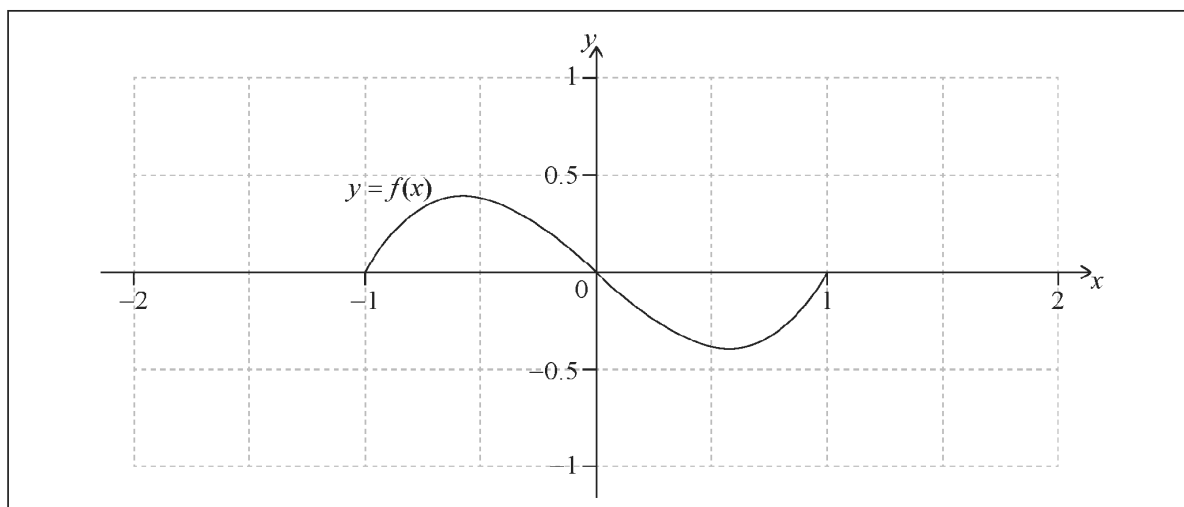
(ii) Find the area of R .

[2]

Another function, g , is defined such that $g(x) = 2f(x - 1)$.

(b) On the following set of axes, the graph of $y = f(x)$ has been drawn. On the same set of axes, sketch the graph of $y = g(x)$.

[2]



The region R from the original graph $y = f(x)$ is rotated through 2π radians about the x -axis to form a solid.

(c) Find the volume of the solid.

[3]

17 - (MATHAI/11_HL_Summer_2024_Q1) - Algebra-functions - Roots

[Maximum mark: 6]

Maan deposited \$100 000 into a savings account with a nominal annual interest rate of $I\%$ **compounded monthly**. At the end of the eighth year, the amount in the account had increased to \$150 000.

- (a) Find the value of I . [3]

Maan withdraws the \$150 000 and places it in an annuity, earning a nominal annual interest rate of 6.1% **compounded monthly**. At the end of each month, Maan will receive a payment of \$1000.

- (b) Find the amount of money remaining in the annuity at the end of 10 years. Express your answer to the nearest dollar. [3]

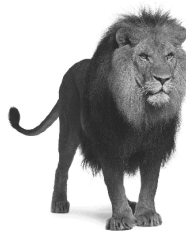
18 - (MATHAI/11_HL_Summer_2024_Q7) - Algebra-functions - Roots

[Maximum mark: 8]

When studying big cats, researchers use a model in which the mass (m kilograms) of an animal is directly proportional to the cube of its shoulder height (h metres).



Cheetah



Lion

A cheetah has a mass of 64 kg and shoulder height of 0.8 metres.

- (a) (i) Use the model to find an expression for m in terms of h .
(ii) Hence find the mass of a different cheetah, with a shoulder height of 0.75 metres. [4]

'Rubner's law' states that the energy needs of an animal (E) are directly proportional to the square of h .

The energy needs of a lion of mass 220 kg are k times the energy needs of a cheetah of mass 64 kg.

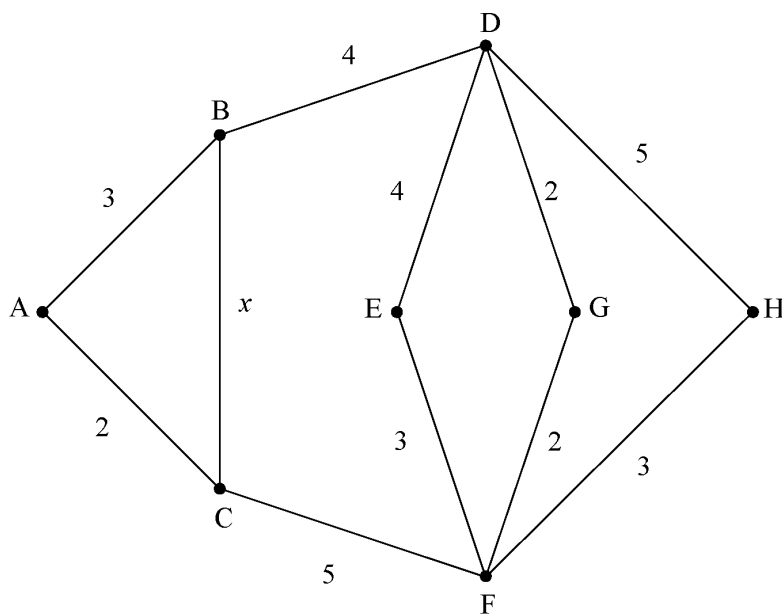
- (b) Find the value of k . [4]

