

IB Diploma

MATHEMATICS AA

HL P1
2017 — 2025

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1 - (MAT/12_HL_Summer_2017_Q2) - Functions - Roots

The function f is defined by $f(x) = 2x^3 + 5$, $-2 \leq x \leq 2$.

- (a) Write down the range of f . [2]
- (b) Find an expression for $f^{-1}(x)$. [2]
- (c) Write down the domain and range of f^{-1} . [2]

2 - (MAT/12_HL_Summer_2017_Q9) - Functions - Roots, Differentiation, Integration

Consider the function f defined by $f(x) = x^2 - a^2$, $x \in \mathbb{R}$ where a is a positive constant.

- (a) Showing any x and y intercepts, any maximum or minimum points and any asymptotes, sketch the following curves on separate axes.

(i) $y = f(x)$;

(ii) $y = \frac{1}{f(x)}$;

(iii) $y = \left| \frac{1}{f(x)} \right|$. [8]

- (b) Find $\int f(x) \cos x \, dx$. [5]

The function g is defined by $g(x) = x\sqrt{f(x)}$ for $|x| > a$.

- (c) By finding $g'(x)$ explain why g is an increasing function. [4]

3 - (MAT/11_HL_Summer_2017_Q11) - Functions - Roots, Graphs, Integration

- (a) (i) Express $x^2 + 3x + 2$ in the form $(x + h)^2 + k$.
- (ii) Factorize $x^2 + 3x + 2$. [2]

Consider the function $f(x) = \frac{1}{x^2 + 3x + 2}$, $x \in \mathbb{R}$, $x \neq -2$, $x \neq -1$.

- (b) Sketch the graph of $f(x)$, indicating on it the equations of the asymptotes, the coordinates of the y -intercept and the local maximum. [5]
- (c) Show that $\frac{1}{x+1} - \frac{1}{x+2} = \frac{1}{x^2 + 3x + 2}$. [1]
- (d) Hence find the value of p if $\int_0^1 f(x) dx = \ln(p)$. [4]
- (e) Sketch the graph of $y = f(|x|)$. [2]
- (f) Determine the area of the region enclosed between the graph of $y = f(|x|)$, the x -axis and the lines with equations $x = -1$ and $x = 1$. [3]

4 - (MAT/11_HL_Summer_2017_Q12) - Functions - Roots, Graphs

Consider the polynomial $P(z) = z^5 - 10z^2 + 15z - 6$, $z \in \mathbb{C}$.

- (a) Write down the sum and the product of the roots of $P(z) = 0$. [2]
- (b) Show that $(z - 1)$ is a factor of $P(z)$. [2]

The polynomial can be written in the form $P(z) = (z - 1)^3(z^2 + bz + c)$.

- (c) Find the value of b and the value of c . [5]
- (d) Hence find the complex roots of $P(z) = 0$. [3]

Consider the function $q(x) = x^5 - 10x^2 + 15x - 6$, $x \in \mathbb{R}$.

- (e) (i) Show that the graph of $y = q(x)$ is concave up for $x > 1$.
- (ii) Sketch the graph of $y = q(x)$ showing clearly any intercepts with the axes. [6]

5 - (MAT/12_HL_Summer_2018_Q10) - Functions - Roots

The function f is defined by $f(x) = \frac{ax + b}{cx + d}$, for $x \in \mathbb{R}$, $x \neq -\frac{d}{c}$.

(a) Find the inverse function f^{-1} , stating its domain. [5]

The function g is defined by $g(x) = \frac{2x - 3}{x - 2}$, $x \in \mathbb{R}$, $x \neq 2$.

(b) (i) Express $g(x)$ in the form $A + \frac{B}{x - 2}$ where A, B are constants.

(ii) Sketch the graph of $y = g(x)$. State the equations of any asymptotes and the coordinates of any intercepts with the axes. [5]

The function h is defined by $h(x) = \sqrt{x}$, for $x \geq 0$.

(c) State the domain and range of $h \circ g$. [4]

6 - (MAT/12_HL_Summer_2019_Q3) - Functions - Roots

[Maximum mark: 5]

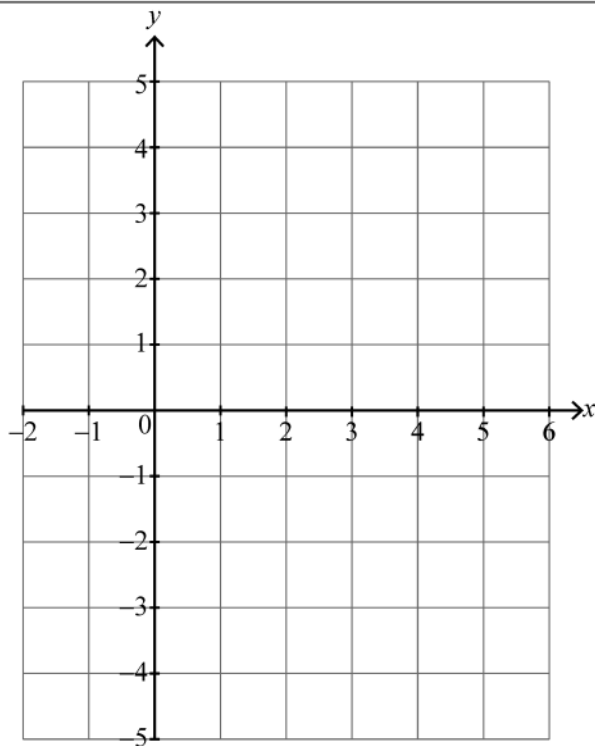
Consider the function $f(x) = x^4 - 6x^2 - 2x + 4$, $x \in \mathbb{R}$.

The graph of f is translated two units to the left to form the function $g(x)$.

Express $g(x)$ in the form $ax^4 + bx^3 + cx^2 + dx + e$ where $a, b, c, d, e \in \mathbb{Z}$.

7 - (MAT/12_HL_Summer_2019_Q5) - Graphs, Functions - Roots

- (a) Sketch the graph of $y = \frac{x-4}{2x-5}$, stating the equations of any asymptotes and the coordinates of any points of intersection with the axes. [5]



- (b) Consider the function $f: x \rightarrow \sqrt{\frac{x-4}{2x-5}}$.

Write down

- (i) the largest possible domain of f ;
(ii) the corresponding range of f . [3]

8 - (MathAA/12_HL_Summer_2021_Q1) - Algebra-functions - Roots

[Maximum mark: 4]

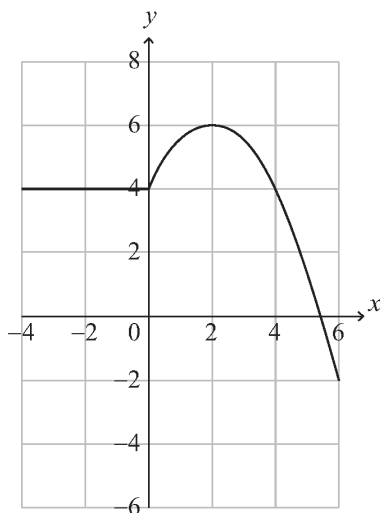
Consider two consecutive positive integers, n and $n + 1$.

