

IB Diploma

MATHEMATICS AA

SL P2
2017 — 2025

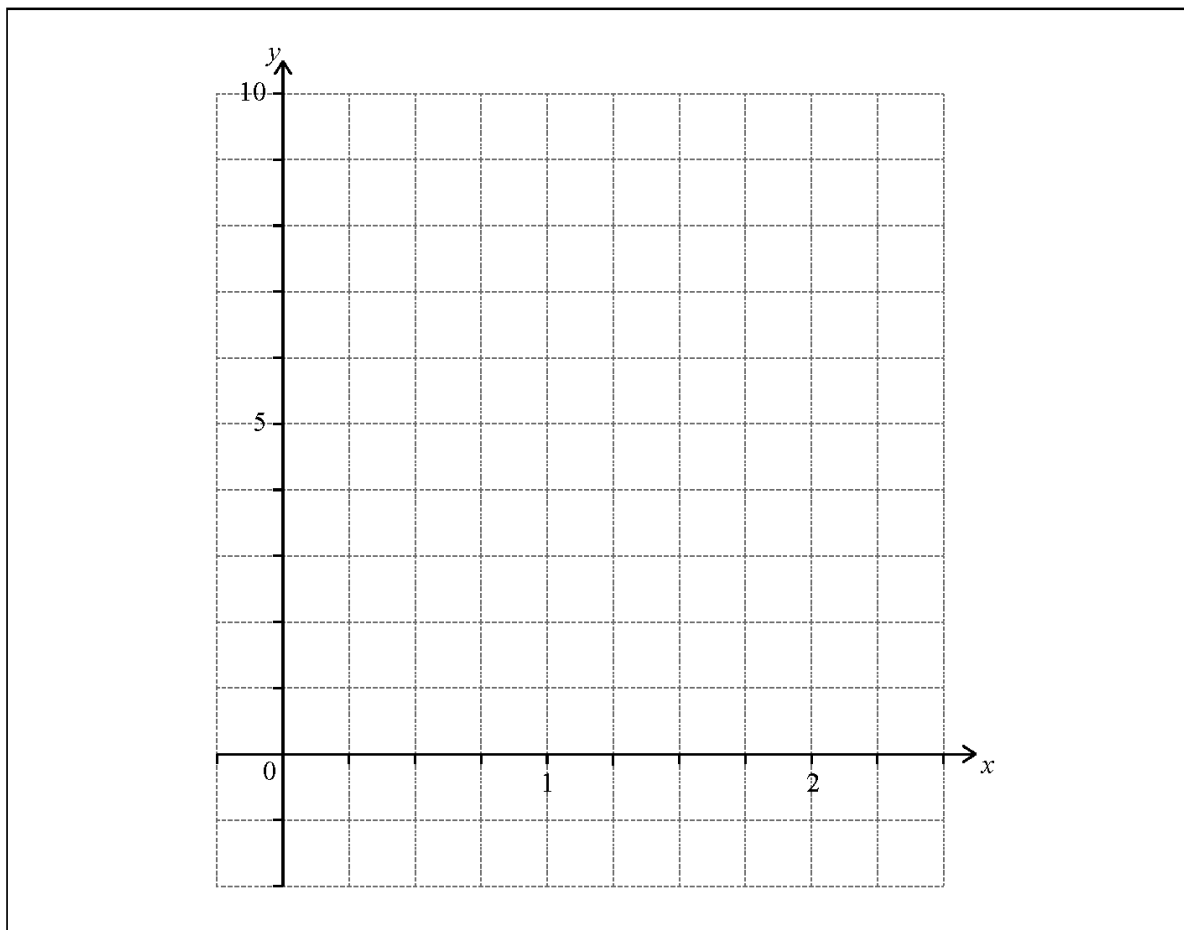
Chapter 1	Functions - Roots	Page 1
Chapter 2	Quadratics	Page 13
Chapter 3	Exponentials - Logarithms	Page 14
Chapter 4	Graphs	Page 20
Chapter 5	Sequences - Series	Page 31
Chapter 6	Complex Numbers	-----
Chapter 7	Permutation - Combination	-----
Chapter 8	Binomial Theorem	Page 42
Chapter 9	Remainder & Factor Theorem	-----
Chapter 10	Mathematical Induction	-----
Chapter 11	Radian	Page 45
Chapter 12	Trigonometry	Page 57
Chapter 13	Matrices	-----
Chapter 14	Vectors - Lines - Planes	Page 103
Chapter 15	Statistics	Page 109
Chapter 16	Probability	Page 136
Chapter 17	Differentiation	Page 172
Chapter 18	Integration	Page 197
Chapter 19	Differential Equations	-----
Chapter 20	Kinematics	Page 209
Chapter 21	Geometry	Page 224
	Answers	Page 234

1 - (MAT/22_SL_Summer_2017_Q6) - Functions - Roots, Graphs

Let $f(x) = x^2 - 1$ and $g(x) = x^2 - 2$, for $x \in \mathbb{R}$.

(a) Show that $(f \circ g)(x) = x^4 - 4x^2 + 3$. [2]

(b) On the following grid, sketch the graph of $(f \circ g)(x)$, for $0 \leq x \leq 2.25$. [3]



(c) The equation $(f \circ g)(x) = k$ has exactly two solutions, for $0 \leq x \leq 2.25$. Find the possible values of k . [3]

2 - (MAT/21_SL_Summer_2019_Q9) - Functions - Roots

Let $f(x) = \frac{16}{x}$. The line L is tangent to the graph of f at $x = 8$.

- (a) Find the gradient of L . [2]

L can be expressed in the form $\mathbf{r} = \begin{pmatrix} 8 \\ 2 \end{pmatrix} + t\mathbf{u}$.

- (b) Find $\mathbf{u}$. [2]

The direction vector of $y = x$ is $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$.

- (c) Find the acute angle between $y = x$ and L . [5]

- (d) (i) Find $(f \circ f)(x)$.

(ii) Hence, write down $f^{-1}(x)$.

- (iii) Hence or otherwise, find the obtuse angle formed by the tangent line to f at $x = 8$ and the tangent line to f at $x = 2$. [7]

3 - (MAT/20_SL_Winter_2019_Q3) - Functions - Roots

[Maximum mark: 7]

Let $f(x) = x - 8$, $g(x) = x^4 - 3$ and $h(x) = f(g(x))$.

(a) Find $h(x)$. [2]

Let C be a point on the graph of h . The tangent to the graph of h at C is parallel to the graph of f .

(b) Find the x -coordinate of C. [5]

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4 - (MAT/20_SL_Winter_2020_Q1) - Functions - Roots

Consider the function $f(x) = x^2 + x + \frac{50}{x}$, $x \neq 0$.

(a) Find $f(1)$. [2]

(b) Solve $f(x) = 0$. [2]

The graph of f has a local minimum at point A.

(c) Find the coordinates of A. [2]

5 - (MAT/20_SL_Winter_2020_Q4) - Functions - Roots

Let $f(x) = 4 - x^3$ and $g(x) = \ln x$, for $x > 0$.