19 - (MATHAI/11_HL_Summer_2024_Q8) - Algebra-functions - Roots

[Maximum mark: 5]

The weights on the following graph represent the lengths of different roads in kilometres.

diagram not to scale



- (a) Write down the vertices with odd degree. [1]

The total length of the roads is $33 + x$ km.

- (b) Find **two** expressions, in terms of x , for the shortest distance required to walk along all of the paths, beginning and ending at the same vertex. Include in your answer the interval of values of x for which each expression is valid. [4]

20 - (MATHAI/11_HL_Summer_2024_Q12) - Algebra-functions - Roots

[Maximum mark: 8]

Roma is told by the manufacturers of their phone that, when charging from 0%, the percentage P of charge can be modelled by $P = 100 - 100 \times 2^{-t}$, where t is the number of hours since the phone started charging.

- (a) Find the time taken for the amount of charge to go from 0% to 40%, according to this model. [2]

Roma decides to test the model. They charge the phone from 0% and record the charge of the phone for different values of t . The results are shown in the following table.

t hours	1	2	3	4
P percent	48	74	86	91

- (b) Find the sum of the square residuals (SS_{res}) when using $P = 100 - 100 \times 2^{-t}$ to fit this data. [4]

Roma now uses a quadratic function to model the charge in the phone. They calculate that, when using this model, the sum of the square residuals for these four points is 2.45.

- (c) State one reason why Roma might prefer to use
- the quadratic model.
 - $P = 100 - 100 \times 2^{-t}$. [2]

21 - (MATHAI/12_HL_Summer_2024_Q1) - Algebra-functions - Roots

[Maximum mark: 9]

Imani invests \$3000 in a bank that pays a nominal annual interest rate of 1.25% compounded monthly.

- (a) Calculate the amount of money Imani will have in the bank at the end of 6 years. Give your answer correct to two decimal places. [3]
- (b) Calculate the number of months it takes until Imani has at least \$3550 in the bank. [2]

Imani uses the \$3550 as a partial payment for a used car costing \$22 000. For the remainder she takes out a loan from a bank.

- (c) Write down the amount of money that Imani takes out as a loan. [1]

The loan is for 8 years and the nominal annual interest rate is 12.6% compounded monthly. Imani will pay the loan in fixed monthly instalments at the end of each month.

- (d) Calculate the amount, correct to the nearest dollar, that Imani will have to pay the bank each month. [3]

22 - (MATHAI/12_HL_Summer_2024_Q5) - Algebra-functions - Roots

[Maximum mark: 8]

The annual growth of a tree is 80% of its growth during the previous year.

This year the tree is 42 m in height and one year ago its height was 37 m.

(a) Calculate the annual growth of the tree in the coming year. [2]

(b) Calculate the height of the tree 6 years from now. Give your answer correct to the nearest cm. [4]

If the tree continues to follow this pattern of growth, its height will never exceed k metres.

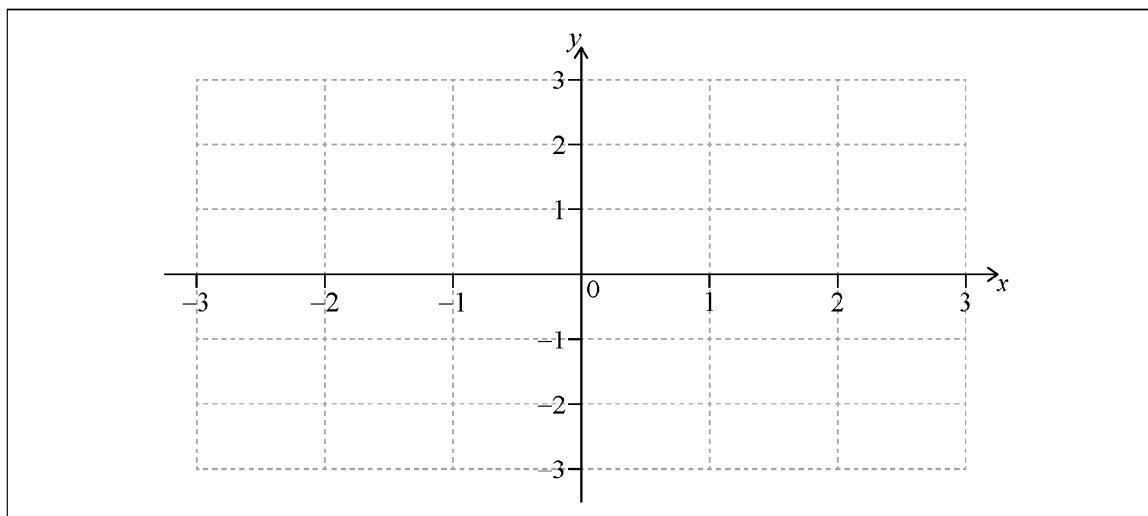
(c) Find the smallest possible value of k . [2]