Show that the difference of their squares is equal to the sum of the two integers.

9 - (MathAA/11_HL_Summer_2021_Q1) - Algebra-functions - Roots

[Maximum mark: 5]

The graph of $y = f(x)$ for $-4 \leq x \leq 6$ is shown in the following diagram.



(a) Write down the value of

(i) $f(2)$;

(ii) $(f \circ f)(2)$.

[2]

(b) Let $g(x) = \frac{1}{2}f(x) + 1$ for $-4 \leq x \leq 6$. On the axes above, sketch the graph of g .

[3]

10 - (MathAA/12_HL_Summer_2021_Q7) - Algebra-functions - Roots

[Maximum mark: 5]

The cubic equation $x^3 - kx^2 + 3k = 0$ where $k > 0$ has roots α , β and $\alpha + \beta$.

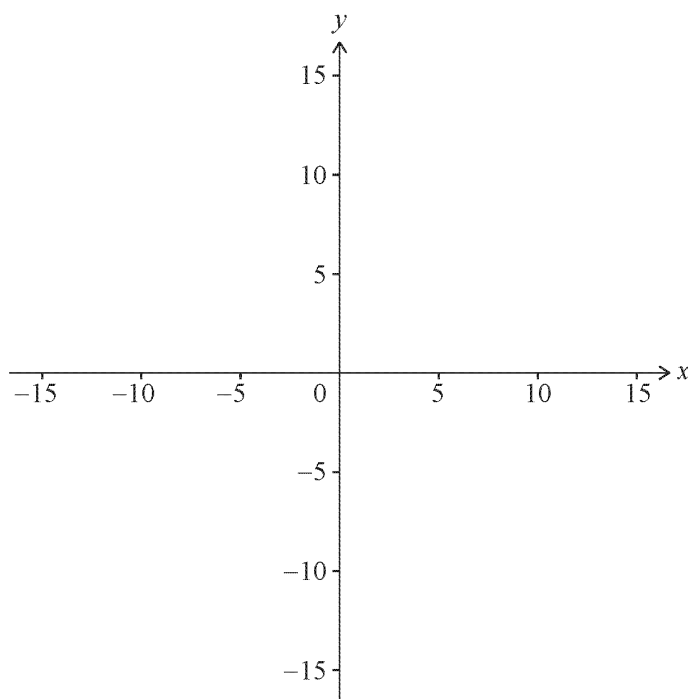
Given that $\alpha\beta = -\frac{k^2}{4}$, find the value of k .

11 - (MathAA/10_HL_Winter_2021_Q2) - Algebra-functions - Roots, Graphs

[Maximum mark: 9]

The function f is defined by $f(x) = \frac{2x+4}{3-x}$, where $x \in \mathbb{R}$, $x \neq 3$.

- (a) Write down the equation of
- (i) the vertical asymptote of the graph of f ;
 - (ii) the horizontal asymptote of the graph of f . [2]
- (b) Find the coordinates where the graph of f crosses
- (i) the x -axis;
 - (ii) the y -axis. [2]
- (c) Sketch the graph of f on the axes below. [1]



The function g is defined by $g(x) = \frac{ax+4}{3-x}$, where $x \in \mathbb{R}$, $x \neq 3$ and $a \in \mathbb{R}$.

- (d) Given that $g(x) = g^{-1}(x)$, determine the value of a . [4]

12 - (MathAA/12_HL_Summer_2022_Q11) - Algebra-functions - Roots

[Maximum mark: 20]

A function f is defined by $f(x) = \frac{1}{x^2 - 2x - 3}$, where $x \in \mathbb{R}$, $x \neq -1$, $x \neq 3$.

- (a) Sketch the curve $y = f(x)$, clearly indicating any asymptotes with their equations. State the coordinates of any local maximum or minimum points and any points of intersection with the coordinate axes. [6]

A function g is defined by $g(x) = \frac{1}{x^2 - 2x - 3}$, where $x \in \mathbb{R}$, $x > 3$.

- (b) The inverse of g is g^{-1} .
- (i) Show that $g^{-1}(x) = 1 + \frac{\sqrt{4x^2 + x}}{x}$.
- (ii) State the domain of g^{-1} . [7]

A function h is defined by $h(x) = \arctan \frac{x}{2}$, where $x \in \mathbb{R}$.

- (c) Given that $(h \circ g)(a) = \frac{\pi}{4}$, find the value of a .
- Give your answer in the form $p + \frac{q}{2}\sqrt{r}$, where $p, q, r \in \mathbb{Z}^+$. [7]

13 - (MathAA/12_HL_Summer_2022_Q9) - Algebra-functions - Roots

[Maximum mark: 5]

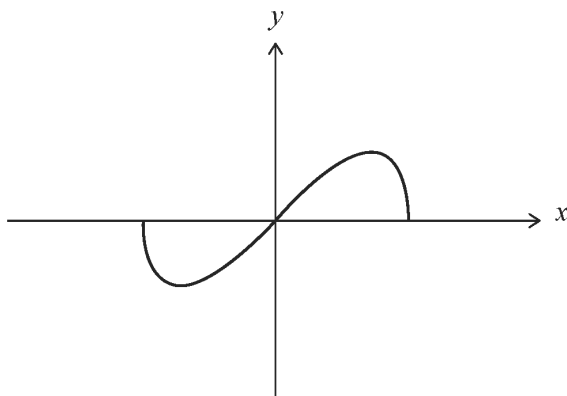
Prove by contradiction that the equation $2x^3 + 6x + 1 = 0$ has no integer roots.

14 - (MathAA/12_HL_Summer_2022_Q6) - Algebra-functions - Roots

[Maximum mark: 8]

A function f is defined by $f(x) = x\sqrt{1-x^2}$ where $-1 \leq x \leq 1$.

The graph of $y = f(x)$ is shown below.



- (a) Show that f is an odd function. [2]

The range of f is $a \leq y \leq b$, where $a, b \in \mathbb{R}$.

- (b) Find the value of a and the value of b . [6]

15 - (MathAA/12_HL_Summer_2022_Q2) - Algebra-functions - Roots

[Maximum mark: 6]

Consider any three consecutive integers, $n-1$, n and $n+1$.

- (a) Prove that the sum of these three integers is always divisible by 3. [2]
- (b) Prove that the sum of the squares of these three integers is never divisible by 3. [4]

16 - (MathAA/11_HL_Summer_2022_Q8) - Algebra-functions - Roots

[Maximum mark: 6]

Consider integers a and b such that $a^2 + b^2$ is exactly divisible by 4. Prove by contradiction that a and b cannot both be odd.

