

IB Diploma

MATHEMATICS AA

SL P1
2017 — 2025

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1 - (MAT/11_SL_Summer_2017_Q2) - Functions - Roots

Let $f(x) = 5x$ and $g(x) = x^2 + 1$, for $x \in \mathbb{R}$.

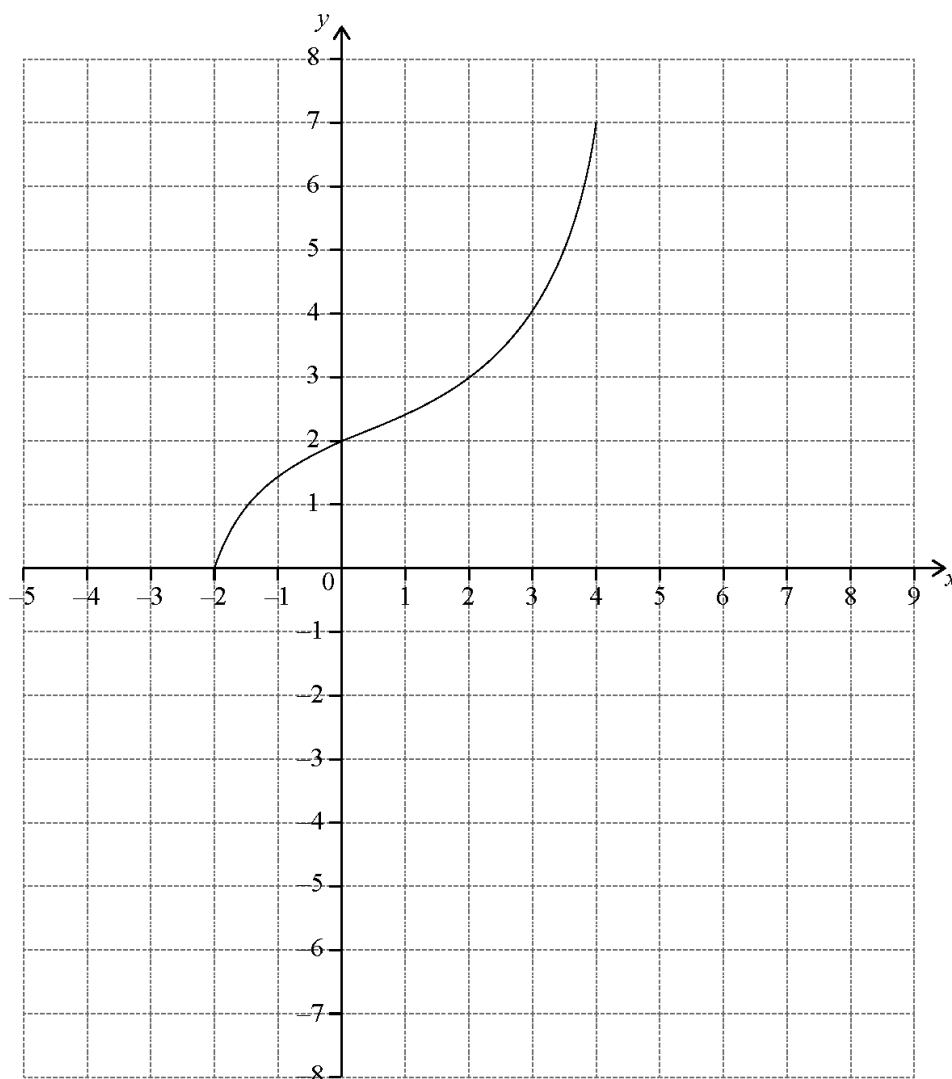
(a) Find $f^{-1}(x)$. [2]

(b) Find $(f \circ g)(7)$. [3]

2 - (MAT/10_SL_Winter_2017_Q3) - Functions - Roots, Graphs

[Maximum mark: 6]

The following diagram shows the graph of a function f , with domain $-2 \leq x \leq 4$.



The points $(-2, 0)$ and $(4, 7)$ lie on the graph of f .

- (a) Write down the range of f . [1]
- (b) Write down
- (i) $f(2)$;
- (ii) $f^{-1}(2)$. [2]
- (c) On the grid opposite, sketch the graph of f^{-1} . [3]

3 - (MAT/10_SL_Winter_2017_Q5) - Functions - Roots

[Maximum mark: 6]

Let $f(x) = 1 + e^{-x}$ and $g(x) = 2x + b$, for $x \in \mathbb{R}$, where b is a constant.

- (a) Find $(g \circ f)(x)$. [2]
- (b) Given that $\lim_{x \rightarrow +\infty} (g \circ f)(x) = -3$, find the value of b . [4]

4 - (MAT/11_SL_Summer_2018_Q1) - Functions - Roots

Let $f(x) = \sqrt{x+2}$ for $x \geq -2$ and $g(x) = 3x - 7$ for $x \in \mathbb{R}$.

- (a) Write down $f(14)$. [1]
- (b) Find $(g \circ f)(14)$. [2]
- (c) Find $g^{-1}(x)$. [3]

5 - (MAT/12_SL_Summer_2018_Q6) - Functions - Roots

[Maximum mark: 7]

Let $f(x) = px^2 + qx - 4p$, where $p \neq 0$. Find the number of roots for the equation $f(x) = 0$. Justify your answer.

6 - (MAT/10_SL_Winter_2018_Q2) - Functions - Roots

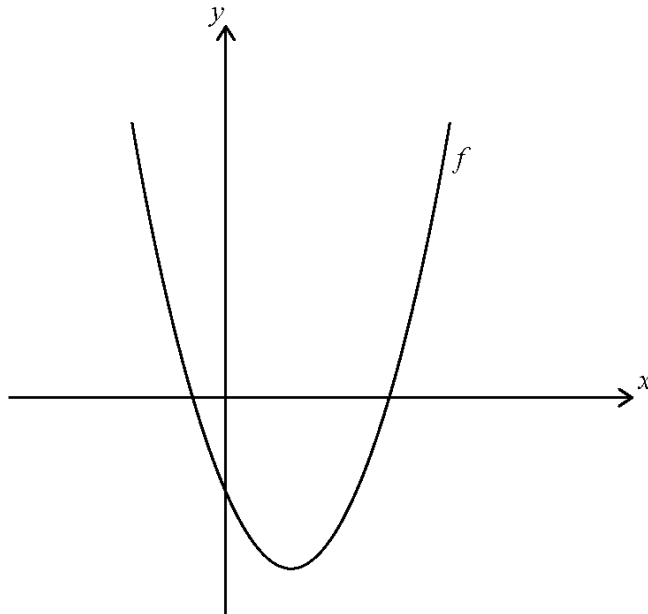
Two functions, f and g , are defined in the following table.

x	-2	1	3	6
$f(x)$	6	3	1	-2
$g(x)$	-7	-2	5	9

- (a) Write down the value of $f(1)$. [1]
- (b) Find the value of $(g \circ f)(1)$. [2]
- (c) Find the value of $g^{-1}(-2)$. [2]

7 - (MAT/10_SL_Winter_2018_Q8) - Functions - Roots, Graphs

Let $f(x) = x^2 - 4x - 5$. The following diagram shows part of the graph of f .



- (a) Find the x -intercepts of the graph of f . [5]
- (b) Find the equation of the axis of symmetry of the graph of f . [2]
- (c) The function can be written in the form $f(x) = (x - h)^2 + k$.
- (i) Write down the value of h .
- (ii) Find the value of k . [4]

The graph of a second function, g , is obtained by a reflection of the graph of f in the y -axis, followed by a translation of $\begin{pmatrix} -3 \\ 6 \end{pmatrix}$.

- (d) Find the coordinates of the vertex of the graph of g . [5]

8 - (MAT/12_SL_Summer_2019_Q3) - Functions - Roots

Consider the function $f(x) = \frac{3x+1}{x-2}$, $x \neq 2$.

- (a) For the graph of f ,
- (i) write down the equation of the vertical asymptote;
- (ii) find the equation of the horizontal asymptote. [3]

Let $g(x) = x^2 + 4$, $x \in \mathbb{R}$.

- (b) Find $(f \circ g)(1)$. [3]

9 - (MAT/11_SL_Summer_2019_Q4) - Functions - Roots

Let $f(x) = \frac{2x-1}{x+3}$, $x \neq -3$.

- (a) Write down the equation of the vertical asymptote of the graph of f . [1]
- (b) Find $f^{-1}(x)$. [3]
- (c) Find the equation of the horizontal asymptote of the graph of f^{-1} . [2]

10 - (MAT/10_SL_Winter_2019_Q10) - Functions - Roots, Differentiation

[Maximum mark: 14]

Let $g(x) = p^x + q$, for $x, p, q \in \mathbb{R}$, $p > 1$. The point $A(0, a)$ lies on the graph of g .

Let $f(x) = g^{-1}(x)$. The point B lies on the graph of f and is the reflection of point A in the line $y = x$.

- (a) Write down the coordinates of B. [2]

The line L_1 is tangent to the graph of f at B.

- (b) Given that $f'(a) = \frac{1}{\ln p}$, find the equation of L_1 in terms of x, p and q . [5]

The line L_2 is tangent to the graph of g at A and has equation $y = (\ln p)x + q + 1$.

The line L_2 passes through the point $(-2, -2)$.

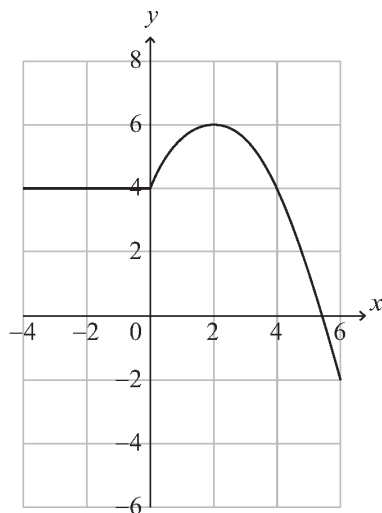
The gradient of the normal to g at A is $\frac{1}{\ln\left(\frac{1}{3}\right)}$.