(a) Find $(f \circ g)(x)$. [2]

(b) (i) Solve the equation $(f \circ g)(x) = x$.

(ii) Hence or otherwise, given that $g(2a) = f^{-1}(2a)$, find the value of a . [5]

6 - (MathAA/22_SL_Summer_2021_Q5) - Algebra-functions - Roots

[Maximum mark: 6]

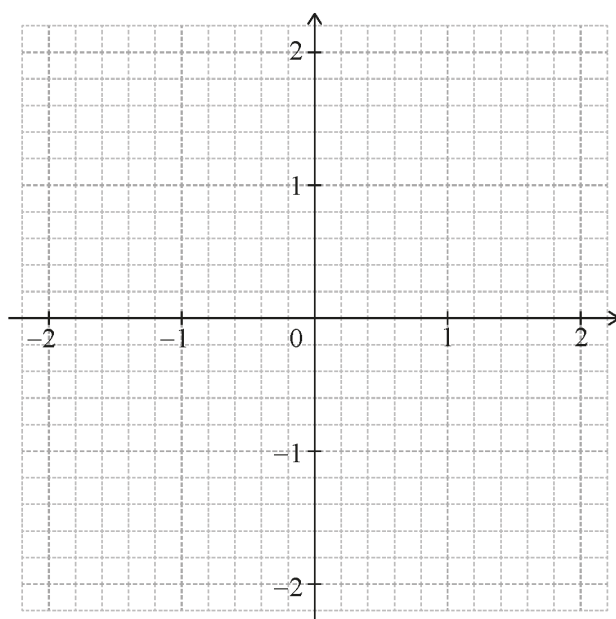
The functions f and g are defined for $x \in \mathbb{R}$ by $f(x) = 6x^2 - 12x + 1$ and $g(x) = -x + c$, where $c \in \mathbb{R}$.

(a) Find the range of f . [2]

(b) Given that $(g \circ f)(x) \leq 0$ for all $x \in \mathbb{R}$, determine the set of possible values for c . [4]

7 - (MathAA/20_SL_Winter_2021_Q2) - Algebra-functions - Roots

[Maximum mark: 5]

Consider the function $f(x) = e^{-x^2} - 0.5$, for $-2 \leq x \leq 2$.(a) Find the values of x for which $f(x) = 0$. [2](b) Sketch the graph of f on the following grid. [3]

8 - (MathAA/21_SL_Summer_2022_Q8) - Algebra-functions - Roots, Integration

[Maximum mark: 16]

The function f is defined by $f(x) = \frac{4x+1}{x+4}$, where $x \in \mathbb{R}$, $x \neq -4$.

- (a) For the graph of f
- (i) write down the equation of the vertical asymptote;
 - (ii) find the equation of the horizontal asymptote. [3]
- (b) (i) Find $f^{-1}(x)$.
- (ii) Using an algebraic approach, show that the graph of f^{-1} is obtained by a reflection of the graph of f in the y -axis followed by a reflection in the x -axis. [8]

The graphs of f and f^{-1} intersect at $x = p$ and $x = q$, where $p < q$.

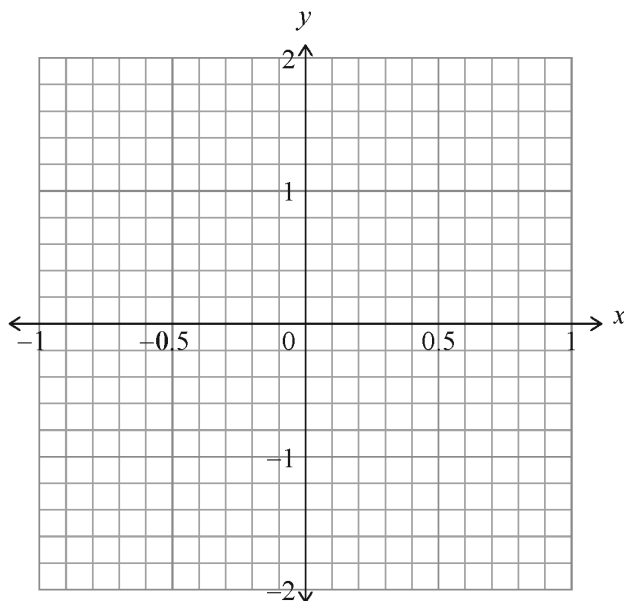
- (c) (i) Find the value of p and the value of q .
- (ii) Hence, find the area enclosed by the graph of f and the graph of f^{-1} . [5]

9 - (MathAA/21_SL_Summer_2023_Q3) - Algebra-functions - Roots

[Maximum mark: 5]

The functions f and g are defined by $f(x) = 2x - x^3$ and $g(x) = \tan x$.

- (a) Find $(f \circ g)(x)$. [2]
- (b) On the following grid, sketch the graph of $y = (f \circ g)(x)$ for $-1 \leq x \leq 1$. Write down and clearly label the coordinates of any local maximum or minimum points. [3]



10 - (MathAA/22_SL_Summer_2023_Q7) - Algebra-functions - Roots, Differentiation

[Maximum mark: 15]

Consider the function $h(x) = \sqrt{4x-2}$, for $x \geq \frac{1}{2}$.

- (a) (i) Find $h^{-1}(x)$, the inverse of $h(x)$, and state its domain. [5]
- (ii) Write down the range of $h^{-1}(x)$.
- (b) The graph of h intersects the graph of h^{-1} at two points. Find the x -coordinates of these two points. [3]
- (c) Find the area enclosed by the graph of h and the graph of h^{-1} . [2]
- (d) Find $h'(x)$. [2]
- (e) Find the value of x for which the graph of h and the graph of h^{-1} have the same gradient. [3]

11 - (MathAA/21_SL_Summer_2024_Q2) - *Algebra-functions - Roots*

[Maximum mark: 7]

Darren buys a car for \$35 000. The value of the car decreases by 15% in the first year.

- (a) Find the value of the car at the end of the first year. [2]

After the first year, the value of the car decreases by 11% in each subsequent year.

- (b) Find the value of Darren's car 10 years after he buys it, giving your answer to the nearest dollar. [2]

When Darren has owned the car for n complete years, the value of the car is less than 10% of its original value.

- (c) Find the least value of n . [3]

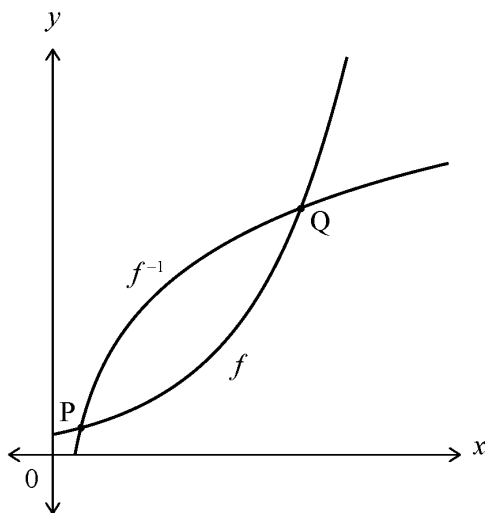
12 - (MathAA/21_SL_Summer_2024_Q7) - Algebra-functions - Roots, Graphs

[Maximum mark: 12]

Consider the function defined by $f(x) = \frac{3}{2}e^{x-2}$, $0 \leq x \leq 4$.