17 - (MathAA/11_HL_Summer_2022_Q3) - Algebra-functions - Roots

[Maximum mark: 7]

Consider the functions $f(x) = \sqrt{3} \sin x + \cos x$ where $0 \leq x \leq \pi$ and $g(x) = 2x$ where $x \in \mathbb{R}$.

- (a) Find $(f \circ g)(x)$. [2]
- (b) Solve the equation $(f \circ g)(x) = 2 \cos 2x$ where $0 \leq x \leq \pi$. [5]

18 - (MathAA/10_HL_Winter_2022_Q8) - *Trigonometry, Algebra-functions - Roots*

[Maximum mark: 5]

Let $f(x) = \cos(x - k)$, where $0 \leq x \leq a$ and $a, k \in \mathbb{R}^+$.

- (a) Consider the case where
- $k = \frac{\pi}{2}$
- .

By sketching a suitable graph, or otherwise, find the largest value of a for which the inverse function f^{-1} exists. [2]

- (b) Find the largest value of
- a
- for which the inverse function
- f^{-1}
- exists in the case where
- $k = \pi$
- . [1]

- (c) Find the largest value of
- a
- for which the inverse function
- f^{-1}
- exists in the case where
- $\pi < k < 2\pi$
- . Give your answer in terms of
- k
- . [2]

19 - (MathAA/10_HL_Winter_2022_Q3) - *Algebra-functions - Roots*

[Maximum mark: 5]

Let a be a constant, where $a > 1$.

- (a) Show that
- $a^2 + \left(\frac{a^2 - 1}{2}\right)^2 = \left(\frac{a^2 + 1}{2}\right)^2$
- . [3]

Consider a right-angled triangle with sides of length a , $\left(\frac{a^2 - 1}{2}\right)$ and $\left(\frac{a^2 + 1}{2}\right)$.

- (b) Find an expression for the area of the triangle in terms of
- a
- . [2]

20 - (MathAA/11_HL_Summer_2023_Q1) - *Algebra-functions - Roots*

[Maximum mark: 7]

The function f is defined by $f(x) = \frac{7x+7}{2x-4}$ for $x \in \mathbb{R}$, $x \neq 2$.

- (a) Find the zero of
- $f(x)$
- . [2]

- (b) For the graph of
- $y = f(x)$
- , write down the equation of

(i) the vertical asymptote;

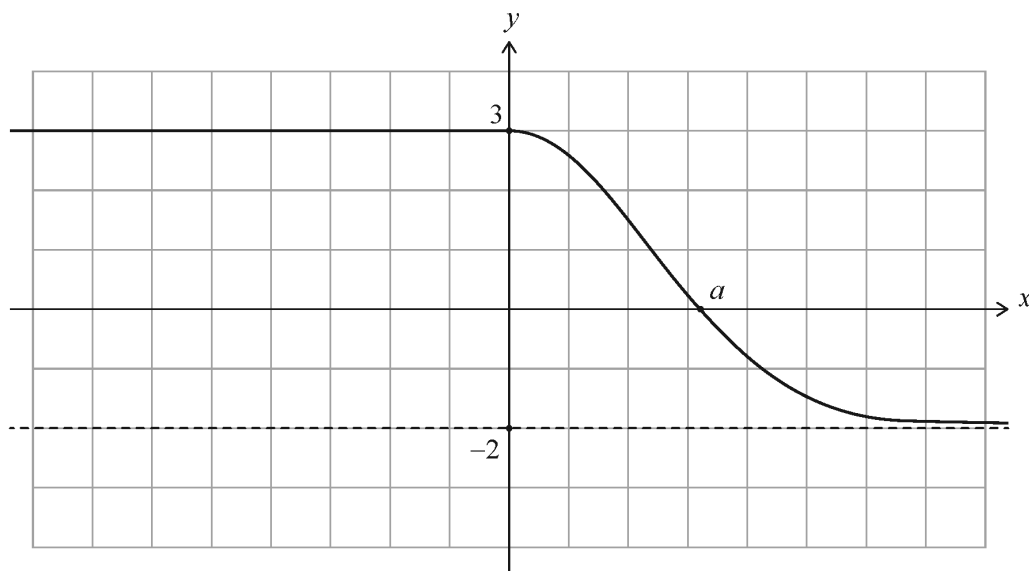
(ii) the horizontal asymptote. [2]

- (c) Find
- $f^{-1}(x)$
- , the inverse function of
- $f(x)$
- . [3]

21 - (MathAA/11_HL_Summer_2023_Q8) - Algebra-functions - Roots

[Maximum mark: 7]

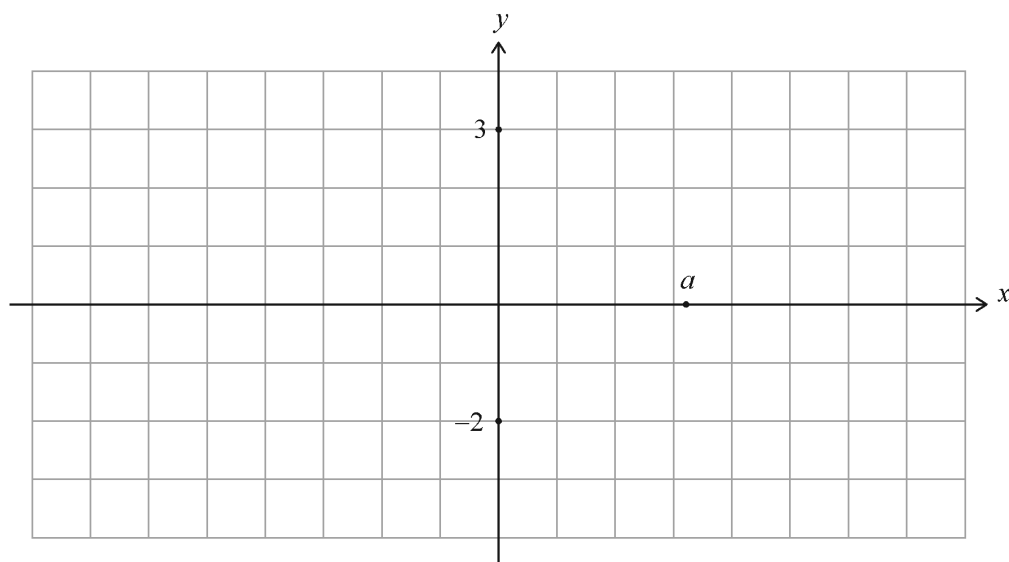
Part of the graph of a function, f , is shown in the following diagram. The graph of $y = f(x)$ has a y -intercept at $(0, 3)$, an x -intercept at $(a, 0)$ and a horizontal asymptote $y = -2$.



Consider the function $g(x) = |f(|x|)|$.

- (a) On the following grid, sketch the graph of $y = g(x)$, labelling any axis intercepts and giving the equation of the asymptote.

[4]



- (b) Find the possible values of k such that $(g(x))^2 = k$ has exactly two solutions.