- (c) Find the equation of L_1 in terms of x . [7]

11 - (MathAA/11_SL_Summer_2021_Q1) - Algebra-functions - Roots

[Maximum mark: 5]

The graph of $y = f(x)$ for $-4 \leq x \leq 6$ is shown in the following diagram.



(a) Write down the value of

(i) $f(2)$;

(ii) $(f \circ f)(2)$.

[2]

(b) Let $g(x) = \frac{1}{2}f(x) + 1$ for $-4 \leq x \leq 6$. On the axes above, sketch the graph of g .

[3]

12 - (MathAA/12_SL_Summer_2021_Q2) - Algebra-functions - Roots

[Maximum mark: 4]

Consider two consecutive positive integers, n and $n + 1$.

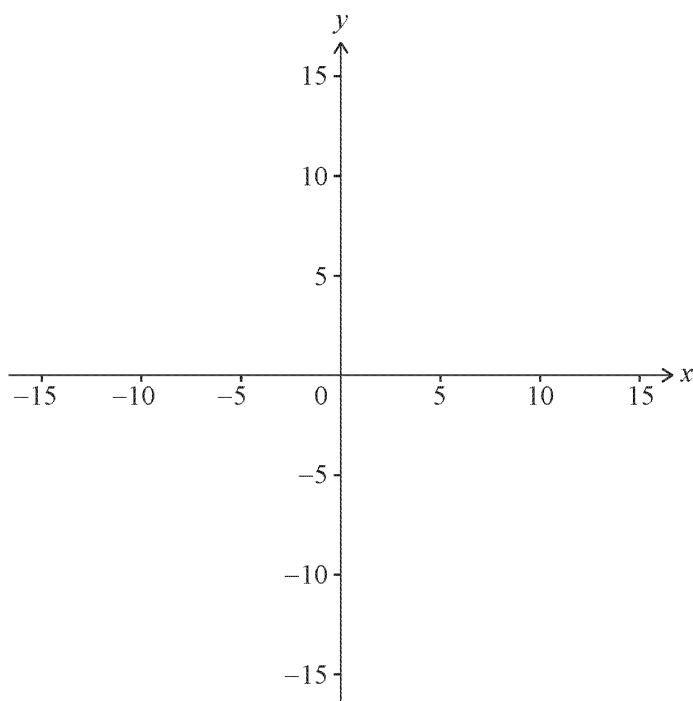
Show that the difference of their squares is equal to the sum of the two integers.

13 - (MathAA/10_SL_Winter_2021_Q3) - *Graphs, Algebra-functions - Roots*

[Maximum mark: 5]

The function f is defined by $f(x) = \frac{2x+4}{3-x}$, where $x \in \mathbb{R}$, $x \neq 3$.

- (a) Write down the equation of
- (i) the vertical asymptote of the graph of f ;
 - (ii) the horizontal asymptote of the graph of f . [2]
- (b) Find the coordinates where the graph of f crosses
- (i) the x -axis;
 - (ii) the y -axis. [2]
- (c) Sketch the graph of f on the axes below. [1]



14 - (MathAA/11_SL_Summer_2022_Q4) - Algebra-functions - Roots, Trigonometry

[Maximum mark: 7]

Consider the functions $f(x) = \sqrt{3} \sin x + \cos x$ where $0 \leq x \leq \pi$ and $g(x) = 2x$ where $x \in \mathbb{R}$.

- (a) Find $(f \circ g)(x)$. [2]
- (b) Solve the equation $(f \circ g)(x) = 2 \cos 2x$ where $0 \leq x \leq \pi$. [5]

15 - (MathAA/12_SL_Summer_2022_Q1) - Algebra-functions - Roots

[Maximum mark: 5]

The following table shows values of $f(x)$ and $g(x)$ for different values of x .

Both f and g are one-to-one functions.

x	-2	0	3	4
$f(x)$	8	4	0	-3
$g(x)$	-5	-2	4	0

- (a) Find $g(0)$. [1]
- (b) Find $(f \circ g)(0)$. [2]
- (c) Find the value of x such that $f(x) = 0$. [2]

16 - (MathAA/12_SL_Summer_2022_Q3) - Algebra-functions - Roots

[Maximum mark: 6]

Consider any three consecutive integers, $n - 1$, n and $n + 1$.

- (a) Prove that the sum of these three integers is always divisible by 3. [2]
- (b) Prove that the sum of the squares of these three integers is never divisible by 3. [4]

17 - (MathAA/10_SL_Winter_2022_Q1) - Algebra-functions - Roots

[Maximum mark: 7]

Let $f(x) = -2x + 3$, for $x \in \mathbb{R}$.

- (a) The graph of a linear function g is parallel to the graph of f and passes through the origin. Find an expression for $g(x)$. [2]
- (b) The graph of a linear function h is perpendicular to the graph of f and passes through the point $(-1, 2)$. Find an expression for $h(x)$. [3]
- (c) Find $(g \circ h)(0)$. [2]

18 - (MathAA/10_SL_Winter_2022_Q4) - Algebra-functions - Roots

[Maximum mark: 5]

Let a be a constant, where $a > 1$.

(a) Show that $a^2 + \left(\frac{a^2-1}{2}\right)^2 = \left(\frac{a^2+1}{2}\right)^2$. [3]

Consider a right-angled triangle with sides of length a , $\left(\frac{a^2-1}{2}\right)$ and $\left(\frac{a^2+1}{2}\right)$.

(b) Find an expression for the area of the triangle in terms of a . [2]

19 - (MathAA/10_SL_Winter_2022_Q7) - Algebra-functions - Roots

[Maximum mark: 16]

(a) The graph of a quadratic function f has its vertex at the point $(3, 2)$ and it intersects the x -axis at $x = 5$. Find f in the form $f(x) = a(x-h)^2 + k$. [3]

The quadratic function g is defined by $g(x) = px^2 + (t-1)x - p$ where $x \in \mathbb{R}$ and $p, t \in \mathbb{R}, p \neq 0$.

(b) In the case where $g(-3) = g(1) = 4$,

(i) find the value of p and the value of t ;

(ii) find the range of g . [7]

(c) The linear function j is defined by $j(x) = -x + 3p$ where $x \in \mathbb{R}$ and $p \in \mathbb{R}, p \neq 0$.

Show that the graphs of $j(x) = -x + 3p$ and $g(x) = px^2 + (t-1)x - p$ have two distinct points of intersection for every possible value of p and t . [6]

20 - (MathAA/11_SL_Summer_2023_Q2) - Algebra-functions - Roots

[Maximum mark: 7]

The function f is defined by $f(x) = \frac{7x+7}{2x-4}$ for $x \in \mathbb{R}, x \neq 2$.

(a) Find the zero of $f(x)$. [2]

(b) For the graph of $y = f(x)$, write down the equation of

(i) the vertical asymptote;

(ii) the horizontal asymptote. [2]

(c) Find $f^{-1}(x)$, the inverse function of $f(x)$. [3]

21 - (MathAA/12_SL_Summer_2023_Q3) - Algebra-functions - Roots

[Maximum mark: 5]

A function f is defined by $f(x) = 1 - \frac{1}{x-2}$, where $x \in \mathbb{R}$, $x \neq 2$.

- (a) The graph of $y = f(x)$ has a vertical asymptote and a horizontal asymptote.

Write down the equation of

- (i) the vertical asymptote;
(ii) the horizontal asymptote.

[2]

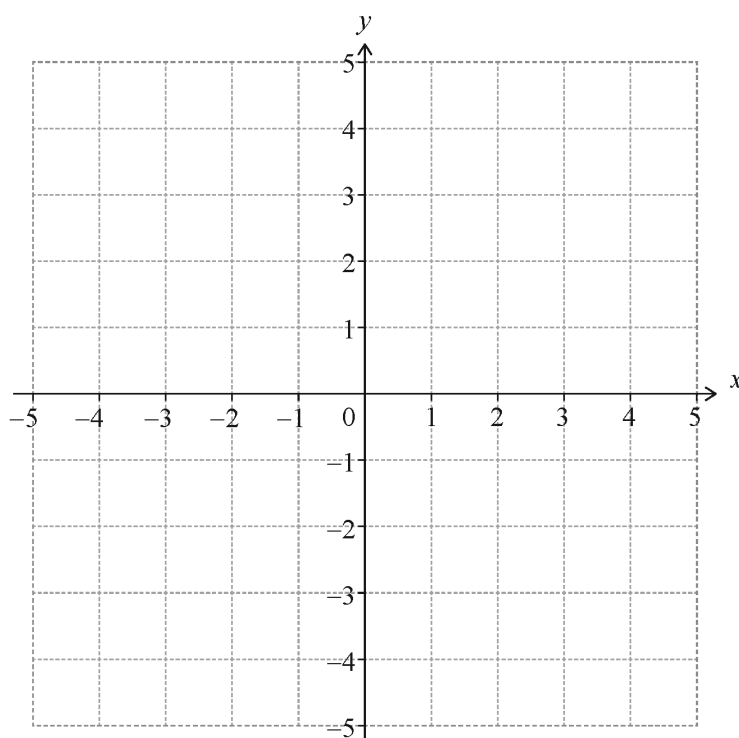
- (b) Find the coordinates of the point where the graph of $y = f(x)$ intersects

- (i) the y -axis;
(ii) the x -axis.

[2]

- (c) On the following set of axes, sketch the graph of $y = f(x)$, showing all the features found in parts (a) and (b).

[1]



22 - (MathAA/12_SL_Summer_2023_Q6) - Algebra-functions - Roots

[Maximum mark: 7]

The functions f and g are defined for $x \in \mathbb{R}$ by

$$f(x) = ax + b, \text{ where } a, b \in \mathbb{Z}$$

$$g(x) = x^2 + x + 3.$$

Find the two possible functions f such that $(g \circ f)(x) = 4x^2 - 14x + 15$.

23 - (MathAA/11_SL_Winter_2023_Q2) - Algebra-functions - Roots

[Maximum mark: 5]

Consider the functions $f(x) = x + 2$ and $g(x) = x^2 - k^2$, where k is a real constant.