- (a) Show that the inverse function is given by $f^{-1}(x) = 2 + \ln\left(\frac{2x}{3}\right)$. [3]

The graphs of f and f^{-1} intersect at two points P and Q, as shown on the following diagram.



- (b) Find PQ. [3]

The graph of f is reflected in the x -axis and then translated parallel to the y -axis by 5 units in the positive direction to give the graph of a function g .

- (c) Write down

- (i) an expression for $g(x)$;
(ii) the domain of g . [3]

- (d) Solve the equation $f(x) = g(x)$. Give your answer in the form $x = a + \ln b$, where $a, b \in \mathbb{Q}$. [3]

13 - (MathAA/21_SL_Winter_2024_Q1) - Algebra-functions - Roots

[Maximum mark: 7]

Consider the function $f(x) = 7\sqrt{x} - x - 7$, where $0 \leq x \leq 60$.

(a) Find the value of

(i) $f(0)$;

(ii) $f(60)$.

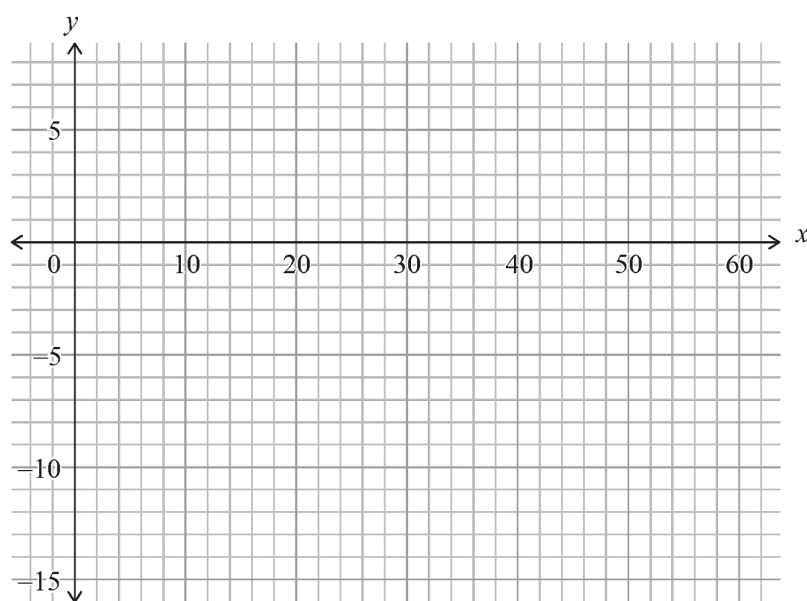
[2]

(b) Find the two roots of $f(x) = 0$.

[2]

(c) Sketch the graph of $y = f(x)$ on the following grid.

[3]



14 - (MathAA/22_SL_Winter_2024_Q1) - Algebra-functions - Roots

[Maximum mark: 7]

Consider the function $f(x) = 11\sqrt{x} - 2x - 11$, where $0 \leq x \leq 20$.

(a) Find the value of

(i) $f(0)$;

(ii) $f(20)$.

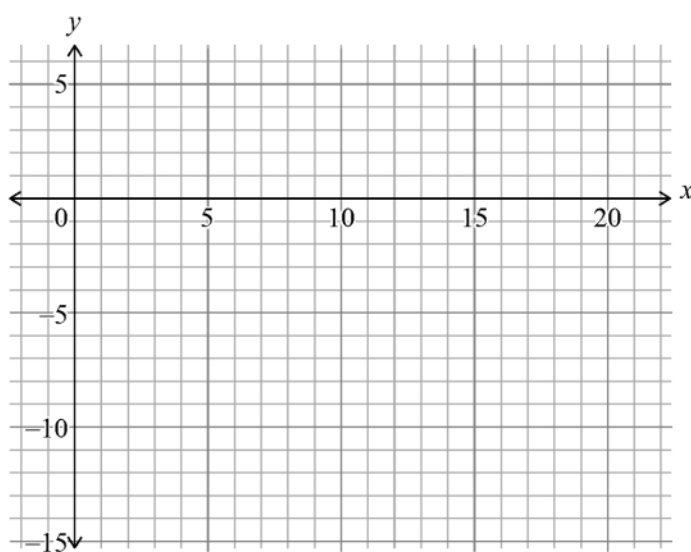
[2]

(b) Find the two roots of $f(x) = 0$.

[2]

(c) Sketch the graph of $y = f(x)$ on the following grid.

[3]



15 - (MathAA/21_SL_Summer_2025_Q1) - Algebra-functions - Roots, Integration

[Maximum mark: 5]

Consider the function $f(x) = \frac{(x-1)^2}{x}$, where $x \in \mathbb{R}$, $x \neq 0$.

(a) Show that $\frac{(x-1)^2}{x} = x - 2 + \frac{1}{x}$.

[2]

(b) Hence, find $\int f(x) dx$.

[3]

16 - (MathAA/21_SL_Summer_2025_Q9) - Algebra-functions - Roots, Differentiation, Graphs

[Maximum mark: 15]

Consider the function $f(x) = \frac{2-2x}{x+2}$, where $x \in \mathbb{R}$, $x \neq -2$.

(a) Show that $f^{-1}(x) = f(x)$. [3]

The point $P\left(k, \frac{2-2k}{k+2}\right)$ is the point on the graph of $y = f(x)$ that is closest to the origin $(0, 0)$.

(b) (i) Find the value of k .

(ii) Hence, write down the coordinates of P . [4]

Consider the function $g(x) = \frac{2-3x}{cx+d}$, where $x \in \mathbb{R}$, $x \neq -\frac{d}{c}$, and $c, d \neq 0$.

The graph of $y = g(x)$ has a vertical asymptote and a horizontal asymptote.

(c) In terms of c and d , write down the equation of

(i) the vertical asymptote;

(ii) the horizontal asymptote. [2]

It is given that $g^{-1}(x) = g(x)$.

(d) Find the value of d . [2]

Consider the case where $c = 1$.

(e) Sketch the graph of $y = \frac{1}{g(x)}$, showing the values of any intercepts with the axes and including any asymptotes, labelled with their equations. [4]

17 - (MathAA/21_SL_Winter_2025_Q7) - Algebra-functions - Roots, Graphs, Differentiation

[Maximum mark: 13]

Consider the function $f(x) = \frac{3x+1}{2x-4}$ where $x \neq 2$.

(a) Write down the equation of

(i) the vertical asymptote of the graph of f ;

(ii) the horizontal asymptote of the graph of f . [2]

(b) Find $f'(x)$. [4]

(c) Hence, explain briefly why f is never increasing. [2]

The point $P(x, f(x))$ is on the graph of f such that $x < 2$.

Let d be the distance between the points P and $Q(3, 5)$.