23 - (MATHAI/12_HL_Summer_2024_Q10) - Algebra-functions - Roots, Graphs

[Maximum mark: 9]

Consider the function $f(x) = x\sqrt{3-x^2}$, $-\sqrt{3} \leq x \leq \sqrt{3}$.

(a) Sketch the graph of $y = f(x)$ on the following pair of axes. [2]



The area between the graph of $y = f(x)$ and the x -axis is rotated through 360° about the x -axis.

(b) (i) Write down an integral that represents this volume.

(ii) Calculate the value of this integral. [4]

The graph of the function f is transformed, to give the graph of the function g , in the following way:

- It is first stretched by scale factor 2, parallel to the x -axis with the y -axis invariant.
- It is then stretched by scale factor 0.5, parallel to the y -axis with the x -axis invariant.

(c) Find the volume obtained when the area between the graph of $y = g(x)$ and the x -axis is rotated through 360° about the x -axis. [3]

ANSWERS

1 - (MATHAI/11_HL_Summer_2021_Q17) - Algebra-functions - Roots

new function is $f(x-a)+b (= \ln(x-a)+b)$

(M1)

$$f(0) = \ln(-a) + b = 1$$

A1

$$f(e^3) = \ln(e^3 - a) + b = 1 + \ln 2$$

A1

$$\ln(-a) = \ln(e^3 - a) - \ln 2$$

(M1)

$$\ln(-a) = \ln\left(\frac{e^3 - a}{2}\right)$$

$$-a = \frac{e^3 - a}{2}$$

$$-2a = e^3 - a$$

$$a = -e^3 \quad (= -20.0855...)$$

A1

$$b = 1 - \ln e^3 = 1 - 3 = -2$$

(M1)A1

Total [7 marks]

2 - (MATHAI/12_HL_Summer_2021_Q2) - Algebra-functions - Roots

(a) $(f(-7) =) 8$ and $(f(7) =) 1$

(A1)

range is $f(x) \leq 1, f(x) \geq 8$

A1A1

Note: Award at most **A1A1A0** if strict inequalities are used.

[3 marks]

(b) interchanging x, y at any stage

(A1)

$$y = 2 - \frac{12}{x+5}$$

$$\frac{12}{x+5} = 2 - y$$

$$\frac{12}{2-y} = x+5$$

(A1)

$$\frac{12}{2-y} - 5 = x$$

$$(f^{-1}(x) =) \frac{12}{2-x} - 5 \quad \left(= \frac{2+5x}{2-x} \right)$$

A1

[3 marks]

(c) range is $-7 \leq f^{-1}(x) \leq 7, f^{-1}(x) \neq -5$

A1

[1 mark]

3 - (MATHAI/12_HL_Summer_2021_Q11) - Algebra-functions - Roots

- (a) Odd vertices are A, B, D, H
Consider pairings:

A1

M1

Note: Award (**M1**) if there are four vertices not necessarily all correct.

AB DH has shortest route AD, DE, EB and DE, EH,
so repeated edges $(19 + 16 + 19) + (16 + 27) = 97$

Note: Condone AB in place of AD, DE, EB giving $56 + (16 + 27) = 99$.

AD BH has shortest route AD and BE, EH,
so repeated edges $19 + (19 + 27) = 65$

AH BD has shortest route AD, DE, EH and BE, ED,
so repeated edges $(19 + 16 + 27) + (19 + 16) = 97$

A2

Note: Award **A1** if only one or two pairings are correctly considered.

so best pairing is AD, BH
weight of route is therefore $582 + 65 = 647$

A1

[5 marks]

- (b) least value of the pairings is 19 therefore repeat AD

R1

B and H

A1

Note: Do not award **R0A1**.

[2 marks]

4 - (MATHAI/10_HL_Winter_2021_Q3) - Algebra-functions - Roots

$$(a) \quad h(4) = \frac{640}{4^2} + 0.5 \quad \text{OR} \quad h(14) = \frac{640}{14^2} + 0.5 \quad (M1)$$

Note: Award **(M1)** for substituting 4 or 14 into h . This can be implicit from seeing 3.77 (3.76530...) or 40.5.

$$3.77 \leq h(x) \leq 40.5 \quad (3.76530... \leq h(x) \leq 40.5) \quad A1A1$$

Note: Award **A1** for both correct endpoints seen, **A1** for the endpoints in a correct interval.