(a) Write down an expression for $(g \circ f)(x)$. [2]

(b) Given that $(g \circ f)(4) = 11$, find the possible values of k . [3]

24 - (MathAA/12_SL_Winter_2023_Q2) - Algebra-functions - Roots

[Maximum mark: 5]

Consider the functions $f(x) = x - 3$ and $g(x) = x^2 + k^2$, where k is a real constant.

(a) Write down an expression for $(g \circ f)(x)$. [2]

(b) Given that $(g \circ f)(2) = 10$, find the possible values of k . [3]

25 - (MathAA/11_SL_Summer_2024_Q8) - Algebra-functions - Roots, Graphs

[Maximum mark: 14]

Consider the function $f(x) = \frac{4x+2}{x-2}$, $x \neq 2$.

- (a) Sketch the graph of $y = f(x)$. On your sketch, indicate the values of any axis intercepts and label any asymptotes with their equations. [5]
- (b) Write down the range of f . [1]

Consider the function $g(x) = x^2 + bx + c$. The graph of g has an axis of symmetry at $x = 2$.

The two roots of $g(x) = 0$ are $-\frac{1}{2}$ and p , where $p \in \mathbb{Q}$.

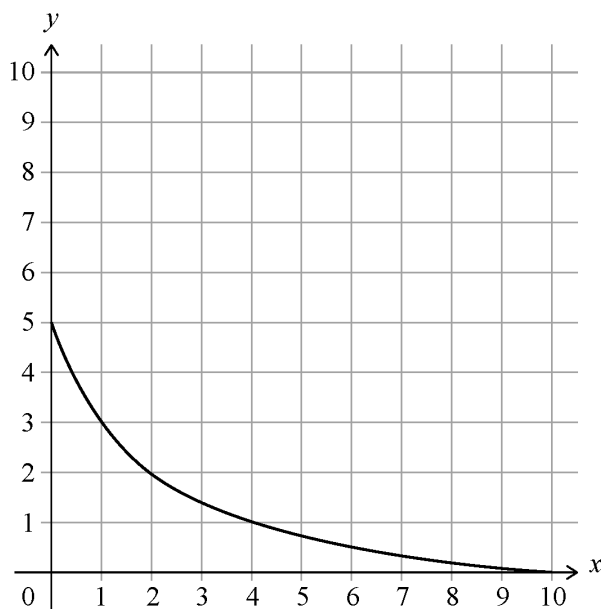
- (c) Show that $p = \frac{9}{2}$. [1]
- (d) Find the value of b and the value of c . [3]
- (e) Find the y -coordinate of the vertex of the graph of $y = g(x)$. [2]
- (f) Find the number of solutions of the equation $f(x) = g(x)$. [2]

26 - (MathAA/12_SL_Summer_2024_Q1) - Algebra-functions - Roots, Graphs

[Maximum mark: 5]

The graph of $y = f(x)$ for $0 \leq x \leq 10$ is shown in the following diagram.

The graph intercepts the axes at $(10, 0)$ and $(0, 5)$.



(a) Write down the value of

(i) $f(4)$;

(ii) $f \circ f(4)$;

(iii) $f^{-1}(3)$.

[3]

(b) On the axes above, sketch the graph of $y = f^{-1}(x)$. Show clearly where the graph intercepts the axes.

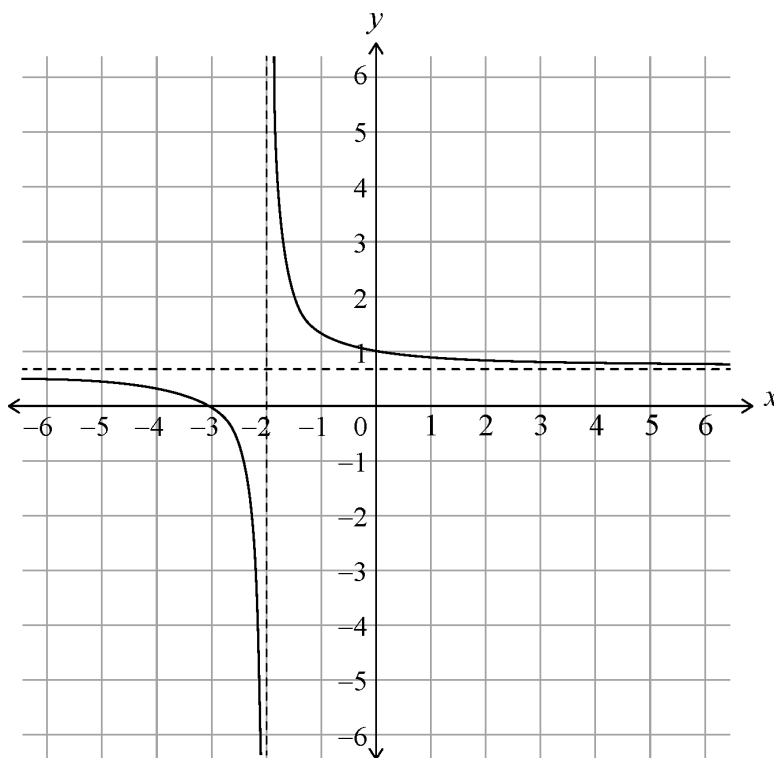
[2]

27 - (MathAA/12_SL_Summer_2024_Q6) - Algebra-functions - Roots, Graphs

[Maximum mark: 8]

A function f is defined by $f(x) = \frac{2(x+3)}{3(x+2)}$, where $x \in \mathbb{R}, x \neq -2$.

The graph $y = f(x)$ is shown below.



(a) Write down the equation of the horizontal asymptote.

[1]

Consider $g(x) = mx + 1$, where $m \in \mathbb{R}, m \neq 0$.

- (b) (i) Write down the number of solutions to $f(x) = g(x)$ for $m > 0$.
- (ii) Determine the value of m such that $f(x) = g(x)$ has only one solution for x .
- (iii) Determine the range of values for m , where $f(x) = g(x)$ has two solutions for $x \geq 0$.

[7]

28 - (MathAA/11_SL_Winter_2024_Q4) - Algebra-functions - Roots

[Maximum mark: 4]

Prove that $(2n + 5)^2 - (2n - 5)^2$ is a multiple of 10 for all $n \in \mathbb{Z}^+$.

29 - (MathAA/11_SL_Winter_2024_Q8) - Algebra-functions - Roots

[Maximum mark: 14]

The function f is defined as $f(x) = \log_2(8x)$, where $x > 0$.

(a) Find the value of

(i) $f(2)$;

(ii) $f\left(\frac{1}{8}\right)$. [3]

(b) Find an expression for $f^{-1}(x)$. [4]

(c) Hence, or otherwise, find $f^{-1}(0)$. [1]

The graph of $y = f(4x^2)$ can be obtained by translating and stretching the graph of $y = \log_2 x$.

(d) Describe these two transformations specifying the order in which they are to be applied. [6]

30 - (MathAA/12_SL_Winter_2024_Q4) - Algebra-functions - Roots

[Maximum mark: 4]

Prove that $(3n + 2)^2 - (3n - 2)^2$ is a multiple of 12 for all $n \in \mathbb{Z}^+$.

31 - (MathAA/12_SL_Winter_2024_Q8) - Algebra-functions - Roots

[Maximum mark: 14]

The function f is defined as $f(x) = \log_2(4x)$, where $x > 0$.

(a) Find the value of

(i) $f(8)$;

(ii) $f\left(\frac{1}{4}\right)$. [3]

(b) Find an expression for $f^{-1}(x)$. [4]

(c) Hence, or otherwise, find $f^{-1}(0)$. [1]

The graph of $y = f(16x^3)$ can be obtained by translating and stretching the graph of $y = \log_2 x$.

(d) Describe these two transformations specifying the order in which they are to be applied. [6]

ANSWERS

1 - (MAT/11_SL_Summer_2017_Q2) - Functions - Roots

- (a) interchanging x and y
eg $x = 5y$

(M1)

$$f^{-1}(x) = \frac{x}{5}$$

A1 N2

[2 marks]

- (b) METHOD 1

attempt to substitute 7 into $g(x)$ or $f(x)$

(M1)

eg $7^2 + 1, 5 \times 7$

$$g(7) = 50$$

(A1)

$$f(50) = 250$$

A1 N2

METHOD 2

attempt to form composite function (in any order)

(M1)

eg $5(x^2 + 1), (5x)^2 + 1$

correct substitution

(A1)

eg $5 \times (7^2 + 1)$

$$(f \circ g)(7) = 250$$

A1 N2

[3 marks]

2 - (MAT/10_SL_Winter_2017_Q3) - Functions - Roots, Graphs

- (a) correct range (do not accept $0 \leq x \leq 7$)
eg $[0, 7], 0 \leq y \leq 7$

A1 N1

[1 mark]

- (b) (i) $f(2) = 3$

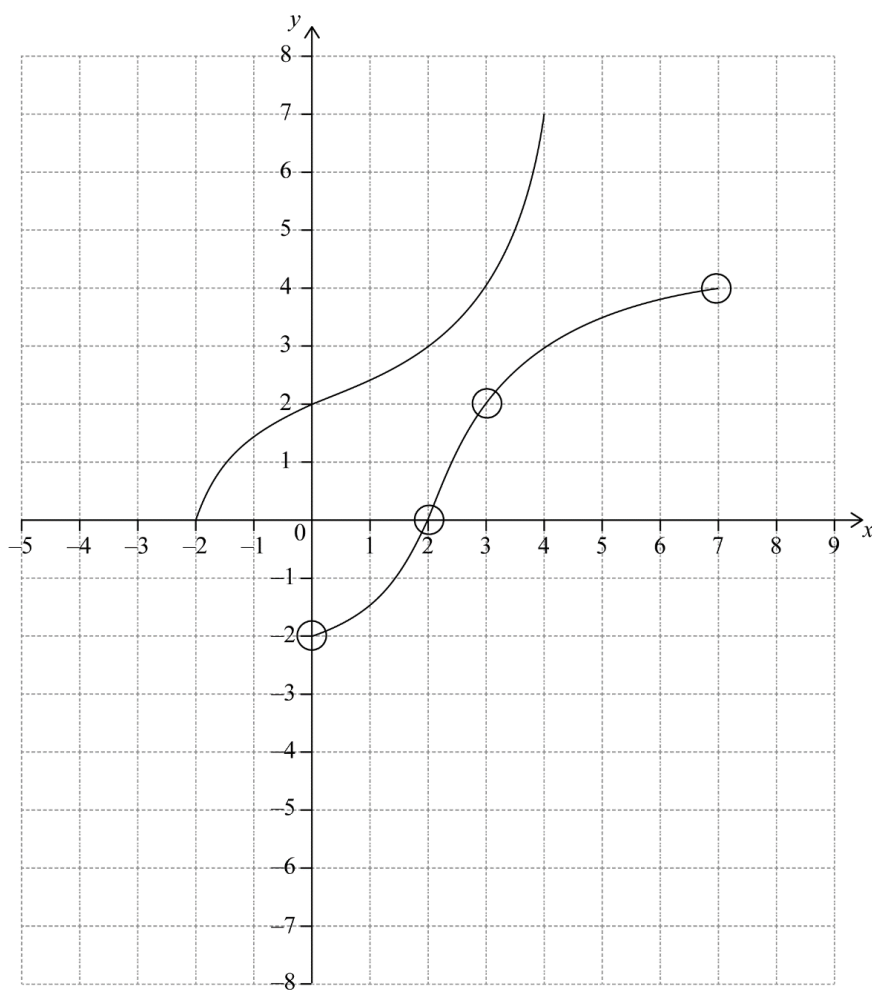
A1 N1

- (ii) $f^{-1}(2) = 0$

A1 N1

[2 marks]