(d) Find an expression for d in terms of x . [2]

(e) Hence, find the coordinates of P such that d is a minimum. [3]

ANSWERS

1 - (MAT/22_SL_Summer_2017_Q6) - Functions - Roots, Graphs

(a) attempt to form composite in either order

(M1)

eg $f(x^2 - 2), (x^2 - 1)^2 - 2$

$$(x^4 - 4x^2 + 4) - 1$$

A1

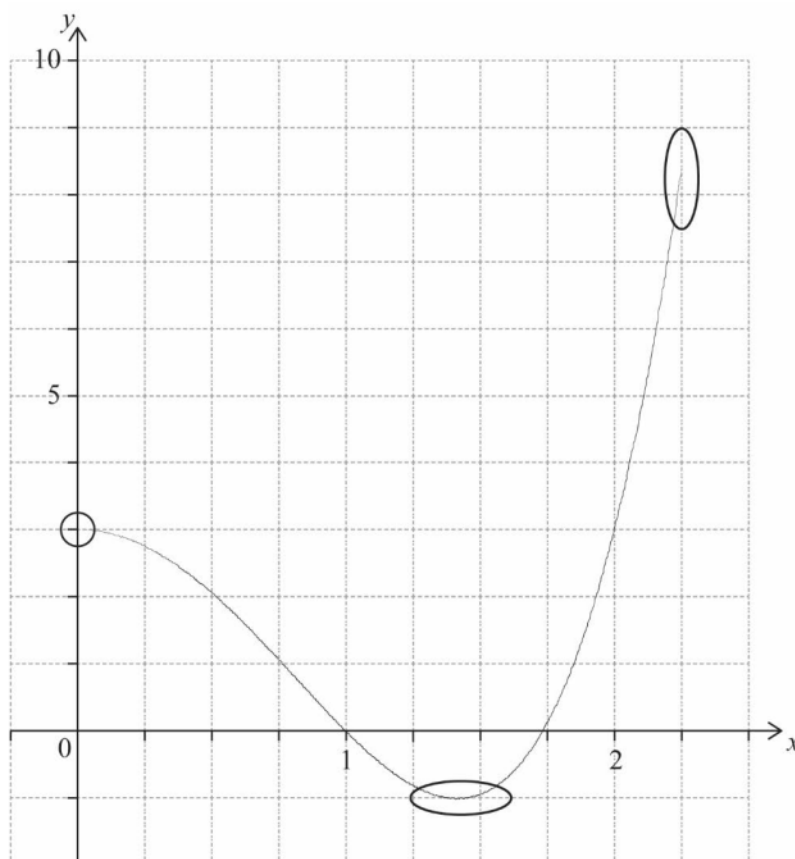
$$(f \circ g)(x) = x^4 - 4x^2 + 3$$

AG

N0

[2 marks]

(b)

A1
A1A1

N3

Note: Award **A1** for approximately correct shape which changes from concave down to concave up. Only if this **A1** is awarded, award the following:
A1 for left hand endpoint in circle **and** right hand endpoint in oval,
A1 for minimum in oval.

[3 marks]

(c) evidence of identifying max/min as relevant points

(M1)

eg $x = 0, 1.41421, y = -1, 3$

correct interval (inclusion/exclusion of endpoints must be correct)

A2

N3

eg $-1 < k \leq 3,]-1, 3], (-1, 3]$

[3 marks]

2 - (MAT/21_SL_Summer_2019_Q9) - Functions - Roots

(a) attempt to find $f'(8)$

(M1)

eg $f'(x)$, y' , $-16x^{-2}$

-0.25 (exact)

A1 N2
[2 marks](b) $u = \begin{pmatrix} 4 \\ -1 \end{pmatrix}$ or any scalar multiple

A2 N2

[2 marks]

(c) correct scalar product and magnitudes

(A1)(A1)(A1)

scalar product $= 1 \times 4 + 1 \times -1$ ($= 3$)

magnitudes $= \sqrt{1^2 + 1^2}, \sqrt{4^2 + (-1)^2}$ ($= \sqrt{2}, \sqrt{17}$)

substitution of **their** values into correct formula

(M1)

eg $\frac{4-1}{\sqrt{1^2+1^2}\sqrt{4^2+(-1)^2}}, \frac{-3}{\sqrt{2}\sqrt{17}}, 2.1112, 120.96^\circ$

$1.03037, 59.0362^\circ$

angle $= 1.03, 59.0^\circ$

A1 N4

[5 marks]

- (d) (i) attempt to form composite
- $(f \circ f)(x)$

(M1)

$$\text{eg } f(f(x)), f\left(\frac{16}{x}\right), \frac{16}{f(x)}$$

correct working

(A1)

$$\text{eg } \frac{16}{16/x}, 16 \times \frac{x}{16}$$

$$(f \circ f)(x) = x$$

A1**N2**

- (ii)
- $f^{-1}(x) = \frac{16}{x}$
- (accept
- $y = \frac{16}{x}, \frac{16}{x}$
-)

A1**N1****Note:** Award **A0** in part (ii) if part (i) is incorrect.

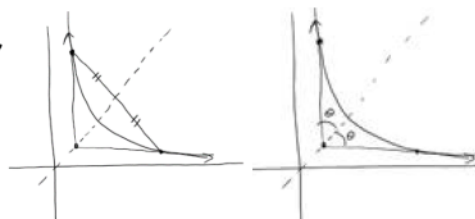
Award **A0** in part (ii) if the candidate has found $f^{-1}(x) = \frac{16}{x}$ by interchanging x and y .

- (iii)
- METHOD 1**

recognition of symmetry about $y = x$ **(M1)**

eg

$$(2, 8) \Leftrightarrow (8, 2)$$

evidence of doubling **their** angle**(M1)**

$$\text{eg } 2 \times 1.03, 2 \times 59.0$$

$$2.06075, 118.072^\circ$$

$$2.06 \text{ (radians)} \text{ (118 degrees)}$$

A1**N2**

3 - (MAT/20_SL_Winter_2019_Q3) - Functions - Roots

- (a) attempt to form composite (in any order)

(M1)

eg $f(x^4 - 3), (x - 8)^4 - 3$

$h(x) = x^4 - 11$

A1 N2

[2 marks]

- (b) recognizing that the gradient of the tangent is the derivative

(M1)

eg h'

correct derivative (seen anywhere)

(A1)

$h'(x) = 4x^3$

correct value for gradient of f (seen anywhere)

(A1)

$f'(x) = 1, m = 1$

setting **their** derivative equal to 1

(M1)

$4x^3 = 1$

0.629960

$x = \sqrt[3]{\frac{1}{4}}$ (exact), 0.630

A1 N3

[5 marks]

4 - (MAT/20_SL_Winter_2020_Q1) - Functions - Roots

- (a) attempt to substitute
- $x = 1$

(M1)

eg $f(1), 1^2 + 1 + \frac{50}{1}$

52 (exact)

A1 N2

[2 marks]

- (b) -4.04932

-4.05

A2 N2

[2 marks]

- (c) (2.76649, 28.4934)

A(2.77, 28.5)

A1A1 N2

[2 marks]