[3]

22 - (MathAA/12_HL_Summer_2023_Q2) - Algebra-functions - Roots

[Maximum mark: 5]

A function f is defined by $f(x) = 1 - \frac{1}{x-2}$, where $x \in \mathbb{R}$, $x \neq 2$.

- (a) The graph of $y = f(x)$ has a vertical asymptote and a horizontal asymptote.

Write down the equation of

- (i) the vertical asymptote;
(ii) the horizontal asymptote.

[2]

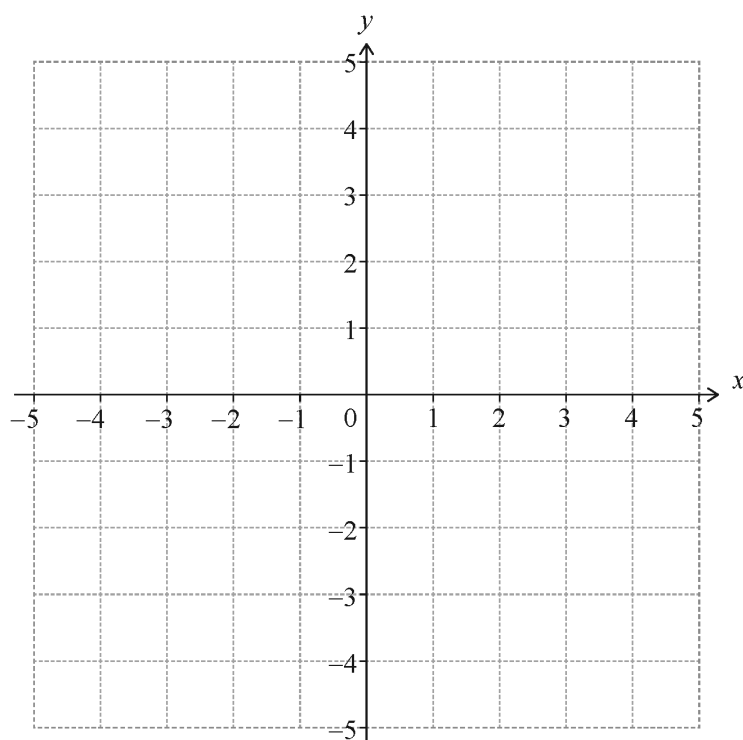
- (b) Find the coordinates of the point where the graph of $y = f(x)$ intersects

- (i) the y -axis;
(ii) the x -axis.

[2]

- (c) On the following set of axes, sketch the graph of $y = f(x)$, showing all the features found in parts (a) and (b).

[1]



23 - (MathAA/12_HL_Summer_2023_Q5) - Algebra-functions - Roots

[Maximum mark: 7]

The functions f and g are defined for $x \in \mathbb{R}$ by

$$f(x) = ax + b, \text{ where } a, b \in \mathbb{Z}$$

$$g(x) = x^2 + x + 3.$$

Find the two possible functions f such that $(g \circ f)(x) = 4x^2 - 14x + 15$.

24 - (MathAA/11_HL_Winter_2023_Q1) - Algebra-functions - Roots

[Maximum mark: 5]

Consider the functions $f(x) = x - 3$ and $g(x) = x^2 + k^2$, where k is a real constant.

(a) Write down an expression for $(g \circ f)(x)$. [2]

(b) Given that $(g \circ f)(2) = 10$, find the possible values of k . [3]

25 - (MathAA/11_HL_Winter_2023_Q9) - Algebra-functions - Roots

[Maximum mark: 8]

Consider the function $f(x) = \frac{\sin^2(kx)}{x^2}$, where $x \neq 0$ and $k \in \mathbb{R}^+$.

(a) Show that f is an even function. [2]

(b) Given that $\lim_{x \rightarrow 0} f(x) = 16$, find the value of k . [6]

26 - (MathAA/12_HL_Winter_2023_Q1) - Algebra-functions - Roots

[Maximum mark: 5]

Consider the functions $f(x) = x - 3$ and $g(x) = x^2 + k^2$, where k is a real constant.

(a) Write down an expression for $(g \circ f)(x)$. [2]

(b) Given that $(g \circ f)(2) = 10$, find the possible values of k . [3]

27 - (MathAA/12_HL_Winter_2023_Q9) - Algebra-functions - Roots

[Maximum mark: 8]

Consider the function $f(x) = \frac{\sin^2(kx)}{x^2}$, where $x \neq 0$ and $k \in \mathbb{R}^+$.

(a) Show that f is an even function. [2]

(b) Given that $\lim_{x \rightarrow 0} f(x) = 16$, find the value of k . [6]

28 - (MathAA/11_HL_Summer_2024_Q10) - Algebra-functions - Roots, Graphs

[Maximum mark: 16]

Consider the function $f(x) = \frac{4x+2}{x-2}$, $x \neq 2$.

- (a) Sketch the graph of $y = f(x)$. On your sketch, indicate the values of any axis intercepts and label any asymptotes with their equations. [5]
- (b) Write down the range of f . [1]

Consider the function $g(x) = x^2 + bx + c$. The graph of g has an axis of symmetry at $x = 2$.

The two roots of $g(x) = 0$ are $-\frac{1}{2}$ and p , where $p \in \mathbb{Q}$.

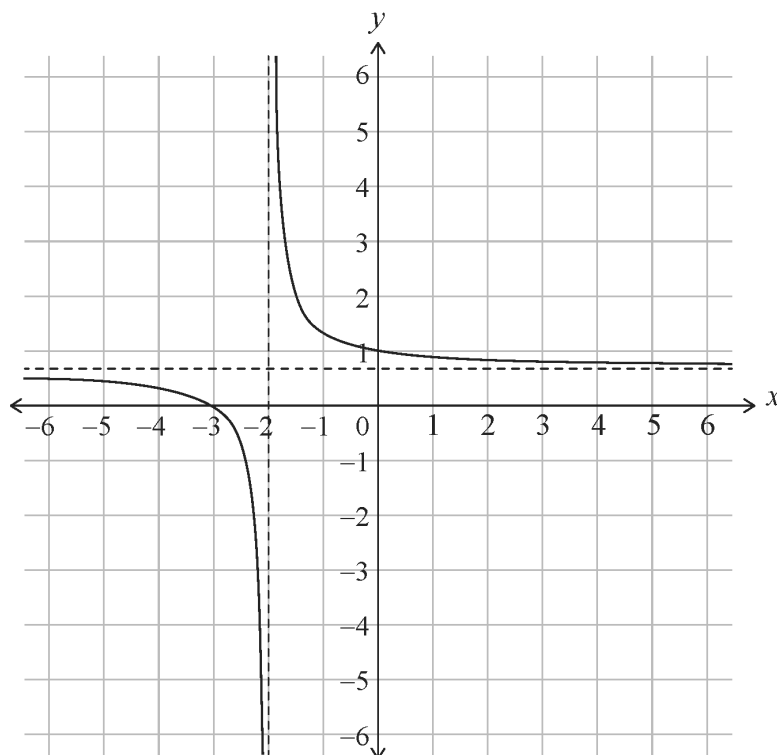
- (c) Show that $p = \frac{9}{2}$. [1]
- (d) Find the value of b and the value of c . [3]
- (e) Find the y -coordinate of the vertex of the graph of $y = g(x)$. [2]
- (f) Find the product of the solutions of the equation $f(x) = g(x)$. [4]

29 - (MathAA/12_HL_Summer_2024_Q5) - Algebra-functions - Roots, Graphs

[Maximum mark: 8]

A function f is defined by $f(x) = \frac{2(x+3)}{3(x+2)}$, where $x \in \mathbb{R}, x \neq -2$.