- (c)



A1A1A1

N3

Notes: Award **A1** for both end points within circles,
A1 for images of (2, 3) and (0, 2) within circles,
A1 for approximately correct reflection in $y = x$, concave up then concave down shape (do not accept line segments).

[3 marks]

3 - (MAT/10_SL_Winter_2017_Q5) - Functions - Roots

- (a) attempt to form composite

(M1)

eg $g(1+e^{-x})$

correct function

A1

N2

eg $(g \circ f)(x) = 2 + b + 2e^{-x}, 2(1+e^{-x}) + b$

[2 marks]

- (b) evidence of
- $\lim_{x \rightarrow \infty} (2 + b + 2e^{-x}) = 2 + b + \lim_{x \rightarrow \infty} (2e^{-x})$

(M1)

eg $2 + b + 2e^{-\infty}$, graph with horizontal asymptote when $x \rightarrow \infty$

Note: Award **M0** if candidate clearly has incorrect limit, such as $x \rightarrow 0$, e^{∞} , $2e^0$.evidence that $e^{-x} \rightarrow 0$ (seen anywhere)

(A1)

eg $\lim_{x \rightarrow \infty} (e^{-x}) = 0$, $1 + e^{-x} \rightarrow 1$, $2(1) + b = -3$, $e^{\text{large negative number}} \rightarrow 0$, graph of $y = e^{-x}$ or

$y = 2e^{-x}$ with asymptote $y = 0$, graph of composite function with asymptote $y = -3$

correct working

(A1)

eg $2 + b = -3$

$b = -5$

A1

N2

[4 marks]

4 - (MAT/11_SL_Summer_2018_Q1) - Functions - Roots

- (a)
- $f(14) = 4$

A1

N1

[1 mark]

- (b) attempt to substitute

(M1)

eg $g(4)$, $3 \times 4 - 7$

5

A1

N2

[2 marks]

- (c) interchanging
- x
- and
- y
- (seen anywhere)

(M1)

eg $x = 3y - 7$

evidence of correct manipulation

(A1)

eg $x + 7 = 3y$

$g^{-1}(x) = \frac{x+7}{3}$

A1

N3

[3 marks]

5 - (MAT/12_SL_Summer_2018_Q6) - Functions - Roots

METHOD 1

evidence of discriminant

(M1)eg $b^2 - 4ac$, Δ

correct substitution into discriminant

(A1)eg $q^2 - 4p(-4p)$

correct discriminant

A1eg $q^2 + 16p^2$ $16p^2 > 0$ (accept $p^2 > 0$)**A1** $q^2 \geq 0$ (do not accept $q^2 > 0$)**A1** $q^2 + 16p^2 > 0$ **A1** f has 2 roots**A1****N0****METHOD 2** y -intercept = $-4p$ (seen anywhere)**A1**if p is positive, then the y -intercept will be negative**A1**an upward-opening parabola with a negative y -intercept**R1**eg sketch that must indicate $p > 0$.if p is negative, then the y -intercept will be positive**A1**a downward-opening parabola with a positive y -intercept**R1**eg sketch that must indicate $p < 0$. f has 2 roots**A2****N0****[7 marks]**

6 - (MAT/10_SL_Winter_2018_Q2) - Functions - Roots

(a) $f(1) = 3$ **A1****N1****[1 mark]**

(b) attempt to form the composite (including value)

(M1)eg $g(3)$, $g(f(1))$ $(g \circ f)(1) = 5$ **A1****N2****[2 marks]**

(c) valid approach

(M1)eg $g(x) = -2$ $g^{-1}(-2) = 1$ **A1****N2****[2 marks]**

7 - (MAT/10_SL_Winter_2018_Q8) - Functions - Roots, Graphs

- (a) valid approach (M1)
 eg $f(x) = 0$, $x^2 - 4x - 5 = 0$
 valid attempt to solve quadratic equation (M1)
 eg factorizing, formula, completing the square
 evidence of correct working (A1)
 eg $(x-5)(x+1)$, $x = \frac{4 \pm \sqrt{16 - 4(-5)}}{2}$
 $x = -1$, $x = 5$ (accept $(-1, 0)$, $(5, 0)$) A1A1 N3
 [5 marks]
- (b) correct working (A1)
 eg $\frac{-(-4)}{2(1)}$, $\frac{-1+5}{2}$
 $x = 2$ (must be an equation with $x =$) A1 N2
 [2 marks]
- (c) (i) $h = 2$ A1 N1
- (ii) METHOD 1
 valid approach (M1)
 eg $f(2)$
 correct substitution (A1)
 eg $(2)^2 - 4(2) - 5$
 $k = -9$ A1 N2
- METHOD 2
 valid attempt to complete the square (M1)
 eg $x^2 - 4x + 4$
 correct working (A1)
 eg $(x^2 - 4x + 4) - 4 - 5$, $(x-2)^2 - 9$
 $k = -9$ A1 N2
 [4 marks]

(d) METHOD 1 (working with vertex)

vertex of f is at $(2, -9)$ **(A1)**

correct horizontal reflection **(A1)**

eg $x = -2, (-2, -9)$

valid approach for translation of **their** x or y value **(M1)**

eg $x-3, y+6, \begin{pmatrix} -2 \\ -9 \end{pmatrix} + \begin{pmatrix} -3 \\ 6 \end{pmatrix}$, one correct coordinate for vertex

vertex of g is $(-5, -3)$ (accept $x = -5, y = -3$) **A1A1 N1N1**

METHOD 2 (working with function)

correct approach for horizontal reflection **(A1)**

eg $f(-x)$

correct horizontal reflection **(A1)**

eg $(-x)^2 - 4(-x) - 5, x^2 + 4x - 5, (-x-2)^2 - 9$

valid approach for translation of **their** x or y value **(M1)**

eg $(x+3)^2 + 4(x+3) - 5 + 6, x^2 + 10x + 22, (x+5)^2 - 3$, one correct coordinate for vertex

vertex of g is $(-5, -3)$ (accept $x = -5, y = -3$) **A1A1 N1N1**

[5 marks]

8 - (MAT/12_SL_Summer_2019_Q3) - Functions - Roots

(a) (i) $x = 2$ (must be an equation)**A1 N1**

(ii) valid approach

(M1)

$$\text{eg } 3 + \frac{7}{x-2}, x \rightarrow \infty, \frac{3x}{x}, \frac{3}{1}, \frac{3 + \frac{1}{x}}{1 - \frac{2}{x}}, \frac{3(x-2)+7}{x-2}$$

 $y = 3$ (must be an equation)**A1 N2****[3 marks]**(b) **METHOD 1**attempt to substitute 1 into $g(x)$ or $f(x)$ **(M1)**

$$\text{eg } 1^2 + 4, \frac{3+1}{1-2}$$

$$g(1) = 5$$

(A1)

$$(f \circ g)(1) = \frac{16}{3}$$

A1 N2**METHOD 2**

attempt to form composite function (in any order)

(M1)

$$\text{eg } \frac{3(x^2 + 4) + 1}{x^2 + 4 - 2}, \left(\frac{3x + 1}{x - 2} \right)^2 + 4$$

correct substitution

(A1)

$$\text{eg } \frac{3(5) + 1}{5 - 2}$$

$$(f \circ g)(1) = \frac{16}{3}$$

A1 N2**[3 marks]**

9 - (MAT/11_SL_Summer_2019_Q4) - Functions - Roots

(a) $x = -3$ (must be an equation)

A1 N1
[1 mark]

(b) interchanging x and y (seen anywhere)

(M1)

eg $x = \frac{2y-1}{y+3}$, $x(y+3) = 2y-1$

evidence of correct manipulation

(A1)

eg $yx + 3x = 2y - 1$, $y(x-2) = -3x - 1$, $2 - \frac{7}{y+3}$

$$f^{-1}(x) = \frac{-3x-1}{x-2} \left(= \frac{3x+1}{2-x}, \frac{7}{2-x} - 3 \right) \text{ (accept } y =)$$

A1 N3**[3 marks]**

(c) valid approach to find horizontal asymptote

(M1)

eg $\frac{-3}{1}$, vert.asymp of f becomes horiz.asymp of f^{-1} , $\frac{-3(x-2)+5}{x-2}$, $x \rightarrow \infty$

$y = -3$ (must be an equation)

A1 N2**[2 marks]**

10 - (MAT/10_SL_Winter_2019_Q10) - Functions - Roots, Differentiation

(a) $B(a, 0)$ (accept $B(q+1, 0)$)**A2 N2**
[2 marks]

(b)

Note: There are many approaches to this part, and the steps may be done in any order. Please check working and award marks in line with the markscheme, noting that candidates may work with the equation of the line before finding a .