5 - (MAT/20_SL_Winter_2020_Q4) - Functions - Roots

- (a) attempt to form composite (in any order)

(M1)

eg $f(\ln x), g(4 - x^3)$

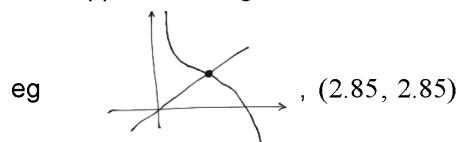
$$(f \circ g)(x) = 4 - (\ln x)^3$$

A1 N2

[2 marks]

- (b) (i) valid approach using GDC

(M1)



2.85056

2.85

A1 N2

- (ii)
- METHOD 1 – (using properties of functions)**

recognizing inverse relationship

(M1)

eg $f(g(2a)) = f(f^{-1}(2a)) (= 2a)$

equating $2a$ to their x from (i)

(A1)

eg $2a = 2.85056$

1.42528

 $a = 1.43$

A1 N2

METHOD 2 – (finding inverse)interchanging x and y (seen anywhere)

(M1)

eg $x = 4 - y^3, f^{-1}(x) = \sqrt[3]{4 - x}$

correct working

(A1)

eg $\sqrt[3]{4 - 2a} = \ln(2a)$, sketch showing intersection of $f^{-1}(2x)$ and $g(2x)$

1.42528

 $a = 1.43$

A1 N2

[5 marks]

6 - (MathAA/22_SL_Summer_2021_Q5) - Algebra-functions - Roots

(a) attempting to find the vertex (M1)

$$x = 1 \text{ OR } y = -5 \text{ OR } f(x) = 6(x-1)^2 - 5$$

range is $y \geq -5$

A1

[2 marks]

(b) METHOD 1

$$(g \circ f)(x) = -(6x^2 - 12x + 1) + c \quad \left(= -\left(6(x-1)^2 - 5\right) + c \right) \quad (\text{A1})$$

EITHER

relating to the range of f OR attempting to find $g(-5)$ (M1)

$$5 + c \leq 0 \quad (\text{A1})$$

OR

attempting to find the discriminant of $(g \circ f)(x)$ (M1)

$$144 + 24(c-1) \leq 0 \quad (120 + 24c \leq 0) \quad (\text{A1})$$

THEN

$$c \leq -5 \quad (\text{A1})$$

[4 marks]

METHOD 2

vertical reflection followed by vertical shift (M1)

new vertex is $(1, 5 + c)$ (A1)

$$5 + c \leq 0 \quad (\text{A1})$$

$$c \leq -5 \quad (\text{A1})$$

[4 marks]

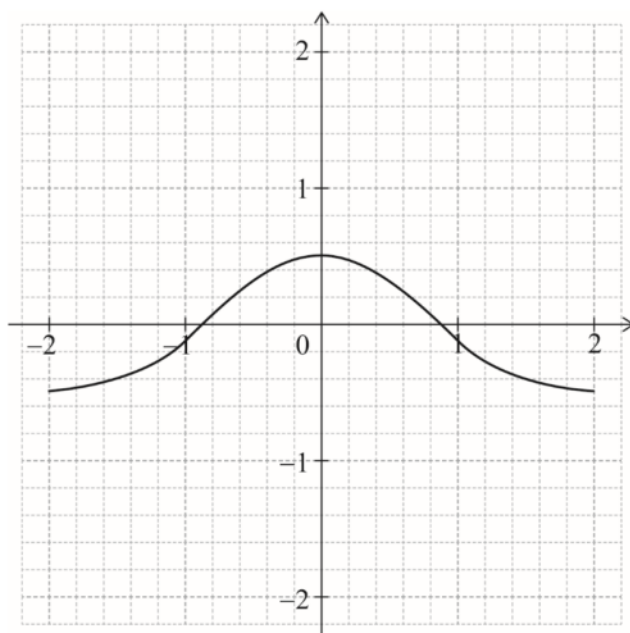
7 - (MathAA/20_SL_Winter_2021_Q2) - Algebra-functions - Roots

(a) $x = -0.832554\dots, x = 0.832554\dots$

$x = -0.833, x = 0.833$

A1A1**[2 marks]**

(b)

**A1A1A1**

Note: Award **A1** for approximately correct shape. Only if this mark is awarded, award **A1** for approximately correct roots and maximum point and **A1** for approximately correct endpoints.

Allow $-1 < x \leq -0.8$, $0.8 \leq x < 1$ for roots, $x = 0$, $0.4 \leq y \leq 0.6$ for maximum and $x = \pm 2$, $-0.6 \leq y \leq -0.4$ for endpoints.

[3 marks]**Total [5 marks]**

8 - (MathAA/21_SL_Summer_2022_Q8) - Algebra-functions - Roots, Integration

(a) (i) $x = -4$

A1

- (ii) attempt to substitute into
- $y = \frac{a}{c}$
- OR table with large values of
- x
- OR sketch of
- f
- showing asymptotic behaviour

(M1)

$y = 4$

A1**[3 marks]**

(b) (i) $y = \frac{4x+1}{x+4}$

attempt to interchange x and y (seen anywhere)**M1**

$xy + 4y = 4x + 1$ OR $xy + 4x = 4y + 1$

(A1)

$xy - 4x = 1 - 4y$ OR $xy - 4y = 1 - 4x$

(A1)

$f^{-1}(x) = \frac{1-4x}{x-4}$ (accept $y = \frac{1-4x}{x-4}$)

A1

- (ii) reflection in
- y
- axis given by
- $f(-x)$

(M1)

$f(-x) = \frac{-4x+1}{-x+4}$

(A1)reflection of their $f(-x)$ in x -axis given by $-f(-x)$ accept "now $-f(x)$ "**M1**

$(-f(-x)) = -\frac{-4x+1}{-x+4}$

$= \frac{-4x+1}{x-4}$ OR $\frac{4x-1}{-x+4}$

A1

$= \frac{1-4x}{x-4} (= f^{-1}(x))$

AG

Note: If the candidate attempts to show the result using a particular coordinate on the graph of f rather than a general coordinate on the graph of f , where appropriate, award marks as follows:

MOA0 for eg $(2,3) \rightarrow (-2,3)$ **MOA0** for $(-2,3) \rightarrow (-2,-3)$ **[8 marks]**

- (c) (i) attempt to solve $f(x) = f^{-1}(x)$ using graph or algebraically

(M1)

$$p = -1 \text{ AND } q = 1$$

A1

Note: Award **(M1)A0** if only one correct value seen.