[3 marks]

$$(b) \quad (i) \quad h(x) = 10 \quad \text{OR} \quad h^{-1}(x) = \sqrt{\frac{640}{x-0.5}} \quad \text{OR} \quad h^{-1}(10) = \sqrt{\frac{640}{10-0.5}} \quad (M1)$$

$$(x =) 8.21 \text{ cm } (8.20782...) \quad A1$$

(ii) a tin that is 10 cm high will have a diameter of 8.21 cm (8.20782...) **A1**

Note: Condone a correct answer expressed as the converse.

$$(iii) \quad 4 \leq h^{-1} \leq 14 \quad A1$$

Note: Accept $4 \leq y \leq 14$. Do not **FT** in this part.

[4 marks]

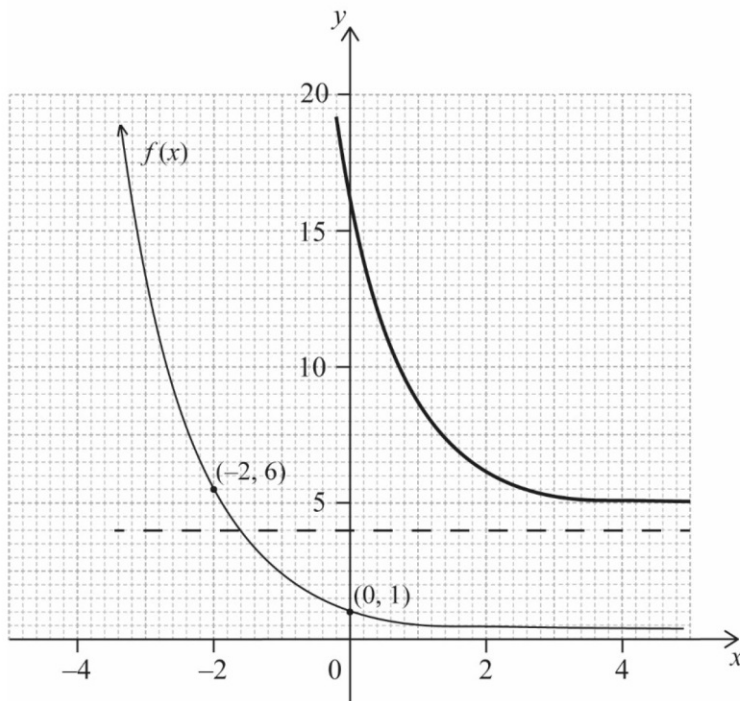
Total: [7 marks]

5 - (MATHAI/10_HL_Winter_2021_Q10) - Algebra-functions - Roots

(a) $g(0) = 16$

M1A1**[2 marks]**

(b)

 y -asymptote ($y = 4$)concave up decreasing curve and passing through $(0, 16)$ **A1****A1****[2 marks]****Total: [4 marks]**

6 - (MATHAI/10_HL_Winter_2021_Q12) - Algebra-functions - Roots

(a) $m = -0.695$ ($-0.695383\dots$); $b = 4.63$ ($4.62974\dots$)

A1A1**[2 marks]**

(b) $\ln x = -0.695(\ln 25) + 4.63$

$\ln x = 2.39288\dots$

$x = 10.9\%$

M1**(A1)****A1****[3 marks]****Total: [5 marks]**

7 - (MATHAI/11_HL_Summer_2022_Q2) - Algebra-functions - Roots

- (a) $x + y + z = 600$ **A1**
 $15x + 10y + 12z = 7816$ **A1**
 $x = 2y$ **A1**

Note: Condone other labelling if clear, e.g. a (adult), c (child) and s (student).
 Accept equivalent, distinct equations e.g. $2y + y + z = 600$.

[3 marks]

- (b) $x = 308, y = 154, z = 138$ **A1A1**

Note: Award **A1** for all three correct values seen, **A1** for correctly labelled as x, y or z .
 Accept answers written in words: e.g. 308 adult tickets.

[2 marks]

8 - (MATHAI/12_HL_Summer_2022_Q5) - Algebra-functions - Roots

- (a) *Accept any one of the following (or equivalent):*
 one minimum and one maximum point
 three x -intercepts or three roots (or zeroes)
 one point of inflexion

R1

Note: Do not accept "S shape" as a justification.

[1 mark]

- (b) (i) $(d) = -5$ **A1**
- (ii) $8 = a + b + c$
 $4 = 8a + 4b + 2c$
 $0 = 27a + 9b + 3c$ **A2**

Note: Award **A2** if all three equations are correct.
 Award **A1** if at least one is correct. Award **A1** for three correct equations that include the letter " d ".

- (iii) $a = 2, b = -12, c = 18$ **A1**

[4 marks]

- (c) equating found expression to zero **(M1)**
 $0 = 2t^3 - 12t^2 + 18t - 5$
 $t = 0.358216..., 1.83174..., 3.81003...$ **(A1)**

(so total time in debt is $3.81003... - 1.83174... + 0.358216 \approx$)
 2.34 (2.33650...) years

A1**[3 marks]**

9 - (MATHAI/12_HL_Summer_2022_Q8) - Radian, Algebra-functions - Roots

$$(a) \quad \pi \times 2^2 \times \frac{30}{360} \quad (M1)$$

$$= 1.047 \text{ cm}^2 \quad A1$$

Note: Do not award the final mark if the answer is not correct to 4 sf.