FINDING a valid attempt to find an expression for a in terms of q **(M1)**

$$g(0) = a, p^0 + q = a$$

$$a = q + 1$$

(A1)**FINDING THE EQUATION OF L_1** **EITHER**

attempt to substitute tangent gradient and coordinates into equation of straight line

(M1)

$$\text{eg } y - 0 = f'(a)(x - a), y = f'(a)(x - (q + 1))$$

correct equation in terms of a and p **(A1)**

$$\text{eg } y - 0 = \frac{1}{\ln(p)}(x - a)$$

ORattempt to substitute tangent gradient and coordinates to find b **(M1)**

$$\text{eg } 0 = \frac{1}{\ln(p)}(a) + b$$

$$b = \frac{-a}{\ln(p)}$$

(A1)**THEN** (must be in terms of **both** p and q)

$$y = \frac{1}{\ln p}(x - q - 1), y = \frac{1}{\ln p}x - \frac{q+1}{\ln p}$$

A1 N3

Note: Award **A0** for final answers in the form $L_1 = \frac{1}{\ln p}(x - q - 1)$.

[5 marks]

(c)

Note: There are many approaches to this part, and the steps may be done in any order. Please check working and award marks in line with the markscheme, noting that candidates may find q in terms of p before finding a value for p .

FINDING p

valid approach to find the gradient of the tangent

(M1)

$$\text{eg } m_1 m_2 = -1, -\frac{1}{\frac{1}{\ln\left(\frac{1}{3}\right)}}, -\ln\left(\frac{1}{3}\right), -\frac{1}{\ln p} = \frac{1}{\ln\left(\frac{1}{3}\right)}$$

correct application of log rule (seen anywhere)

(A1)

$$\text{eg } \ln\left(\frac{1}{3}\right)^{-1}, -(\ln(1) - \ln(3))$$

correct equation (seen anywhere)

A1

$$\text{eg } \ln p = \ln 3, p = 3$$

FINDING q correct substitution of $(-2, -2)$ into L_2 equation**(A1)**

$$\text{eg } -2 = (\ln p)(-2) + q + 1$$

$$q = 2 \ln p - 3, q = 2 \ln 3 - 3 \text{ (seen anywhere)}$$

A1**FINDING L_1** correct substitution of **their** p and q into **their** L_1 **(A1)**

$$\text{eg } y = \frac{1}{\ln 3}(x - (2 \ln 3 - 3) - 1)$$

$$y = \frac{1}{\ln 3}(x - 2 \ln 3 + 2), y = \frac{1}{\ln 3}x - \frac{2 \ln 3 - 2}{\ln 3}$$

A1**N2**

Note: Award **A0** for final answers in the form $L_1 = \frac{1}{\ln 3}(x - 2 \ln 3 + 2)$.

[7 marks]

11 - (MathAA/11_SL_Summer_2021_Q1) - Algebra-functions - Roots

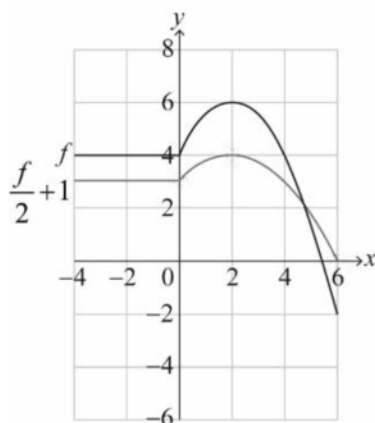
(a) (i) $f(2) = 6$

A1

(ii) $(f \circ f)(2) = -2$

A1**[2 marks]**

(b)

**M1A1A1**

Note: Award **M1** for an attempt to apply any vertical stretch or vertical translation, **A1** for a correct horizontal line segment between -4 and 0 (located roughly at $y = 3$), **A1** for a correct concave down parabola including max point at $(2, 4)$ and for correct end points at $(0, 3)$ and $(6, 0)$ (within circles). Points do not need to be labelled.

[3 marks]

12 - (MathAA/12_SL_Summer_2021_Q2) - Algebra-functions - Roots

attempt to subtract squares of integers

(M1)

$$(n+1)^2 - n^2$$

EITHERcorrect order of subtraction and correct expansion of $(n+1)^2$, seen anywhere**A1A1**

$$= n^2 + 2n + 1 - n^2 (= 2n + 1)$$

OR

correct order of subtraction and correct factorization of difference of squares

A1A1

$$= (n+1-n)(n+1+n) (= 2n+1)$$

THEN

$$= n + n + 1 = \text{RHS}$$

A1

Note: Do not award final **A1** unless all previous working is correct.

which is the sum of n and $n+1$ **AG**

Note: If expansion and order of subtraction are correct, award full marks for candidates who find the sum of the integers as $2n+1$ and then show that the difference of the squares (subtracted in the correct order) is $2n+1$.

[4 marks]**13** - (MathAA/10_SL_Winter_2021_Q3) - Graphs, Algebra-functions - Roots

(a) (i) $x = 3$

A1

(ii) $y = -2$

A1**[2 marks]**

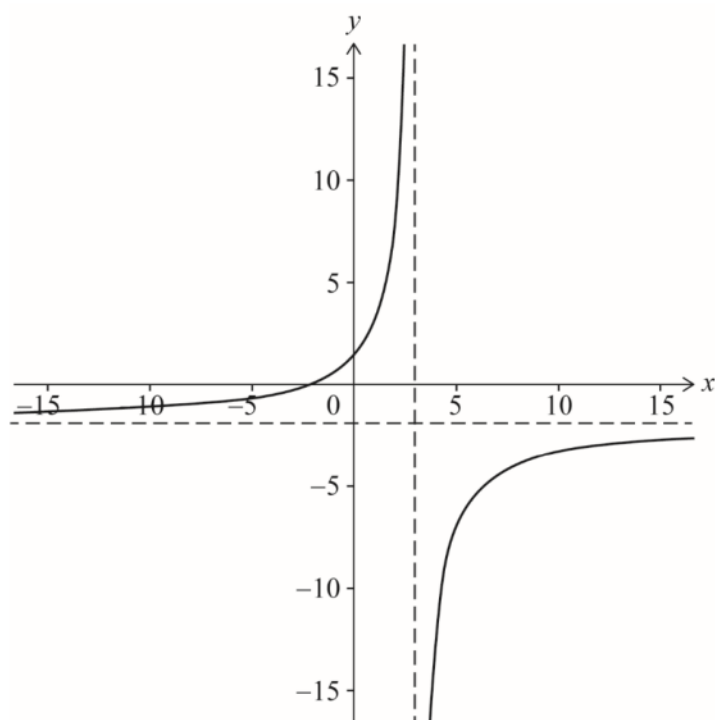
(b) (i) $(-2, 0)$ (accept $x = -2$)

A1

(ii) $\left(0, \frac{4}{3}\right)$ (accept $y = \frac{4}{3}$ and $f(0) = \frac{4}{3}$)

A1**[2 marks]**

(c)

**A1**

Note: Award **A1** for completely correct shape: two branches in correct quadrants with asymptotic behaviour.

[1 mark]**Total [5 marks]**

14 - (MathAA/11_SL_Summer_2022_Q4) - Algebra-functions - Roots, Trigonometry

$$(a) \quad (f \circ g)(x) = f(2x) \quad (A1)$$

$$f(2x) = \sqrt{3} \sin 2x + \cos 2x \quad A1$$

[2 marks]

$$(b) \quad \sqrt{3} \sin 2x + \cos 2x = 2 \cos 2x$$

$$\sqrt{3} \sin 2x = \cos 2x$$

recognising to use $\tan$ or $\cot$ M1

$$\tan 2x = \frac{1}{\sqrt{3}} \text{ OR } \cot 2x = \sqrt{3} \text{ (values may be seen in right triangle)} \quad (A1)$$

$$\left(\arctan\left(\frac{1}{\sqrt{3}}\right) = \right) \frac{\pi}{6} \text{ (seen anywhere) (accept degrees)} \quad (A1)$$

$$2x = \frac{\pi}{6}, \frac{7\pi}{6}$$

$$x = \frac{\pi}{12}, \frac{7\pi}{12} \quad A1A1$$

Note: Do not award the final **A1** if any additional solutions are seen.
Award **A1A0** for correct answers in degrees.
Award **A0A0** for correct answers in degrees with additional values.

[5 marks]

15 - (MathAA/12_SL_Summer_2022_Q1) - Algebra-functions - Roots

$$(a) \quad g(0) = -2 \quad A1$$

[1 mark]

$$(b) \quad \text{evidence of using composite function} \quad (M1)$$

$$f(g(0)) \text{ OR } f(-2)$$

$$(f \circ g)(0) = 8 \quad A1$$

[2 marks]

$$(c) \quad x = 3 \quad A2$$

[2 marks]

16 - (MathAA/12_SL_Summer_2022_Q3) - Algebra-functions - Roots

$$(a) \quad (n-1) + n + (n+1) \quad (A1)$$

$$= 3n \quad A1$$

which is always divisible by 3 AG

[2 marks]

$$(b) \quad (n-1)^2 + n^2 + (n+1)^2 \quad (= n^2 - 2n + 1 + n^2 + n^2 + 2n + 1) \quad A1$$

attempts to expand either $(n-1)^2$ or $(n+1)^2$ (do not accept $n^2 - 1$ or $n^2 + 1$) (M1)

$$= 3n^2 + 2 \quad A1$$

demonstrating recognition that 2 is not divisible by 3 or $\frac{2}{3}$ seen after correct

expression divided by 3 R1

$3n^2$ is divisible by 3 and so $3n^2 + 2$ is never divisible by 3

OR the first term is divisible by 3, the second is not

$$\text{OR } 3\left(n^2 + \frac{2}{3}\right) \quad \text{OR } \frac{3n^2 + 2}{3} = n^2 + \frac{2}{3}$$

hence the sum of the squares is never divisible by 3 AG

[4 marks]

17 - (MathAA/10_SL_Winter_2022_Q1) - Algebra-functions - Roots

- (a) gradient of g is -2 (may be seen in function, do not accept $-2x + 3$) (A1)

$$g(x) = -2x \quad \text{A1}$$

[2 marks]

- (b) gradient is $\frac{1}{2}$ (may be seen in function) (A1)

attempt to substitute **their** gradient and $(-1, 2)$ into any form of equation for straight line (M1)

$$y - 2 = \frac{1}{2}(x + 1) \text{ OR } 2 = \frac{1}{2} \cdot (-1) + c$$

$$h(x) = \frac{1}{2}(x + 1) + 2 \left(= \frac{1}{2}x + \frac{5}{2} \right) \quad \text{A1}$$

[3 marks]

- (c) $(g \circ h)(x) = -2\left(\frac{1}{2}x + \frac{5}{2}\right)$ OR $h(0) = \frac{5}{2}$ OR $g\left(\frac{5}{2}\right)$ (A1)

$$(g \circ h)(0) = -5 \quad \text{A1}$$

[2 marks]

18 - (MathAA/10_SL_Winter_2022_Q4) - Algebra-functions - Roots

(a)

Note: Award a maximum of **M1A0A0** if the candidate manipulates both sides of the equation (such as moving terms from one side to the other).