The graph $y = f(x)$ is shown below.



(a) Write down the equation of the horizontal asymptote.

[1]

Consider $g(x) = mx + 1$, where $m \in \mathbb{R}, m \neq 0$.

- (b) (i) Write down the number of solutions to $f(x) = g(x)$ for $m > 0$.
- (ii) Determine the value of m such that $f(x) = g(x)$ has only one solution for x .
- (iii) Determine the range of values for m , where $f(x) = g(x)$ has two solutions for $x \geq 0$.

[7]

30 - (MathAA/12_HL_Summer_2024_Q6) - Algebra-functions - Roots

[Maximum mark: 5]

A farmer grows two types of apples—cooking apples and eating apples. The weights of the apples, in grams, can be modelled as normal distributions with the following parameters.

Apple type	Mean μ	Standard deviation σ
Eating	100 g	20 g
Cooking	140 g	40 g

For each type of apple you can assume that 95 % of the weights are within two standard deviations of the mean.

- (a) Find the percentage of eating apples that have a weight greater than 140 g. [1]

The farmer grows a large number of apples of which 80 % are eating apples.

Both types of apples are picked and randomly mixed together in a cleaning machine.

After cleaning, the machine separates out those that have a weight greater than 140 g into a container.

- (b) An apple is randomly selected from this container. Find the probability that it is an eating apple. Give your answer in the form $\frac{c}{d}$, where $c, d \in \mathbb{Z}^+$. [4]

31 - (MathAA/12_HL_Summer_2024_Q7) - Algebra-functions - Roots

[Maximum mark: 7]

A function $g(x)$ is defined by $g(x) = 2x^3 - 7x^2 + dx - e$, where $d, e \in \mathbb{R}$.

α, β and γ are the three roots of the equation $g(x) = 0$ where $\alpha, \beta, \gamma \in \mathbb{R}$.

- (a) Write down the value of $\alpha + \beta + \gamma$. [1]

A function $h(z)$ is defined by $h(z) = 2z^5 - 11z^4 + rz^3 + sz^2 + tz - 20$, where $r, s, t \in \mathbb{R}$.

α, β and γ are also roots of the equation $h(z) = 0$.

It is given that $h(z) = 0$ is satisfied by the complex number $z = p + 3i$.

- (b) Show that $p = 1$. [3]

It is now given that $h\left(\frac{1}{2}\right) = 0$, and $\alpha, \beta \in \mathbb{Z}^+$, $\alpha < \beta$ and $\gamma \in \mathbb{Q}$.

- (c) (i) Find the value of the product $\alpha\beta$.
(ii) Write down the value of α and the value of β . [3]

32 - (MathAA/1_HL_Winter_2024_Q3) - Algebra-functions - Roots

[Maximum mark: 4]

Prove that $(3n + 2)^2 - (3n - 2)^2$ is a multiple of 12 for all $n \in \mathbb{Z}^+$.

33 - (MathAA/1_HL_Winter_2024_Q6) - Algebra-functions - Roots

[Maximum mark: 5]

(a) Solve $2x^2 - 15x + 18 < 0$. [3]

(b) The function f is defined by $f(x) = \sqrt{2x^2 - 15x + 18}$, where $x \in \mathbb{R}$, $x \leq k$.

Find the greatest value of k for which f^{-1} exists, justifying your answer. [2]

34 - (MathAA/11_HL_Summer_2025_Q1) - Differentiation, Algebra-functions - Roots, Graphs

[Maximum mark: 5]

Consider the function $f(x) = \frac{4x^3}{3} - 16x$, where $x \in \mathbb{R}$.

The graph of $y = f(x)$ has a local minimum point at (p, q) where $p > 0$.

Find the value of p and the value of q .

35 - (MathAA/11_HL_Summer_2025_Q10) - Quadratics, Algebra-functions - Roots, Exponentials - Logarithms

[Maximum mark: 18]

The function f is defined by $f(x) = 5(x + 1)(x + 3)$, where $x \in \mathbb{R}$.

(a) Write $f(x)$ in the form $a(x - h)^2 + k$, where $a, h, k \in \mathbb{Z}$. [4]

(b) Sketch the graph of $y = f(x)$, showing the values of any intercepts with the axes and the coordinates of the vertex. [4]

(c) Solve the inequality $f(x) \leq 40$. [4]

The function g is defined by $g(x) = \ln x$, where $x \in \mathbb{R}$, $x > 0$.

(d) (i) Write down an expression for $(f \circ g)(x)$.

(ii) Solve the inequality $(f \circ g)(x) \leq 40$. [3]

(e) Find the domain of $g \circ f$. [3]

ANSWERS

1 - (MAT/12_HL_Summer_2017_Q2) - Functions - Roots

(a) $-11 \leq f(x) \leq 21$

A1A1

Note: **A1** for correct end points, **A1** for correct inequalities.

[2 marks]

(b) $f^{-1}(x) = \sqrt[3]{\frac{x-5}{2}}$

(M1)A1

[2 marks]

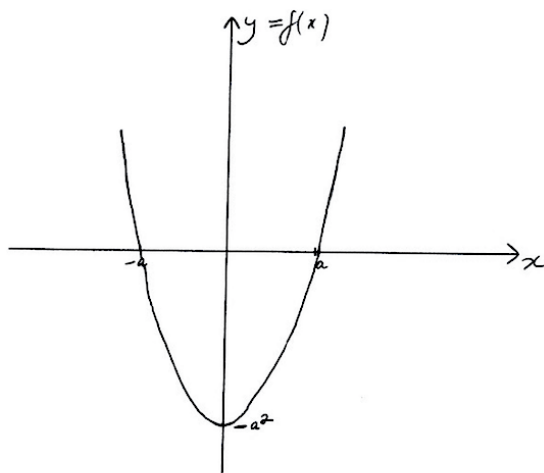
(c) $-11 \leq x \leq 21, -2 \leq f^{-1}(x) \leq 2$

A1A1

[2 marks]