[2 marks]

(b) attempt to substitute any two values from 1.5, 2.5, 25 or 35 into area of sector formula (M1)

$$\left(\text{upper bound} = \pi \times 2.5^2 \times \frac{35}{360} = \right) 1.91 \text{ cm}^2 \quad (1.90895...) \quad A1$$

$$\left(\text{lower bound} = \pi \times 1.5^2 \times \frac{25}{360} = \right) 0.491 \text{ cm}^2 \quad (0.490873...) \quad A1$$

Note: Given the nature of the question, accept correctly rounded **OR** correctly truncated 3 significant figure answers.

[3 marks]

$$(c) \quad \left(\frac{1.047 - 1.90895...}{1.90895...} \times 100 = \right) 45.2 \text{ (\%)} \quad (45.1532...) \quad A1$$

$$\left(\frac{1.047 - 0.490873...}{0.490873...} \times 100 = \right) 113 \text{ (\%)} \quad (113.293...) \quad A1$$

so the largest percentage error is 113 % A1

Note: Accept 45.1 (%) (45.1428), from use of full accuracy answers. Given the nature of the question, accept correctly rounded **OR** correctly truncated 3 significant figure answers. Award **A0A1A0** if 113% is the only value found.

[3 marks]

10 - (MATHAI/12_HL_Summer_2022_Q10) - Algebra-functions - Roots, Exponentials - Logarithms

$$(a) \quad y = \ln\left(\frac{1}{x-2}\right)$$

an attempt to isolate x (or y if switched) (M1)

$$e^y = \frac{1}{x-2}$$

$$x-2 = e^{-y}$$

$$x = e^{-y} + 2$$

switching x and y (seen anywhere) M1

$$f^{-1}(x) = e^{-x} + 2 \quad A1$$

[3 marks]

(b) sketch of $f(x)$ and $f^{-1}(x)$ (M1)

$$x = 2.12 \quad (2.12002...) \quad A1$$

[2 marks]

11 - (MATHAI/11_HL_Summer_2023_Q3) - Algebra-functions - Roots

- (a) attempt to use distance formula for points D and A (M1)

$$DA = \sqrt{11^2 + 7^2}$$

$$= 13.0 \text{ (miles)} \text{ (13.0384..., } \sqrt{170} \text{)} \quad \text{A1}$$

Note: Accept 13 miles. Award **MOAO** for finding the equation of the line DA.

DA may be seen in part (b) but this should not be accepted as answer for part (a).

[2 marks]

- (b) $(DB = \sqrt{13^2 + 5^2} =) \text{ 13.9 (13.9283..., } \sqrt{194} \text{)}$ AND

$$(DC = \sqrt{4^2 + 12^2} =) \text{ 12.6 (12.6491..., } \sqrt{160} \text{)} \quad \text{A1}$$

recognizing closest town is best estimate (M1)

(town C is closest)

$$30^\circ \text{C} \quad \text{A1}$$

Note: If their DA from part (a) is the shortest length, then allow **FT** in (b).

[3 marks]

12 - (MATHAI/12_HL_Summer_2023_Q6) - Algebra-functions - Roots

(a) EITHER

$$\frac{4}{3}\pi(3.4)^3$$

(A1)

Multiplying their volume by $\frac{4}{5}$

(M1)

OR

$$\frac{4}{3}\pi(3.4)^3$$

(A1)

Subtracting $\frac{1}{5}$ of their volume

(M1)

$$\left(\frac{4}{3}\pi(3.4)^3 - \frac{1}{5} \times \frac{4}{3}\pi(3.4)^3\right)$$

Note: The **M1** can be awarded for a final answer of 32.9272... seen without working.

THEN

$$132 \text{ cm}^3 \text{ (131.708... cm}^3\text{)}$$

A1

[3 marks]

(b) $\pi \times 3 \times 11$

(A1)

$$103.672... \text{ (cm}^2\text{)} \text{ OR } 33\pi \text{ (cm}^2\text{)}$$

$$104 \text{ (cm}^2\text{)}$$

A1

[2 marks]

13 - (MATHAI/10_HL_Winter_2023_Q1) - Algebra-functions - Roots

(a) attempt to use the term formula for a geometric sequence

(M1)

$$4 \times (0.8)^2$$

$$= 2.56 \text{ (mm)} \left(\frac{64}{25}\right)$$

A1

[2 marks]

(b) attempt to use the sum formula for a geometric sequence

(M1)

$$\frac{4 \times (1 - 0.8^{12})}{1 - 0.8}$$

$$= 18.6 \text{ (mm)} \text{ (18.6256...)}$$

A1

[2 marks]

(c) attempt to use infinite geometric sum

(M1)

$$\text{e.g. } \frac{4}{1 - 0.8} \text{ OR } 20$$

adding 25 to their expression or value

(M1)