METHOD 1 (working with LHS)

attempting to expand $(a^2 - 1)^2$ (do not accept $a^4 + 1$ or $a^4 - 1$) **(M1)**

$$\text{LHS} = a^2 + \frac{a^4 - 2a^2 + 1}{4} \text{ or } \frac{4a^2 + a^4 - 2a^2 + 1}{4} \quad \textbf{A1}$$

$$= \frac{a^4 + 2a^2 + 1}{4} \quad \textbf{A1}$$

$$= \left(\frac{a^2 + 1}{2} \right)^2 (= \text{RHS}) \quad \textbf{AG}$$

Note: Do not award the final **A1** if further working contradicts the AG.

METHOD 2 (working with RHS)

attempting to expand $(a^2 + 1)^2$ **(M1)**

$$\text{RHS} = \frac{a^4 + 2a^2 + 1}{4}$$

$$= \frac{4a^2 + a^4 - 2a^2 + 1}{4} \quad \textbf{A1}$$

$$= a^2 + \frac{a^4 - 2a^2 + 1}{4} \quad \textbf{A1}$$

$$= a^2 + \left(\frac{a^2 - 1}{2} \right)^2 (= \text{LHS}) \quad \textbf{AG}$$

Note: Do not award the final **A1** if further working contradicts the AG.

[3 marks]

- (b) recognise base and height as a and $\left(\frac{a^2-1}{2}\right)$ (may be seen in diagram) **(M1)**

correct substitution into triangle area formula **A1**

$$\text{Area} = \frac{a}{2} \left(\frac{a^2-1}{2} \right) \text{ (or equivalent) } \left(= \frac{a(a^2-1)}{4} = \frac{a^3-a}{4} \right)$$

[2 marks]

19 - (MathAA/10_SL_Winter_2022_Q7) - Algebra-functions - Roots

- (a) correct substitution of $h=3$ and $k=2$ into $f(x)$ **(A1)**

$$f(x) = a(x-3)^2 + 2$$

correct substitution of $(5,0)$ **(A1)**

$$0 = a(5-3)^2 + 2 \quad \left(a = -\frac{1}{2} \right)$$

Note: The first two A marks are independent.

$$f(x) = -\frac{1}{2}(x-3)^2 + 2 \quad \textbf{A1}$$

[3 marks]

- (b) (i) **METHOD 1**

correct substitution of $(1, 4)$ **(A1)**

$$p + (t-1) - p = 4$$

$$t = 5 \quad \textbf{A1}$$

substituting their value of t into $9p - 3(t-1) - p = 4$ **(M1)**

$$8p - 12 = 4$$

$$p = 2 \quad \textbf{A1}$$

METHOD 2

correct substitution of ONE of the coordinates $(-3, 4)$ or $(1, 4)$ **(A1)**

$$9p - 3(t - 1) - p = 4 \quad \text{OR} \quad p + (t - 1) - p = 4$$

valid attempt to solve their two equations **(M1)**

$$p = 2, t = 5 \quad \textbf{A1A1}$$

$$(g(x) = 2x^2 + 4x - 2)$$

(ii) attempt to find the x -coordinate of the vertex **(M1)**

$$x = \frac{-3+1}{2} (= -1) \quad \text{OR} \quad \frac{-4}{2 \times 2} \quad \text{OR} \quad 4x + 4 = 0 \quad \text{OR} \quad 2(x+1)^2 - 4$$

y -coordinate of the vertex $= -4$ **(A1)**

correct range **A1**

$$[-4, +\infty[\quad \text{OR} \quad y \geq -4 \quad \text{OR} \quad g \geq -4 \quad \text{OR} \quad [-4, \infty)$$

[7 marks]

(c) equating the two functions or equations **(M1)**

$$g(x) = j(x) \quad \text{OR} \quad px^2 + (t-1)x - p = -x + 3p$$

$$px^2 + tx - 4p = 0 \quad \textbf{(A1)}$$

attempt to find discriminant (do not accept only in quadratic formula) **(M1)**

$$\Delta = t^2 + 16p^2 \quad \textbf{A1}$$

$\Delta = t^2 + 16p^2 > 0$, because $t^2 \geq 0$ and $p^2 > 0$, therefore the sum will be positive **R1R1**

Note: Award **R1** for recognising that Δ is positive and **R1** for the reason.

There are two distinct points of intersection between the graphs of g and j . **AG**

[6 marks]

20 - (MathAA/11_SL_Summer_2023_Q2) - Algebra-functions - Roots

(a) recognizing $f(x) = 0$ (M1)

$$x = -1$$
 A1

[2 marks]

(b) (i) $x = 2$ (must be an equation with x) A1(ii) $y = \frac{7}{2}$ (must be an equation with y) A1

[2 marks]

(c) EITHER

interchanging x and y (M1)

$$2xy - 4x = 7y + 7$$

correct working with y terms on the same side: $2xy - 7y = 4x + 7$ (A1)

OR

$$2yx - 4y = 7x + 7$$

correct working with x terms on the same side: $2yx - 7x = 4y + 7$ (A1)interchanging x and y OR making x the subject $x = \frac{4y+7}{2y-7}$ (M1)

THEN

$$f^{-1}(x) = \frac{4x+7}{2x-7} \text{ (or equivalent) } \left(x \neq \frac{7}{2} \right)$$
 A1

[3 marks]

21 - (MathAA/12_SL_Summer_2023_Q3) - Algebra-functions - Roots

(a) (i) $x = 2$

A1

(ii) $y = 1$

A1

[2 marks]

(b) (i) $\left(0, \frac{3}{2}\right)$

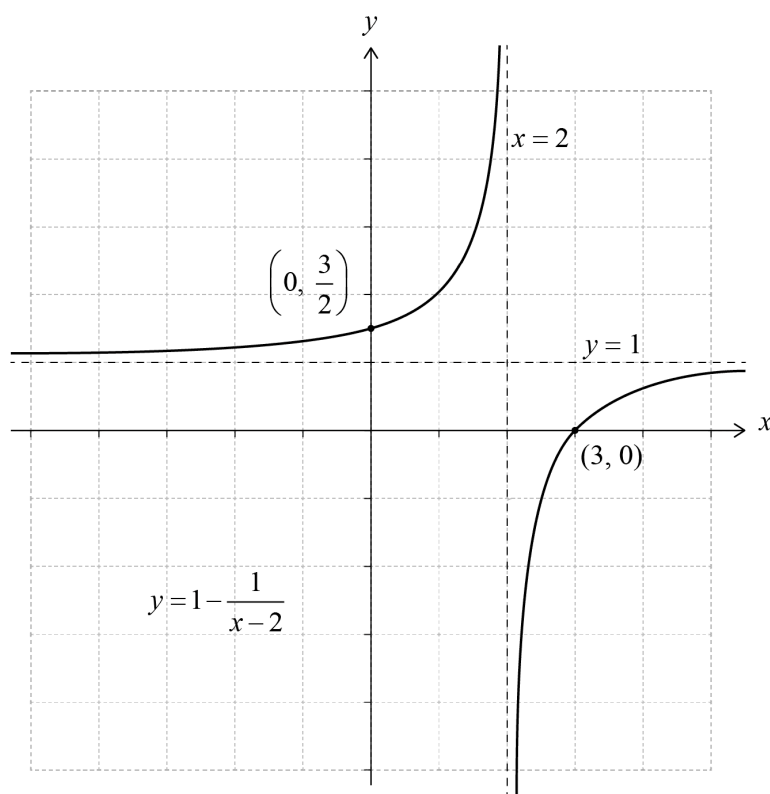
A1

(ii) $(3, 0)$

A1

[2 marks]

(c)



two correct branches with correct asymptotic behaviour and intercepts clearly shown

A1

[1 mark]

22 - (MathAA/12_SL_Summer_2023_Q6) - Algebra-functions - Roots

attempts to form $(g \circ f)(x)$ (M1)

$$[f(x)]^2 + f(x) + 3 \quad \text{OR} \quad (ax + b)^2 + ax + b + 3$$

$$a^2x^2 + 2abx + b^2 + ax + b + 3 (= 4x^2 - 14x + 15) \quad \text{(A1)}$$

equates their corresponding terms to form at least one equation (M1)

$$a^2x^2 = 4x^2 \quad \text{OR} \quad a^2 = 4 \quad \text{OR} \quad 2abx + ax = -14x \quad \text{OR} \quad 2ab + a = -14 \quad \text{OR} \quad b^2 + b + 3 = 15$$

$$a = \pm 2 \text{ (seen anywhere)} \quad \text{A1}$$

attempt to use $2ab + a = -14$ to pair the correct values (seen anywhere) (M1)

$$f(x) = 2x - 4 \text{ (accept } a = 2 \text{ with } b = -4), f(x) = -2x + 3 \text{ (accept } a = -2 \text{ with } b = 3) \quad \text{A1A1}$$

[7 marks]

23 - (MathAA/11_SL_Winter_2023_Q2) - Algebra-functions - Roots

(a) attempt to form $(g \circ f)(x)$ (M1)

$$((g \circ f)(x)) = (x + 2)^2 - k^2 \quad (= x^2 + 4x + 4 - k^2) \quad \text{A1}$$