2 - (MAT/12_HL_Summer_2017_Q9) - Functions - Roots, Differentiation, Integration

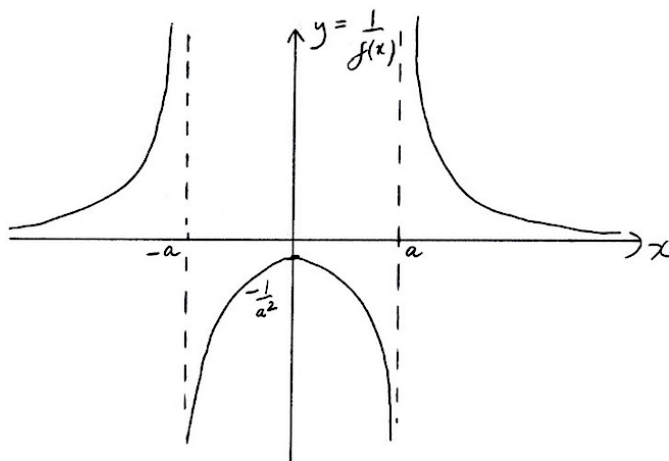
(a) (i)



A1 for correct shape

A1 for correct x and y intercepts and minimum point

(ii)



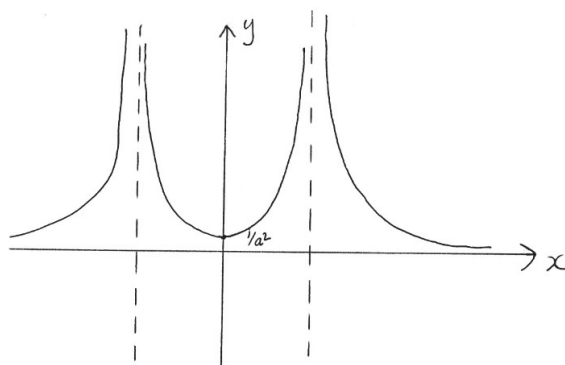
A1 for correct shape

A1 for correct vertical asymptotes

A1 for correct implied horizontal asymptote

A1 for correct maximum point

(iii)

**A1** for reflecting negative branch from (ii) in the x -axis**A1** for correctly labelled minimum point**[8 marks]**(b) **EITHER**

attempt at integration by parts

$$\begin{aligned}\int (x^2 - a^2) \cos x dx &= (x^2 - a^2) \sin x - \int 2x \sin x dx \\ &= (x^2 - a^2) \sin x - 2 \left[-x \cos x + \int \cos x dx \right] \\ &= (x^2 - a^2) \sin x + 2x \cos x - 2 \sin x + c\end{aligned}$$

(M1)**A1A1****A1****A1****OR**

$$\int (x^2 - a^2) \cos x dx = \int x^2 \cos x dx - \int a^2 \cos x dx$$

attempt at integration by parts

$$\int x^2 \cos x dx = x^2 \sin x - \int 2x \sin x dx$$

$$= x^2 \sin x - 2 \left[-x \cos x + \int \cos x dx \right]$$

$$= x^2 \sin x + 2x \cos x - 2 \sin x$$

$$- \int a^2 \cos x dx = -a^2 \sin x$$

$$\int (x^2 - a^2) \cos x dx = (x^2 - a^2) \sin x + 2x \cos x - 2 \sin x + c$$

(M1)**A1A1****A1****A1****[5 marks]**

(c) $g(x) = x(x^2 - a^2)^{\frac{1}{2}}$

$$g'(x) = (x^2 - a^2)^{\frac{1}{2}} + \frac{1}{2}x(x^2 - a^2)^{-\frac{1}{2}}(2x)$$

M1A1A1

Note: Method mark is for differentiating the product. Award **A1** for each correct term.

$$g'(x) = (x^2 - a^2)^{\frac{1}{2}} + x^2(x^2 - a^2)^{-\frac{1}{2}}$$

both parts of the expression are positive hence $g'(x)$ is positive

R1

and therefore g is an increasing function (for $|x| > a$)

AG**[4 marks]**

3 - (MAT/11_HL_Summer_2017_Q11) - Functions - Roots, Graphs, Integration

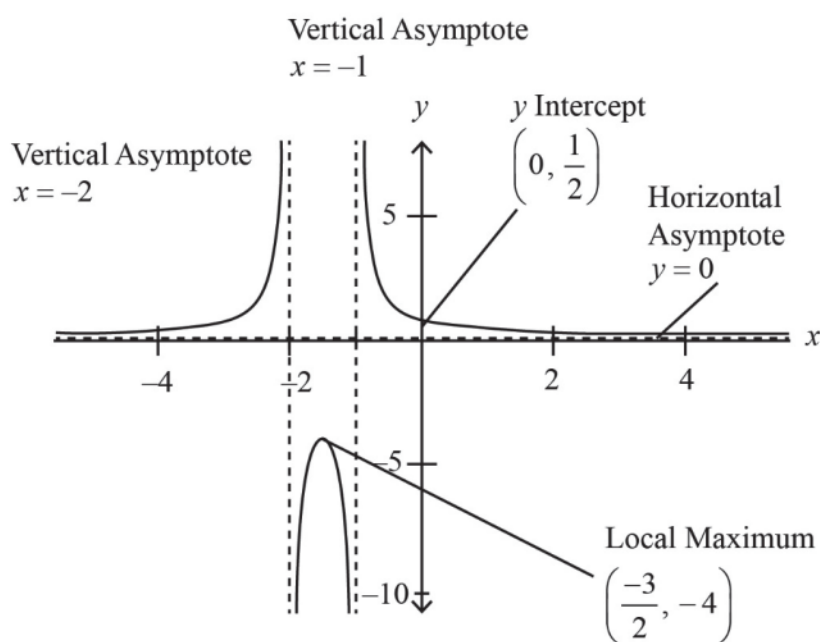
(a) (i) $x^2 + 3x + 2 = \left(x + \frac{3}{2}\right)^2 - \frac{1}{4}$

A1

(ii) $x^2 + 3x + 2 = (x + 2)(x + 1)$

A1**[2 marks]**

(b)

**A1** for the shape**A1** for the equation $y = 0$ **A1** for asymptotes $x = -2$ and $x = -1$ **A1** for coordinates $\left(-\frac{3}{2}, -4\right)$ **A1** y-intercept $\left(0, \frac{1}{2}\right)$ **[5 marks]**