(maximum width = $25 + 20$)

$$45 \text{ (mm)}$$

A1

[3 marks]

[Total 7 marks]

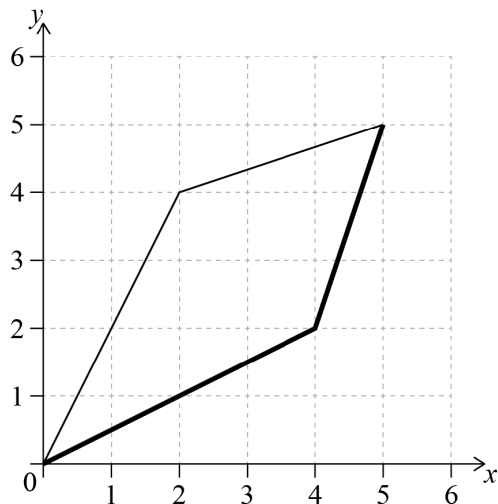
14 - (MATHAI/10_HL_Winter_2023_Q2) - Algebra-functions - Roots

(a) 4

A1

[1 mark]

(b)



A1A1

Note: Award **A1** for passing through (0, 0) and (4, 2),
A1 for passing through (4, 2) and (5, 5).

[2 marks]

(c) attempt to solve $y = 3x - 1$ for x OR changing variables

(M1)

$$(g^{-1}(x)) = \frac{x+1}{3}$$

A1

[2 marks]

(d) sketch of $g(x)$ or $g^{-1}(x)$, algebraic approach

(M1)

$$\frac{1}{2}x = \frac{x+1}{3}$$

$$(x =) 2$$

A1

[2 marks]

[Total 7 marks]

15 - (MATHAI/10_HL_Winter_2023_Q3) - Algebra-functions - Roots

- (a) (upper bound =) 0.525 (m)
(lower bound =) 0.515 (m)

A1
A1

Note: Accept an answer in interval notation or written as an inequality.

[2 marks]

- (b) **METHOD 1 Convert REC to linear metres**

attempt to convert REC to metres using their lower bound
 440×0.515 (= 226.6) **OR** 280×0.515 (= 144.2) seen

(M1)

attempt to use the formula for the volume of a right pyramid

(M1)

$$(V =) \frac{1}{3}(440 \times 0.515)^2(280 \times 0.515)$$

(A1)

$$2470000 \text{ (m}^3\text{)} \text{ (2468106.051..., } 2.47 \times 10^6\text{)}$$

A1

METHOD 2 Convert REC to cubic metres

attempt to use the formula for the volume of a right pyramid

(M1)

$$(V =) \frac{1}{3}(440)^2(280) \text{ (=18069333.33...)}$$

attempt to convert 1 cubic REC to cubic metres using their lower bound
 (1 cubic REC =) 0.515^3

(M1)

$$(V =) \frac{1}{3}(440)^2(280) \times (0.515)^3$$

(A1)

$$2470000 \text{ (m}^3\text{)} \text{ (2468106.051..., } 2.47 \times 10^6\text{)}$$

A1

[4 marks]

16 - (MATHAI/10_HL_Winter_2023_Q11) - Algebra-functions - Roots, Integration

(a) (i) EITHER

$$(\text{area of } R \Rightarrow) \int_{-1}^1 |x^3 - x| dx$$

A1

OR

$$(\text{area of } R \Rightarrow) 2 \times \int_{-1}^0 x^3 - x dx \quad \text{OR} \quad (\text{area of } R \Rightarrow) -2 \times \int_0^1 x^3 - x dx$$

A1

OR

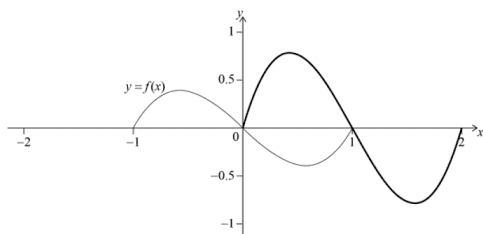
$$(\text{area of } R \Rightarrow) \int_{-1}^0 x^3 - x dx - \int_0^1 x^3 - x dx$$

A1(ii) (area of $R \Rightarrow$) 0.5**A1**

Note: Follow through from part (a)(i) only if answer is greater than zero.

[2 marks]

(b)

**A1A1**

Note: Award **A1** for sketch with correct shape on $[0, 2]$, **A1** for vertical stretch x2. Condone max/min of g extending to 1 / -1.