[2 marks]

(b) substituting $x = 4$ into their $(g \circ f)(x)$ and setting their expression = 11 (M1)

$$(4 + 2)^2 - k^2 = 11 \quad \text{OR} \quad 4^2 + 4(4) + 4 - k^2 = 11$$

$$k^2 = 25 \quad \text{OR} \quad -k^2 = -25 \quad \text{(A1)}$$

$$k = \pm 5 \quad \text{A1}$$

[3 marks]

24 - (MathAA/12_SL_Winter_2023_Q2) - Algebra-functions - Roots

(a) attempt to form $(g \circ f)(x)$

(M1)

$$((g \circ f)(x)) = (x-3)^2 + k^2 \quad (= x^2 - 6x + 9 + k^2)$$

A1

[2 marks]

(b) substituting $x = 2$ into their $(g \circ f)(x)$ and setting their expression = 10

(M1)

$$(2-3)^2 + k^2 = 10 \quad \text{OR} \quad 2^2 - 6(2) + 9 + k^2 = 10$$

$$k^2 = 9$$

(A1)

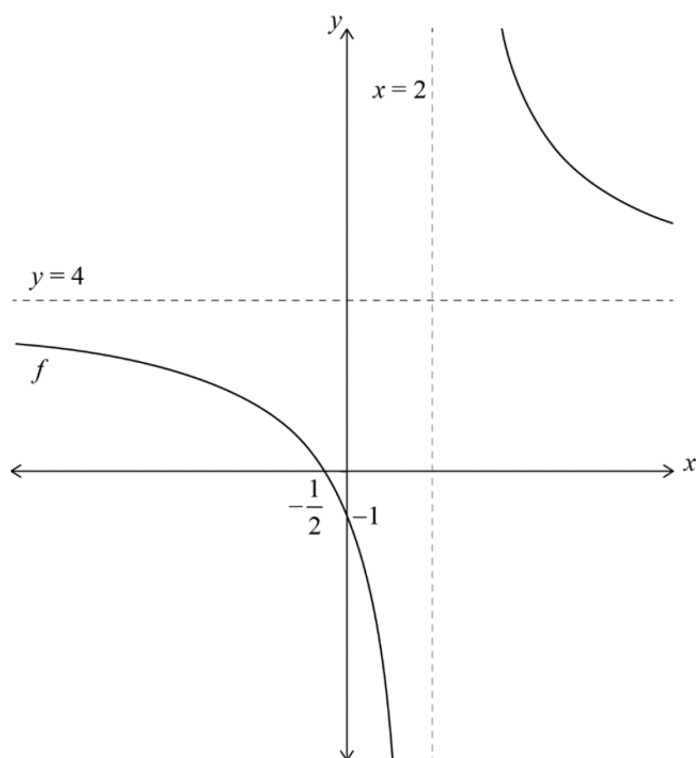
$$k = \pm 3$$

A1

[3 marks]

25 - (MathAA/11_SL_Summer_2024_Q8) - Algebra-functions - Roots, Graphs

(a)

vertical asymptote $x = 2$ sketched and labelled with correct equation**A1**horizontal asymptote $y = 4$ sketched and labelled with correct equation**A1**

For an approximate rational function shape:

labelled intercepts $-\frac{1}{2}$ on x -axis, -1 on y -axis**A1A1**

two branches in correct opposite quadrants with correct asymptotic behaviour

A1

Note: These marks may be awarded independently.

[5 marks](b) $y \neq 4$ (or equivalent)**A1****[1 mark]**

$$(c) \quad 2 + \frac{5}{2} \text{ OR } \left(-\frac{1}{2}\right) + 2 \times \frac{5}{2} \text{ OR } \frac{-\frac{1}{2} + p}{2} = 2 \text{ OR } -4 = -p + \frac{1}{2}$$

A1

$$p = \frac{9}{2}$$

AG**[1 mark]****(d) METHOD 1**

attempt to substitute both roots to form a quadratic

(M1)**EITHER**

$$\left(x + \frac{1}{2}\right)\left(x - \frac{9}{2}\right) \text{ OR } x^2 - \left(-\frac{1}{2} + \frac{9}{2}\right)x + \left(-\frac{1}{2} \times \frac{9}{2}\right)$$

$$= x^2 - 4x - \frac{9}{4}$$

A1A1

$$\left(b = -4, c = -\frac{9}{4}\right)$$

Note: Award **A1** for each correct value. They may be embedded or stated explicitly.

OR

$$(2x+1)(2x-9) = 4\left(x^2 - 4x - \frac{9}{4}\right)$$

$$b = -4, c = -\frac{9}{4}$$

A1A1

Note: Award **A1** for each correct value. They must be stated explicitly.

METHOD 2

$$-\frac{b}{2} = 2 \text{ OR } 4 + b = 0 \Rightarrow b = -4$$

A1attempt to form a valid equation to find c using their b **(M1)**

$$\left(-\frac{1}{2}\right)^2 + -4\left(-\frac{1}{2}\right) + c = 0 \text{ OR } \left(\frac{9}{2}\right)^2 + -4\left(\frac{9}{2}\right) + c = 0$$

$$c = -\frac{9}{4}$$

A1

METHOD 3attempt to form two valid equations in b and c **(M1)**

$$\left(-\frac{1}{2}\right)^2 + b\left(-\frac{1}{2}\right) + c = 0, \left(\frac{9}{2}\right)^2 + b\left(\frac{9}{2}\right) + c = 0$$

$$b = -4, c = -\frac{9}{4}$$

A1A1**METHOD 4**attempt to write $g(x)$ in the form $(x-h)^2 + k$ and substitute for x, h and $g(x)$ **(M1)**

$$\left(-\frac{1}{2} - 2\right)^2 + k = 0 \Rightarrow k = -\frac{25}{4}$$

$$(x-2)^2 - \frac{25}{4}$$

$$= x^2 - 4x - \frac{9}{4}$$

A1A1

$$\left(b = -4, c = -\frac{9}{4}\right)$$

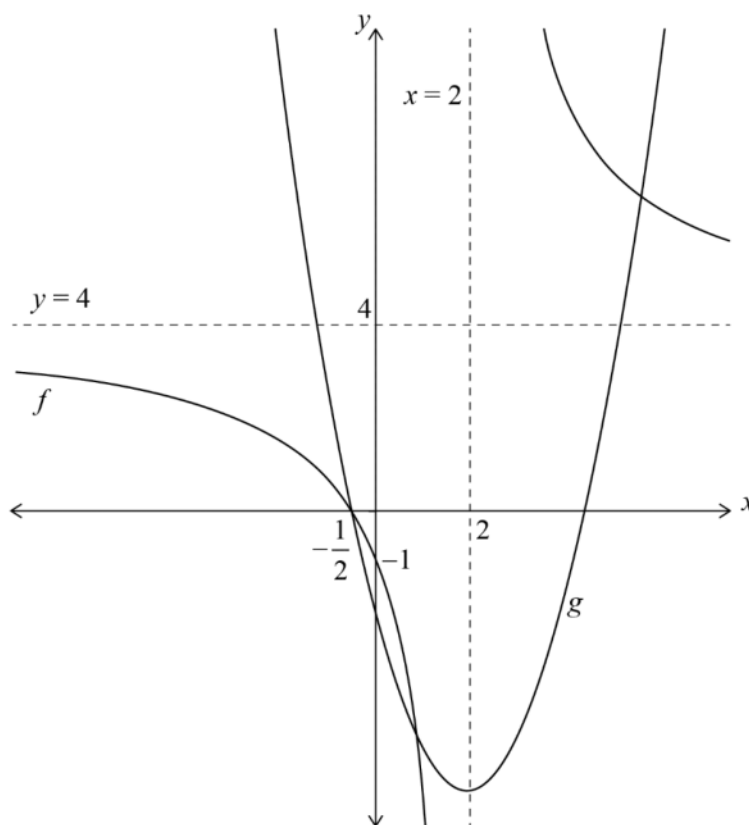
Note: Award **A1** for each correct value. They may be embedded or stated explicitly.

[3 marks](e) attempt to substitute $x = 2$ into their $g(x)$ ORcomplete the square on their $g(x)$ (may be seen in part (d))**(M1)**

$$y = -\frac{25}{4}$$

A1**[2 marks]**

(f)



both graphs sketched on same axes and identifying points of intersection

(M1)

3 solutions

A1

Note: Exception to **FT**: If the candidate's graph in part (a) is incorrect, the **M1** may be awarded for a sketch of their graph from part (a) and $g(x)$. Do not award the final **A1** in this case.

[2 marks]**Total [14 marks]**

26 - (MathAA/12_SL_Summer_2024_Q1) - Algebra-functions - Roots, Graphs

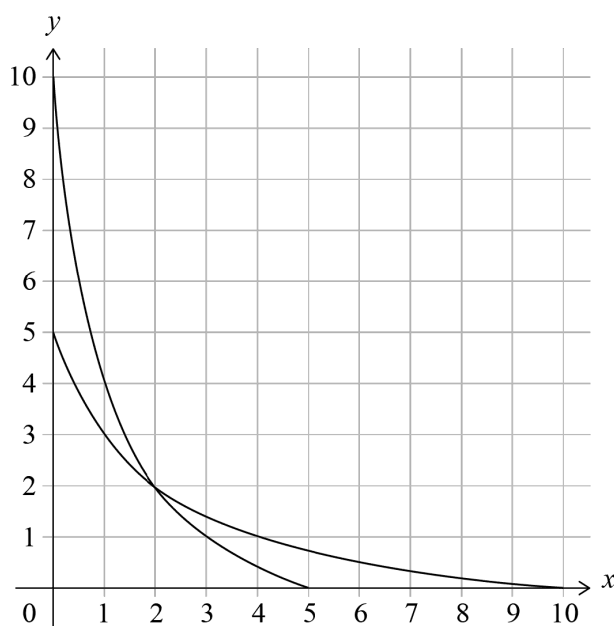
(a) (i) $f(4) = 1$ A1

(ii) $f \circ f(4) = 3$ A1

(iii) $f^{-1}(3) = 1$ A1

[3 marks]

(b)



concave up curve with y intercept at $(0, 10)$ and x intercept at $(5, 0)$ A1

curve passes through $(2, 2)$ OR through $(1, 4)$ and $(3, 1)$ A1

Note: Do not award the second mark unless the first mark has been awarded. (Do not award **A0A1**).