- (ii) attempt to set up an integral to find area between f and f^{-1}

(M1)

$$\int_{-1}^1 \left(\frac{4x+1}{x+4} - \frac{1-4x}{x-4} \right) dx$$

(A1)

$$= 0.675231\dots$$

$$= 0.675$$

A1

[5 marks]

9 - (MathAA/21_SL_Summer_2023_Q3) - Algebra-functions - Roots

- (a) attempt to substitute g into f

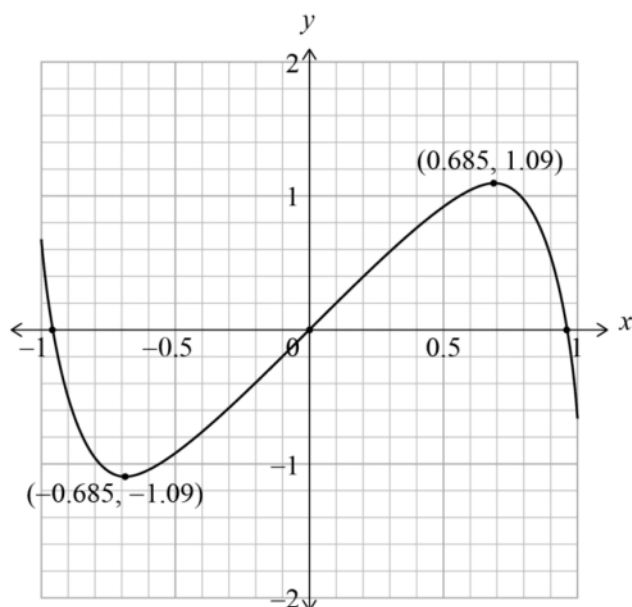
(M1)

$$(f \circ g)(x) = 2 \tan x - \tan^3 x$$

A1

[2 marks]

(b)

**A1A1A1**

Note: **A1** for approximately correct odd function passing through the origin with a maximum above $y = 1$ and a minimum below $y = -1$.

A1 for endpoints at $x = \pm 1$ and y in the intervals $[0.6, 0.8]$ and $[-0.8, -0.6]$

A1 for maximum in approximately correct position and labelled

$(0.685, 1.09)$ AND minimum in approximately correct position and labelled

$(-0.685, -1.09)$. For approximate position, allow $-0.8 \leq x \leq -0.6$,

$-1.2 \leq y \leq -1$ for minimum and $0.6 \leq x \leq 0.8$, $1 \leq y \leq 1.2$ for maximum. If

the candidate gives the coordinates of extrema below their sketch, only

award this mark if extrema are marked in the correct interval (eg by a dot).

[3 marks]

10 - (MathAA/22_SL_Summer_2023_Q7) - Algebra-functions - Roots, Differentiation

(a) (i) swapping x and y , or $h(h^{-1}(x)) = x$ (M1)

$$h^{-1}(x) = \frac{x^2 + 2}{4} \quad \text{A1}$$

recognizing range of h is domain of h^{-1} (M1)

Domain: $x \geq 0$ A1

(ii) range of h^{-1} is $y \geq \frac{1}{2}$ A1

[5 marks]

(b) $\sqrt{4x-2} = \frac{x^2+2}{4}$ OR $\sqrt{4x-2} = x$ OR $\frac{x^2+2}{4} = x$ (M1)

$$x = 0.585786..., x = 3.414213... (= 2 + \sqrt{2})$$

$$x = 0.586, x = 3.41 \quad \text{A1A1}$$

[3 marks]

(c) attempt to form integral of the difference between $h(x)$ and their h^{-1} , using their limits from part (b) (M1)

$$\int_{0.585786...}^{3.414213...} (h(x) - h^{-1}(x)) dx \quad \text{OR} \quad \int_{0.585786...}^{3.414213...} \left(\sqrt{4x-2} - \frac{x^2+2}{4} \right) dx \quad \text{OR}$$

$$6.5996632... - 4.7140452...$$

$$1.88561...$$

$$\text{area} = 1.89 \quad \text{A1}$$

[2 marks]

- (d) attempt to use chain rule or power rule

(M1)

$$h'(x) = 4 \cdot \frac{1}{2} (4x-2)^{-\frac{1}{2}}$$

$$h'(x) = \frac{2}{\sqrt{4x-2}}$$

A1

[2 marks]

- (e) **EITHER**

$$(h^{-1})'(x) = \frac{x}{2}$$

(A1)

equating their $h'(x)$ to the derivative of their $h^{-1}(x)$ and attempting to solve for x

(M1)

$$\frac{2}{\sqrt{4x-2}} = \frac{x}{2}$$

OR

finding intersection of graphs of their derivatives

(M2)

THEN

1.772776...

$x = 1.77$

A1

[3 marks]

11 - (MathAA/21_SL_Summer_2024_Q2) - Algebra-functions - Roots

- (a) recognition that a 15% loss leaves 85% OR finding 15% and subtracting from original **(M1)**

$$0.85 \times 35000 \text{ OR } 35000 - 0.15 \times 35000$$

$$= (\$)29750$$

A1

Note: Accept $(\$)29800$.

[2 marks]

- (b) **EITHER**

$$29750 \times 0.89^9$$

(A1)

OR

$$N = 9$$

$$I\% = -11$$

$$PV = \mp 29750$$

(A1)

THEN

$$\text{value}(FV) = (\$)10423$$

A1

Note: For this **A1** the answer must be rounded to the nearest dollar.

Accept $(\$)10441$ from using 3 sf answer from part (a).

[2 marks]

(c) **METHOD 1**

attempt to solve the inequality (or equation) $29750 \times 0.89^{n-1} < 3500$ OR table of values **(M1)**

19.3643... OR $(n = 19 \Rightarrow) 3651.80...$ OR $(n = 20 \Rightarrow) 3250.10...$ **(A1)**

Note: For candidates using (\$)29800, $n > 19.3787...$, $(n = 19 \Rightarrow) 3657.93...$,
 $(n = 20 \Rightarrow) 3255.56...$

$n = 20$

A1

[3 marks]

METHOD 2

use of the finance app with $I\% = -11$, $PV = \mp 29750$, $FV = \pm 3500$

OR $29750 \times 0.89^N < 3500$ (condone the use of n or x) **(M1)**

$(N =) 18.3643...$ **(A1)**

Note: For candidates using (\$)29800, $N = 18.3787...$

$n = 20$

A1

[3 marks]

Total [7 marks]

12 - (MathAA/21_SL_Summer_2024_Q7) - Algebra-functions - Roots, Graphs

(a) **METHOD 1**attempt to interchange x and y **M1**

Note: This **M1** may be awarded at any stage in the working.

attempt to rearrange using definition of natural log or take the natural log of both sides

M1

$$\frac{2x}{3} = e^{y-2} \Rightarrow \ln\left(\frac{2x}{3}\right) = y-2 \quad \text{OR} \quad x = \frac{3}{2}e^{y-2} \Rightarrow \ln(x) = \ln\left(\frac{3}{2}\right) + y-2$$

A1

$$y = 2 + \ln\left(\frac{2x}{3}\right)$$

$$\text{so } f^{-1}(x) = 2 + \ln\left(\frac{2x}{3}\right)$$

AG**[3 marks]****METHOD 2**attempt to verify that $(f \circ f^{-1})(x) = x$ **M1**

$$(f \circ f^{-1})(x) = \frac{3}{2}e^{\ln\left(\frac{2x}{3}\right)+2-2} = \frac{3}{2}e^{\ln\left(\frac{2x}{3}\right)}$$

attempt to use definition of natural log

M1

$$(f \circ f^{-1})(x) = \frac{3}{2} \times \frac{2x}{3}$$

A1

$$(f \circ f^{-1})(x) = x$$

AG**[3 marks]**

- (b) $(0.264456..., 0.264456...)$ AND $(2.51799..., 2.51799...)$ (A1)

Note: Award **A1** for 0.264456... and 2.51799... seen.

attempt to put their values in distance formula or use of the isosceles right-angled triangle (M1)

$$\begin{aligned} &\sqrt{(2.51799... - 0.264456...)^2 + (2.51799... - 0.264456...)^2} \text{ OR} \\ &\sqrt{2} \times (2.51799... - 0.264456...) \\ &= 3.18689... \\ &= 3.19 \end{aligned}$$

A1
[3 marks]

- (c) (i) $g(x) = -\frac{3}{2}e^{x-2} + 5$ OR $g(x) = -f(x) + 5$ A1A1

Note: Award **A1** for each correct term.