[2 marks](c) attempt to use $\pi \int y^2 dx$ **(M1)**

$$\text{volume} = \pi \int_{-1}^1 (x^3 - x)^2 dx$$

(A1)

$$\text{volume} = 0.479 \text{ (cubic units)} \left(= 0.478718\dots, \frac{16\pi}{105} \right)$$

A1**[3 marks]**

17 - (MATHAI/11_HL_Summer_2024_Q1) - Algebra-functions - Roots

(a) EITHER

$$N = 96$$

$$PV = \mp 100000$$

$$FV = \pm 150000$$

$$P/Y = 12$$

$$C/Y = 12$$

(M1)(A1)

OR

$$N = 8$$

$$PV = \mp 100000$$

$$FV = \pm 150000$$

$$P/Y = 1$$

$$C/Y = 12$$

(M1)(A1)

OR

$$150000 = 100000 \left(1 + \frac{I}{100 \times 12} \right)^{12 \times 8}$$

(M1)(A1)

Note: Award **M1** for an attempt to use a financial app (at least 3 entries, not necessarily correct) or an attempt to use a compound interest formula.
Award **A1** for all entries correct in financial app or correct substitution in compound interest formula.

THEN

$$I = 5.08 \text{ (5.07903...)}$$

A1**[3 marks]**(b) $N = 120$

$$I\% = 6.1$$

$$PV = \mp 150000$$

$$PMT = \pm 1000$$

$$P/Y = 12$$

$$C/Y = 12$$

(M1)(A1)

Note: Award **M1** for an attempt to use a financial app (at least 3 entries, not necessarily correct). Award **A1** for all entries correct in financial app (condone missing +/- sign if the correct final answer is seen).

$$FV = (\$) 110867$$

A1

Note: Answer must be correct to nearest dollar to award the final **A1**. Award **(M1)(A1)A0** for an unsupported final answer to a greater degree of accuracy eg. (\$) 110866.70...
Award **M1A1A0** for a truncated answer of 110866 if no working is shown.

[3 marks]

18 - (MATHAI/11_HL_Summer_2024_Q7) - Algebra-functions - Roots

(a) (i) $m = ah^3$ (M1)
 $64 = a \times 0.8^3 \Rightarrow a = 125$ (A1)

$$m = 125h^3 \quad \text{A1}$$

Note: The final answer must be written as m in terms of h to award the final A1.

(ii) $m = (125(0.75)^3) = 52.7 \text{ (kg)} \quad (52.7343\dots)$ A1
[4 marks]

(b) **EITHER (finding the height of the lion)**

$(220 = 125h^3)$
 $h = \sqrt[3]{1.76} \quad (1.20736\dots)$ (A1)

$E = k_1 h^2$ (seen anywhere) (A1)

$k = \frac{(\sqrt[3]{1.76})^2}{0.8^2}$ (M1)

$= 2.28 \quad (2.27769\dots)$ A1

OR (finding a formula for E in terms of m)

$(m = 125h^3)$
 $E = k_1 h^2$ (A1)

$E = k_2 m^{\frac{2}{3}}$ (A1)

$k = \frac{220^{\frac{2}{3}}}{64^{\frac{2}{3}}}$ (M1)

$= 2.28 \quad (2.27769\dots)$ A1

[4 marks]

19 - (MATHAI/11_HL_Summer_2024_Q8) - Algebra-functions - Roots

(a) B and C A1
[1 mark]

(b) correct intervals seen ($x \leq 5$ (or $x < 5$) AND $x \geq 5$ (or $x > 5$)) A1

Note: The case of $x = 5$ must be included for this A1 to be awarded.

attempt to add edges to $33 + x$ (M1)

(If $x < 5$ (or $x \leq 5$) then repeat BC and) length is $33 + 2x$ A1

(If $x > 5$ (or $x \geq 5$) then repeat AB and AC and) length is $(33 + x + 5)38 + x$ A1

Note: If the intervals are not explicit, award at most A0(M1)A1A1.

[4 marks]

20 - (MATHAI/11_HL_Summer_2024_Q12) - Algebra-functions - Roots

(a) $40 = 100 - 100 \times 2^{-t}$
 0.737 (hours) (0.736965...)

(A1)

A1

Note: Accept 44.2 minutes.

[2 marks]

(b) values of P are 50, 75, 87.5, 93.75

(A1)

$$SS_{res} = (50 - 48)^2 + (75 - 74)^2 + (87.5 - 86)^2 + (93.75 - 91)^2$$

(M1)(A1)

$$= 14.8 \text{ (14.8125)}$$

A1

[4 marks]

(c) (i) The sum of the square residuals is smaller so it is a better fit

R1

(ii) Accept a valid argument in favour of model P or against the quadratic model.

R1

e.g. any one of the following:

quadratic has no asymptote

quadratic will begin to go down

quadratic will become negative

quadratic might not go through (0, 0)

model P is the manufacturer's model

[2 marks]

21 - (MATHAI/12_HL_Summer_2024_Q1) - Algebra-functions - Roots

(a) EITHER

$N = 72$

$I = 1.25$

$PV = -3000$

OR

$N = 6$

$I = 1.25$

$PV = -3000$

$P/Y = 12$

$C/Y = 12$

$P/Y = 1$

$C/Y = 12$

(M1)(A1)

Note: Award **M1** for attempt to use finance app on GDC, **A1** for all entries correct.