[2 marks]

Total [5 marks]

27 - (MathAA/12_SL_Summer_2024_Q6) - Algebra-functions - Roots, Graphs

(a) $y = \frac{2}{3}$ (must be written as equation with $y = ,$)

A1**[1 mark]**

(b) (i) 2

A1

(ii) **EITHER**

$$\frac{2(x+3)}{3(x+2)} = mx + 1$$

attempt to expand to obtain a quadratic equation

(M1)

$$2x + 6 = 3mx^2 + 6mx + 3x + 6$$

$$3mx^2 + (6m+1)x = 0 \quad \text{OR} \quad 3mx^2 + 6mx + x = 0$$

A1

recognition that discriminant $\Delta = 0$ for one solution

(M1)

$$(6m+1)^2 = 0$$

OR

$$\frac{2(x+3)}{3(x+2)} = mx + 1$$

attempt to expand to obtain a quadratic equation

(M1)

$$2x + 6 = 3mx^2 + 6mx + 3x + 6$$

$$3mx^2 + (6m+1)x = 0 \quad \text{OR} \quad 3mx^2 + 6mx + x = 0$$

A1attempt to solve their quadratic for x and equating their solutions**(M1)**

$$x(3mx + 6m + 1) = 0$$

$$x = 0 \quad \text{OR} \quad x = -\frac{6m+1}{3m} (= 0)$$

$$-\frac{6m+1}{3m} = 0$$

ORattempt to find $f'(x)$ using the quotient rule**(M1)**

$$f'(x) = \frac{2}{3} \left(\frac{(x+2) - (x+3)}{(x+2)^2} \right) = \left(\frac{-2}{3(x+2)^2} \right) \quad \text{OR} \quad \frac{2(3x+6) - 3(2x+6)}{(3x+6)^2} \quad \text{or}$$

equivalent

A1recognition that m is the derivative of $f(x)$ at $x = 0$ **(M1)****THEN**

$$\Rightarrow m = -\frac{1}{6}$$

A1

(iii)

Note: In this part, FT may be awarded only for values of m between -1 and 0 .

$$-\frac{1}{6} < m < 0$$

A2

Note: Award **A1** for only $m > -\frac{1}{6}$. Award **A1** for only $m < 0$.

[7 marks]**Total [8 marks]**

28 - (MathAA/11_SL_Winter_2024_Q4) - Algebra-functions - Roots

METHOD 1attempt to expand $(2n+5)^2 - (2n-5)^2$ **M1****Note:** Award **M0** for invalid attempts such as $(2n+5)^2 = 4n^2 + 25$. $= 4n^2 + 20n + 25 - (4n^2 - 20n + 25)$ or equivalent **A1** $= 40n$ OR $20n + 20n$ **A1** $= 10(4n)$ OR $\frac{40}{10} = 4$ OR $\frac{40n}{4} = 10n$ OR $20n + 20n = 10(2n + 2n)$ (or equivalent) **R1**so is a multiple of 10 **AG****Note:** Do not award the **R1** unless both **A** marks have been awarded.**METHOD 2**use of $a^2 - b^2 = (a+b)(a-b)$ where $a = 2n+5$, $b = 2n-5$ **M1** $= (2n+5+2n-5)(2n+5-2n+5)$ **A1A1****Note:** Award **A1** for each correct bracket. $= 4n \times 10 (= 40n)$ **R1**so is a multiple of 10 **AG****Note:** Do not award the **R1** unless both **A** marks have been awarded.**[4 marks]**

29 - (MathAA/11_SL_Winter_2024_Q8) - Algebra-functions - Roots

(a) (i) substitution of $x = 2$ in part (i) or $x = \frac{1}{8}$ in part (ii) (M1)

$$\log_2(8 \times 2) \text{ OR } \log_2(16)$$

$$\log_2(16) = 4 \quad \text{A1}$$

(ii) $f\left(\frac{1}{8}\right) = 0 \quad \text{A1}$

[3 marks]

(b) swap x and y (M1)

$$x = \log_2(8y) \text{ OR } x = 3 + \log_2 y$$

attempt to write as exponential (M1)

$$2^x = 8y \quad \text{(A1)}$$

$$\left(f^{-1}(x) = \frac{2^x}{8} \left(= 2^{x-3}\right) \left(\text{accept } y = \frac{2^x}{8} \text{ or } y = 2^{x-3}\right) \right) \quad \text{A1}$$

[4 marks]

(c) $\frac{1}{8} \quad \text{A1}$

[1 mark]

(d) **METHOD 1**

$$f(4x^2) = \log_2(8 \times 4x^2)$$

attempt to use addition rule for logs (M1)

$$\log_2 8 + \log_2 4 + \log_2 x^2 \text{ OR } \log_2 32 + \log_2 x^2 \text{ (or equivalent)} \quad (A1)$$

attempt to use exponent property for logarithms (M1)

$$f(4x^2) = 5 + 2 \log_2 x \text{ (or equivalent)} \quad A1$$

the graph of g must be vertically stretched (dilated) by a scale factor of 2 and then vertically translated (shifted) 5 units upwards. A2

METHOD 2

$$f(4x^2) = \log_2(8 \times 4x^2)$$

attempt to write argument as a power (M1)

$$\log_2(32x^2) = \log_2\left((\sqrt{32}x)^2\right) \text{ (or equivalent)} \quad (A1)$$

attempt to use exponent property for logarithms (M1)

$$f(4x^2) = 2 \log_2(\sqrt{32}x) \text{ (or equivalent)} \quad A1$$

EITHER

the graph of g must be vertically stretched (dilated) by a scale factor of 2 and stretched (dilated) horizontally by a scale factor of $\frac{1}{\sqrt{32}}$. A1

OR

the graph of g must be stretched (dilated) horizontally by a scale factor of $\frac{1}{\sqrt{32}}$ and vertically stretched (dilated) by a scale factor of 2. A1

Note: In this method, the final mark is **A1**, as the question specifically asks for a translation and a stretch.

[6 marks]

[Total: 14 marks]

30 - (MathAA/12_SL_Winter_2024_Q4) - Algebra-functions - Roots

METHOD 1attempt to expand $(3n+2)^2 - (3n-2)^2$ **M1****Note:** Award **M0** for invalid attempts such as $(3n+2)^2 = 9n^2 + 4$.

$$= 9n^2 + 12n + 4 - (9n^2 - 12n + 4) \text{ or equivalent}$$

A1

$$= 24n \text{ OR } 12n + 12n$$

A1

$$= 12(2n) \text{ OR } \frac{24}{12} = 2 \text{ OR } \frac{24n}{2} = 12n \text{ OR } 12n + 12n = 12(n+n) \text{ (or equivalent)}$$

R1

so is a multiple of 12

AG**Note:** Do not award the **R1** unless both **A** marks have been awarded.**METHOD 2**use of $a^2 - b^2 = (a+b)(a-b)$ where $a = 3n+2$, $b = 3n-2$ **M1**

$$= (3n+2+3n-2)(3n+2-3n+2)$$

$$= 6n \times 4$$

A1

$$= 24n$$

A1

$$= 12(2n) \text{ OR } \frac{24n}{12} = 2n \text{ OR } \frac{24n}{2} = 12n \text{ OR } 12n + 12n = 12(n+n) \text{ (or equivalent)}$$

R1

so is a multiple of 12

AG**Note:** Do not award the **R1** unless both **A** marks have been awarded.**[4 marks]**

31 - (MathAA/12_SL_Winter_2024_Q8) - Algebra-functions - Roots

(a) (i) substitution of $x = 8$ in part (i) or $x = \frac{1}{4}$ in part (ii) **(M1)**

$$\log_2(4 \times 8) \text{ OR } \log_2(32)$$

$$\log_2(32) = 5 \quad \mathbf{A1}$$

(ii) $f\left(\frac{1}{4}\right) = 0 \quad \mathbf{A1}$

[3 marks]

(b) swap x and y **(M1)**

$$x = \log_2(4y) \text{ OR } x = 2 + \log_2 y$$

attempt to write as exponential **(M1)**

$$2^x = 4y \quad \mathbf{(A1)}$$

$$\left(f^{-1}(x) = \frac{2^x}{4} \right) (= 2^{x-2}) \quad \mathbf{A1}$$

[4 marks]

(c) $\frac{1}{4} \quad \mathbf{A1}$

[1 mark]

(d) **METHOD 1**

$$f(16x^3) = \log_2(4 \times 16x^3)$$

attempt to use addition rule for logs **(M1)**

$$\log_2 4 + \log_2 16 + \log_2 x^3 \text{ OR } \log_2 64 + \log_2 x^3 \text{ (or equivalent)} \quad \textbf{(A1)}$$

attempt to use exponent property for logarithms **(M1)**

$$f(16x^3) = 6 + 3 \log_2 x \text{ (or equivalent)} \quad \textbf{A1}$$

the graph of g must be vertically stretched (dilated) by a scale factor of 3 and then vertically translated (shifted) 6 units upwards. **A2**

METHOD 2 (5 marks maximum)

$$f(16x^3) = \log_2(4 \times 16x^3)$$

attempt to write argument as a power **(M1)**

$$\log_2(64x^3) = \log_2((4x)^3) \text{ (or equivalent)} \quad \textbf{(A1)}$$

attempt to use exponent property for logarithms **(M1)**

$$f(16x^3) = 3 \log_2(4x) \text{ (or equivalent)} \quad \textbf{A1}$$

EITHER

the graph of g must be vertically stretched (dilated) by a scale factor of 3 and stretched (dilated) horizontally by a scale factor of $\frac{1}{4}$. **A1**

OR

the graph of g must be stretched (dilated) horizontally by a scale factor of $\frac{1}{4}$ and vertically stretched (dilated) by a scale factor of 3. **A1**

Note: In this method, the final mark is **A1**, as the question specifically asks for a translation and a stretch.

[6 marks]

[Total: 14 marks]

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