(c)
$$\frac{1}{x+1} - \frac{1}{x+2} = \frac{(x+2) - (x+1)}{(x+1)(x+2)}$$

$$= \frac{1}{x^2 + 3x + 2}$$

M1**AG****[1 mark]**

(d) $\int_0^1 \frac{1}{x+1} - \frac{1}{x+2} dx$

$$= [\ln(x+1) - \ln(x+2)]_0^1$$

A1

$$= \ln 2 - \ln 3 - \ln 1 + \ln 2$$

M1

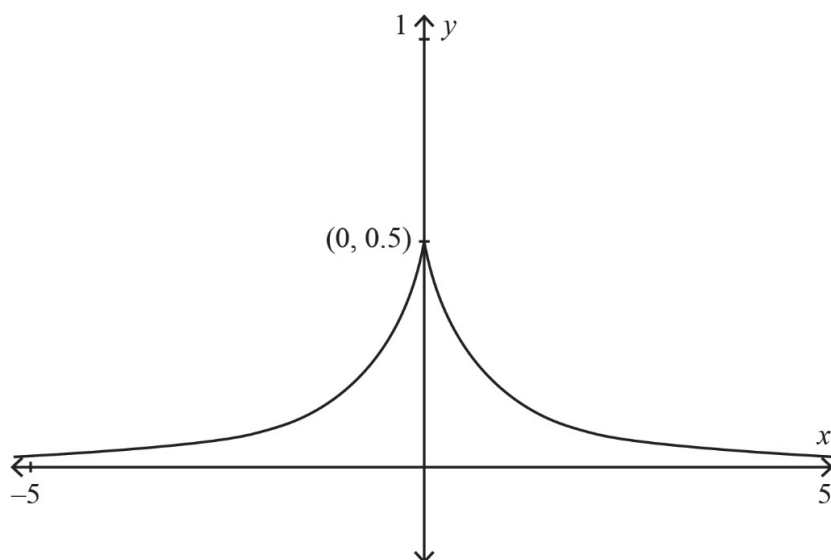
$$= \ln\left(\frac{4}{3}\right)$$

M1A1

$$\therefore p = \frac{4}{3}$$

[4 marks]

(e)



symmetry about the y -axis
correct shape

M1**A1**

Note: Allow **FT** from part (b).

[2 marks]

(f) $2 \int_0^1 f(x) dx$

(M1)(A1)

$$= 2 \ln\left(\frac{4}{3}\right)$$

A1

Note: Do not award **FT** from part (e).

[3 marks]

4 - (MAT/11_HL_Summer_2017_Q12) - Functions - Roots, Graphs

- (a) sum = 0
product = 6

A1

A1

[2 marks]

- (b) $P(1) = 1 - 10 + 15 - 6 = 0$
 $\Rightarrow (z - 1)$ is a factor of $P(z)$

M1A1

AG

Note: Accept use of division to show remainder is zero.

[2 marks]

- (c) **METHOD 1**

$$(z - 1)^3(z^2 + bz + c) = z^5 - 10z^2 + 15z - 6$$

(M1)

by inspection $c = 6$

A1

$$(z^3 - 3z^2 + 3z - 1)(z^2 + bz + 6) = z^5 - 10z^2 + 15z - 6$$

(M1)(A1)

$$b = 3$$

A1

METHOD 2

α, β are two roots of the quadratic

$$b = -(\alpha + \beta), c = \alpha\beta$$

(A1)

from part (a) $1 + 1 + 1 + \alpha + \beta = 0$

(M1)

$$\Rightarrow b = 3$$

A1

$$1 \times 1 \times 1 \times \alpha\beta = 6$$

(M1)

$$\Rightarrow c = 6$$

A1

Note: Award **FT** if $b = -7$ following through from their sum = 10.

METHOD 3

$$(z^5 - 10z^2 + 15z - 6) \div (z - 1) = z^4 + z^3 + z^2 - 9z + 6$$

(M1)A1

Note: This may have been seen in part (b).

$$z^4 + z^3 + z^2 - 9z + 6 \div (z - 1) = z^3 + 2z^2 + 3z - 6$$

(M1)

$$z^3 + 2z^2 + 3z - 6 \div (z - 1) = z^2 + 3z + 6$$

A1A1

[5 marks]

(d) $z^2 + 3z + 6 = 0$

M1

$$z = \frac{-3 \pm \sqrt{9 - 4 \cdot 6}}{2}$$

M1

$$= \frac{-3 \pm \sqrt{-15}}{2}$$

$$z = -\frac{3}{2} \pm \frac{i\sqrt{15}}{2}$$

A1(or $z = 1$)

Notes: Award the second **M1** for an attempt to use the quadratic formula or to complete the square.
Do not award **FT** from (c).

[3 marks]

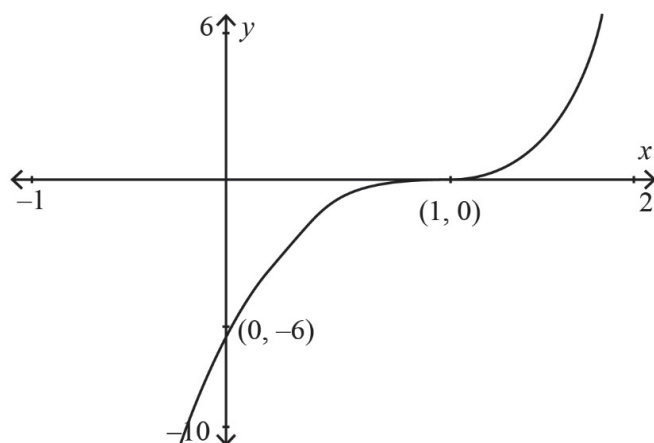
(e) (i) $\frac{d^2 y}{dx^2} = 20x^3 - 20$

M1A1

for $x > 1$, $20x^3 - 20 > 0 \Rightarrow$ concave up

R1AG

(ii)



x -intercept at $(1, 0)$

A1

y -intercept at $(0, -6)$

A1

stationary point of inflexion at $(1, 0)$ with correct curvature either side

A1**[6 marks]**

5 - (MAT/12_HL_Summer_2018_Q10) - Functions - Roots

- (a) attempt to make x the subject of $y = \frac{ax+b}{cx+d}$

M1

$$y(cx+d) = ax+b$$

A1

$$x = \frac{dy-b}{a-cy}$$

A1

$$f^{-1}(x) = \frac{dx-b}{a-cx},$$

A1

Note: Do not allow $y =$ in place of $f^{-1}(x)$.

$$x \neq \frac{a}{c}, \quad (x \in \mathbb{R})$$

A1

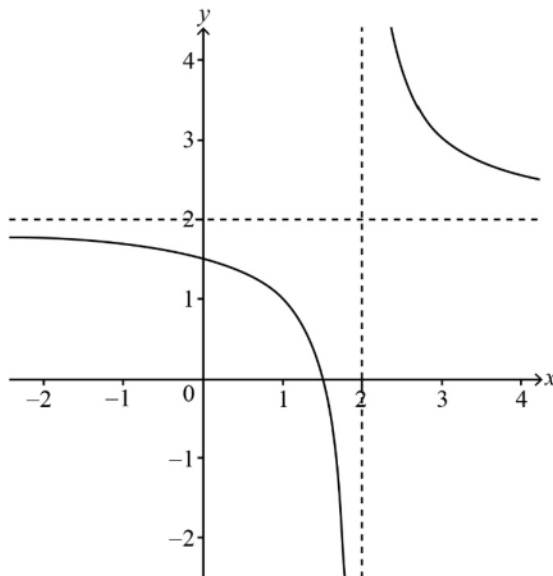
Note: The final **A** mark is independent.