- (ii) $0 \leq x \leq 4$ A1
[3 marks]

- (d) $\frac{3}{2}e^{x-2} = -\frac{3}{2}e^{x-2} + 5$ OR $f(x) = -f(x) + 5$

attempt to collect together terms in e^{x-2} or $f(x)$ (M1)

$$3e^{x-2} = 5 \text{ OR } 2f(x) = 5$$

$$e^{x-2} = \frac{5}{3} \text{ OR } x = f^{-1}\left(\frac{5}{2}\right) \quad \text{(A1)}$$

$$x = 2 + \ln\left(\frac{5}{3}\right) \quad \text{A1}$$

$$\left(a = 2, b = \frac{5}{3}\right)$$

Note: Award **A1** for each correct term given in exact form.

[3 marks]
Total [12 marks]

13 - (MathAA/21_SL_Winter_2024_Q1) - Algebra-functions - Roots

(a) (i) $f(0) = -7$

A1

(ii) $-12.7782\dots$

$$f(60) = -12.8 \left(= 7\sqrt{60} - 67 = 14\sqrt{15} - 67 \right)$$

A1

[2 marks]

(b) attempt to find at least one root

(M1)

$$x = 1.46098\dots \text{ and } x = 33.5390\dots$$

$$x = 1.46 \text{ and } x = 33.5$$

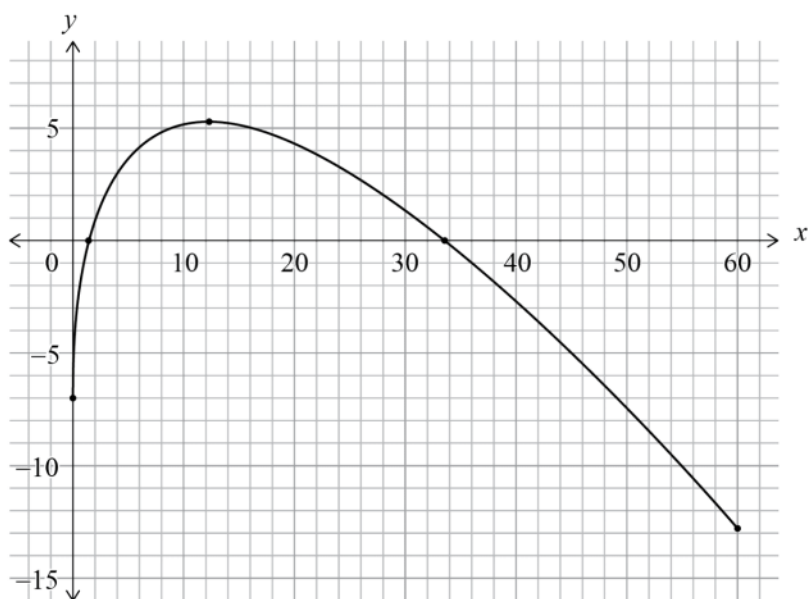
A1

[2 marks]

continued...

Question 1 continued

(c)

**A1A1A1****Note:** Follow through from their part (a).

Award **A1** for endpoints at approximately $(0, -7)$ and $(60, -12.8)$. Allow for y -intercept $-8 < y < -6$ and for the right endpoint in the interval $x = 60$, $-13 < y < -12$.

The following two **A** marks may only be awarded if the approximate shape is correct.

Award **A1** for x -intercepts at approximately $x = 1.5$ and $x = 33.5$ and award **A1** for maximum at approximately $(12.2, 5.3)$. Allow for x -intercepts in the intervals $0 < x < 2$, $32 < x < 34$, and maximum in the intervals $10 < x < 14$, $4.5 < y < 6$.

[3 marks]**Total [7 marks]**

14 - (MathAA/22_SL_Winter_2024_Q1) - Algebra-functions - Roots

(a) (i) $f(0) = -11$

A1

(ii) $-1.80650\dots$

$$f(20) = -1.81 \left(= 11\sqrt{20} - 51 = 22\sqrt{5} - 51 \right)$$

A1

[2 marks]

(b) attempt to find at least one root

(M1)

$$x = 1.72622\dots \text{ and } x = 17.5237\dots$$

$$x = 1.73 \text{ and } x = 17.5$$

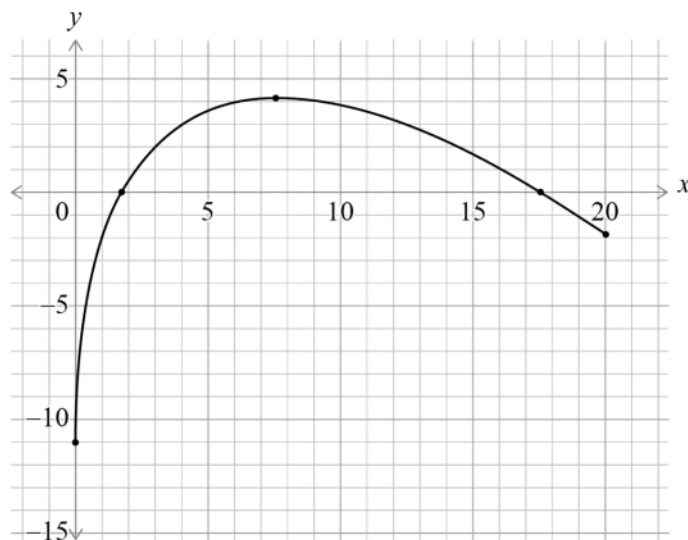
A1

[2 marks]

continued...

Question 1 continued.

(c)



A1A1A1

Note: Follow through from their part (a).

Award **A1** for endpoints at approximately $(0, -11)$ and $(20, -1.8)$. Allow for y -intercept $-12 < y < -10$ and for the right endpoint in the interval $x = 20, -2 < y < -1$.

The following two **A** marks may only be awarded if the approximate shape is correct.

Award **A1** for x -intercepts at approximately $x = 1.7$ and $x = 17.5$ and award **A1** for maximum at approximately $(7.6, 4.1)$. Allow for x -intercepts in the intervals $1 < x < 2$, $17 < x < 18$, and maximum in the intervals $6.5 < x < 8.5$, $3.5 < y < 5$.