OR

$$3000 \left(1 + \frac{1.25}{1200} \right)^{72}$$

(M1)(A1)

Note: Award **M1** for using CI formula, **A1** for correct entries.

THEN

$FV = (\$) 3233.53$

A1

Note: Answer must be correct to 2 dp for the final **A1** to be awarded.

[3 marks]

(b) EITHER

$I = 1.25$

$PV = -3000$

$FV = 3550$

OR

$I = 1.25$

$PV = -3000$

$FV = 3550$

$P/Y = 12$

$C/Y = 12$

$P/Y = 1$

$C/Y = 12$

(A1)

Note: Award **A1** for all entries correct and opposite signs for PV and FV values.

OR

$$3550 = 3000 \left(1 + \frac{1.25}{1200} \right)^{12N}$$

(A1)

Note: Award **A1** for all entries correct.

THEN

$N = 162 \text{ (161.686...)} \text{ (months)}$

A1

[2 marks]

(c) 18450

A1

[1 mark]

(d) **EITHER**

$N = 96$

$I = 12.6$

$PV = -18450$

$FV = 0$

$P/Y = 12$

$C/Y = 12$

$N = 96$

$I = 1.05$

$PV = -18450$

$FV = 0$

$P/Y = 1$

$C/Y = 1$

OR**(M1)(A1)**

Note: Award **M1** for attempt to use finance app on GDC, **A1** for $N=96$, $PV=-18450$ and $FV=0$,

THEN

$PMT = (\$) 306$

A1

Note: The answer must be correct to the nearest dollar for the final **A1** to be awarded.

[3 marks]

22 - (MATHAI/12_HL_Summer_2024_Q5) - Algebra-functions - Roots

(a) recognizing that the growth in one year is the difference in the two heights

$5 \times 0.8 = 4 \text{ (m)}$

(M1)**A1****[2 marks]**(b) recognizing geometric sequence, $r = 0.8$

attempt to find total height by adding initial height to a term in series

(M1)**(M1)****EITHER**

$$42 + \frac{4(1 - (0.8)^6)}{1 - (0.8)}$$

(A1)**OR**

$$37 + \frac{5(1 - (0.8)^7)}{1 - (0.8)}$$

(A1)**THEN**

$= 56.7571 \dots$

$= 56.76 \text{ (m)} \text{ OR } 5676 \text{ (cm)}$

A1

Note: Accept an answer in cm or in m to two decimal places.

[4 marks]

(c) attempt to use infinite geometric series to find the total growth of the tree

(M1)

e.g. $\frac{5}{1-0.8}$ OR $\frac{4}{1-0.8}$

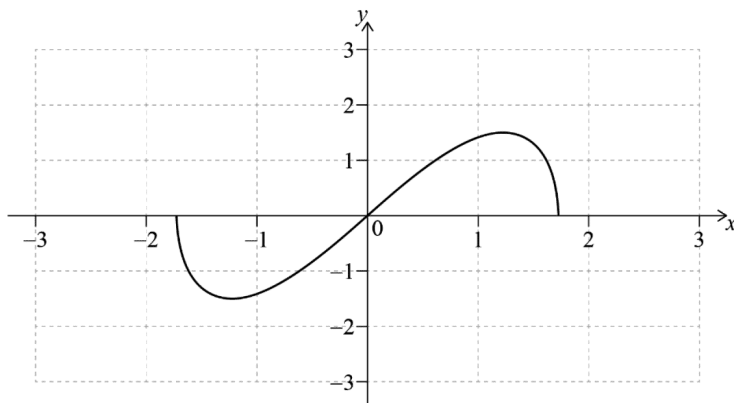
$$\left(S_{\infty} = 37 + \frac{5}{1-0.8}, S_{\infty} = 42 + \frac{4}{1-0.8} \right)$$

$k = 62$

A1**[2 marks]**

23 - (MATHAI/12_HL_Summer_2024_Q10) - Algebra-functions - Roots, Graphs

(a)

**A1A1**

Note: Award **A1** smooth curve, through origin, with maximum in 1st quadrant and minimum in 3rd quadrant (approximately correct position).
Award **A1** correct (approximate) position of endpoints and rotational symmetry.
Condone a (small) gap between end of curve and the x -axis.

[2 marks]

(b) (i) $2\pi \int_0^{\sqrt{3}} (x\sqrt{3-x^2})^2 dx$ OR $\pi \int_{-\sqrt{3}}^{\sqrt{3}} (x\sqrt{3-x^2})^2 dx$ (or equivalent) **A1A1**

Note: Award **A1** for integral with correct limits in the correct places, **A1** for completely correct expression;
 dx must be included. Condone a correct answer presented as two integrals,
despite the demand of the question: $\pi \left(\int_{-\sqrt{3}}^0 (x\sqrt{3-x^2})^2 dx + \int_0^{\sqrt{3}} (x\sqrt{3-x^2})^2 dx \right)$.

(ii) $13.1 \left(13.0593..., \frac{12\pi\sqrt{3}}{5} \right)$

A2**[4 marks]**

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