[3 marks]

Total [7 marks]

15 - (MathAA/21_SL_Summer_2025_Q1) - Algebra-functions - Roots, Integration

(a) **METHOD 1**

attempt to expand brackets on the numerator

M1

$$(x-1)^2 = x^2 - 2x + 1$$

$$\frac{(x-1)^2}{x} = \frac{x^2 - 2x + 1}{x} \left(= \frac{x^2}{x} - \frac{2x}{x} + \frac{1}{x} \right)$$

A1

$$= x - 2 + \frac{1}{x}$$

AG**METHOD 2**

attempt to express terms with a common denominator

M1

$$x - 2 + \frac{1}{x} = \frac{x^2}{x} - \frac{2x}{x} + \frac{1}{x}$$

$$= \frac{x^2 - 2x + 1}{x}$$

A1

$$= \frac{(x-1)^2}{x}$$

AG**[2 marks]**

$$(b) \quad \int f(x) dx = \frac{x^2}{2} - 2x + \ln|x| (+c)$$

A1A1A1
Note: Award **A1** for each correct term.
[3 marks]**Total[5 marks]**

16 - (MathAA/21_SL_Summer_2025_Q9) - Algebra-functions - Roots, Differentiation, Graphs

(a) **METHOD 1**attempt to rearrange and swap x and y (at any stage)**(M1)**

$$x = \frac{2-2y}{y+2} \Rightarrow xy + 2x = -2y + 2$$

$$xy + 2y = 2 - 2x \text{ OR } y(x+2) = 2 - 2x$$

A1

$$f^{-1}(x) = \frac{2-2x}{x+2} \text{ OR } y = \frac{2-2x}{x+2}$$

A1

$$\text{So } f^{-1}(x) = f(x)$$

AG**METHOD 2**attempt to find $f(f(x))$ **(M1)**

$$f(f(x)) = \frac{2 - 2\left(\frac{2-2x}{x+2}\right)}{\left(\frac{2-2x}{x+2}\right) + 2}$$

$$= \frac{2(x+2) - 2(2-2x)}{(2-2x) + 2(x+2)} \left(= \frac{6x}{6} \right)$$

A1

$$= x$$

A1

$$\text{so } f^{-1}(x) = f(x)$$

AG**[3 marks]**

(b) (i) **EITHER**

$$OP = \sqrt{k^2 + \left(\frac{2-2k}{k+2}\right)^2} \quad (\text{A1})$$

attempt to minimise distance (M1)**OR**recognition that P lies on $y = x$ due to symmetry (M1)

$$k = -4.44948... \text{ or } k = 0.449489... \quad (\text{A1})$$

THENminimum when $k = 0.449489...$

$$k = 0.449 \quad \text{A1}$$

Note: If no working seen, award (M0)(A0)A0 for 0.45.

(ii) coordinates of P (0.449,0.449) A1

Note: Award **A1FT** for $f(k)$, using their k .

[4 marks]

$$(c) \quad (i) \quad x = -\frac{d}{c} \quad \text{A1}$$

$$(ii) \quad y = -\frac{3}{c} \quad \text{A1}$$

Note: Answers in (i) and (ii) must be correct equations.

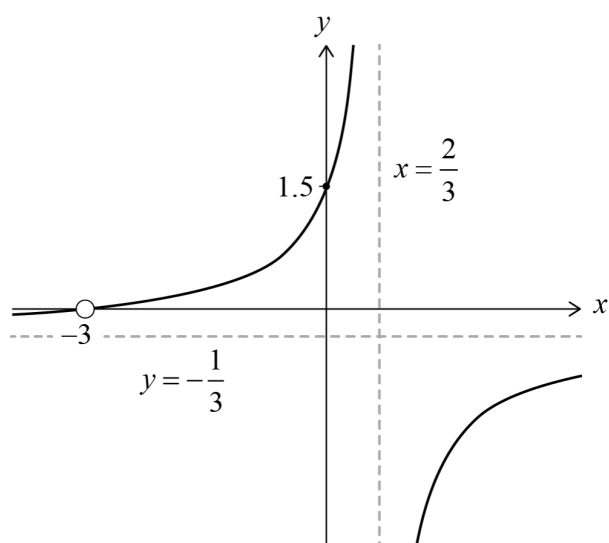
[2 marks]

$$(d) \quad g^{-1}(x) = \frac{2-dx}{cx+3} \text{ OR } -\frac{d}{c} = -\frac{3}{c} \quad (\text{A1})$$

$$d = 3 \quad \text{A1}$$

[2 marks]

(e)



Only if shape is approximately correct for this rational function (i.e. two branches in opposite corners and displaying appropriate asymptotic behavior), award the following marks independently.

asymptotes $x = \frac{2}{3}$ and $y = -\frac{1}{3}$ drawn with correct labels

A1A1

labelled point discontinuity (or x -intercept) at $(-3, 0)$, labelled y -intercept at $(0, \frac{3}{2})$

A1A1

Note: Award **A1FTA1FT**, as appropriate, for their value of d (i.e. x -intercept $= -d$, y -intercept $\frac{d}{2}$).

[4 marks]**Total [15 marks]**

17 - (MathAA/21_SL_Winter_2025_Q7) - Algebra-functions - Roots, Graphs, Differentiation

(a) (i) $x = 2$ **A1**

(ii) $y = \frac{3}{2}$ **A1**

Note: Award **A1A0** for answers $x \neq 2$ and $y \neq \frac{3}{2}$.

[2 marks]

(b) **EITHER**

attempt to use the quotient rule **(M1)**

$$f'(x) = \frac{(2x-4) \times 3 - (3x+1) \times 2}{(2x-4)^2} \quad \textbf{A1A1}$$

Note: Award **A1** for each term in the numerator with correct signs, provided correct denominator is seen.

OR

attempt to use the product rule **(M1)**

$$f'(x) = 3(2x-4)^{-1} + (-1)(3x+1)(2x-4)^{-2} \times 2 \quad \textbf{A1A1}$$

Note: Award **A1** for each term.

THEN

$$f'(x) = \frac{-14}{(2x-4)^2} \quad \textbf{A1}$$

[4 marks]

- (c) recognising $f'(x) < 0$ (when f is decreasing), **OR** $(2x-4)^2 > 0$

OR the gradient (function) is negative (may be seen on a labelled graph of f') **A1**

for all values of x (with $x \neq 2$) **OR** always (or equivalent) **A1**

f is never increasing **AG**

Note: Do not accept a sketch and argument based on the graph of f .

[2 marks]

- (d) attempt to use the distance formula

(M1)

$$d = \sqrt{(x-3)^2 + \left(\frac{3x+1}{2x-4} - 5\right)^2}$$

A1

Note: Accept $d^2 = (x-3)^2 + (f(x)-5)^2$.

[2 marks]

- (e) attempt to find value of x that gives minimum value of d

(M1)

graph of d , $d' = 0$,

$$(x =) -0.305218..., (f(-0.305218...)) = -0.0182941...$$

P has coordinates $(-0.305, -0.0183)$ **A1A1**

Note: Award **A1** for each value.

[3 marks]

Total [13 marks]

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