[5 marks]

- (b) (i) $g(x) = 2 + \frac{1}{x-2}$

A1A1

(ii)



hyperbola shape, with single curves in second and fourth quadrants and third quadrant blank, including vertical asymptote $x=2$

A1

horizontal asymptote $y=2$

A1

intercepts $\left(\frac{3}{2}, 0\right), \left(0, \frac{3}{2}\right)$

A1**[5 marks]**

- (c) the domain of $h \circ g$ is $x \leq \frac{3}{2}, x > 2$

A1A1

the range of $h \circ g$ is $y \geq 0, y \neq \sqrt{2}$

A1A1**[4 marks]**

6 - (MAT/12_HL_Summer_2019_Q3) - Functions - Roots

$$g(x) = f(x+2) = (x+2)^4 - 6(x+2)^2 - 2(x+2) + 4 \quad \text{M1}$$

$$\text{attempt to expand } (x+2)^4 \quad \text{M1}$$

$$(x+2)^4 = x^4 + 4(2x^3) + 6(2^2x^2) + 4(2^3x) + 2^4 \quad \text{(A1)}$$

$$= x^4 + 8x^3 + 24x^2 + 32x + 16 \quad \text{A1}$$

$$g(x) = x^4 + 8x^3 + 24x^2 + 32x + 16 - 6(x^2 + 4x + 4) - 2x - 4 + 4$$

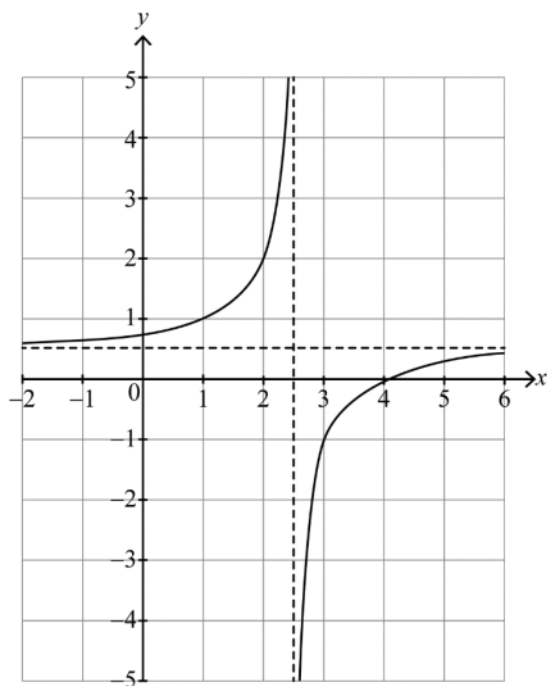
$$= x^4 + 8x^3 + 18x^2 + 6x - 8 \quad \text{A1}$$

Note: For correct expansion of $f(x-2) = x^4 - 8x^3 + 18x^2 - 10x$ award max **M0M1(A1)A0A1**.

[5 marks]

7 - (MAT/12_HL_Summer_2019_Q5) - Graphs, Functions - Roots

(a)



correct shape: two branches in correct quadrants with asymptotic behaviour **A1**

crosses at $(4, 0)$ and $(0, \frac{4}{5})$ **A1A1**

asymptotes at $x = \frac{5}{2}$ and $y = \frac{1}{2}$ **A1A1**

[5 marks]

(b) (i) $x < \frac{5}{2}, x \geq 4$

A1A1

(ii) $f(x) \geq 0, f(x) \neq \frac{1}{\sqrt{2}} \quad (f(x) \in \mathbb{R})$

A1

Note: Follow through from their graph, as long as it is a rectangular hyperbola.

Note: Allow range expressed in terms of y .

[3 marks]

8 - (MathAA/12_HL_Summer_2021_Q1) - Algebra-functions - Roots

attempt to subtract squares of integers

(M1)

$$(n+1)^2 - n^2$$

EITHER

correct order of subtraction and correct expansion of $(n+1)^2$, seen anywhere **A1A1**

$$= n^2 + 2n + 1 - n^2 (= 2n + 1)$$

OR

correct order of subtraction and correct factorization of difference of squares **A1A1**

$$= (n+1-n)(n+1+n) (= 2n+1)$$

THEN

$$= n + n + 1 = \text{RHS}$$

A1

Note: Do not award final **A1** unless all previous working is correct.

which is the sum of n and $n+1$

AG

Note: If expansion and order of subtraction are correct, award full marks for candidates who find the sum of the integers as $2n+1$ and then show that the difference of the squares (subtracted in the correct order) is $2n+1$.

[4 marks]

9 - (MathAA/11_HL_Summer_2021_Q1) - Algebra-functions - Roots

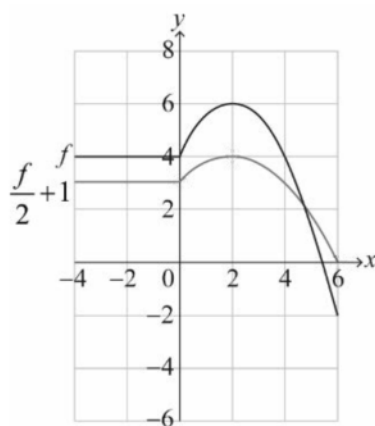
(a) (i) $f(2) = 6$

A1

(ii) $(f \circ f)(2) = -2$

A1**[2 marks]**

(b)

**M1A1A1**

Note: Award **M1** for an attempt to apply any vertical stretch or vertical translation, **A1** for a correct horizontal line segment between -4 and 0 (located roughly at $y = 3$), **A1** for a correct concave down parabola including max point at $(2, 4)$ and for correct end points at $(0, 3)$ and $(6, 0)$ (within circles). Points do not need to be labelled.

[3 marks]

10 - (MathAA/12_HL_Summer_2021_Q7) - Algebra-functions - Roots

$$\alpha + \beta + \alpha + \beta = k$$

(A1)

$$\alpha + \beta = \frac{k}{2}$$

$$\alpha\beta(\alpha + \beta) = -3k$$

(A1)

$$\left(-\frac{k^2}{4}\right)\left(\frac{k}{2}\right) = -3k \left(-\frac{k^3}{8} = -3k\right)$$

M1

attempting to solve $-\frac{k^3}{8} + 3k = 0$ (or equivalent) for k

(M1)

$$k = 2\sqrt{6} \quad (= \sqrt{24}) \quad (k > 0)$$

A1

Note: Award **A0** for $k = \pm 2\sqrt{6}$ ($\pm\sqrt{24}$).

[5 marks]

11 - (MathAA/10_HL_Winter_2021_Q2) - Algebra-functions - Roots, Graphs

(a) (i) $x = 3$

A1

(ii) $y = -2$

A1**[2 marks]**

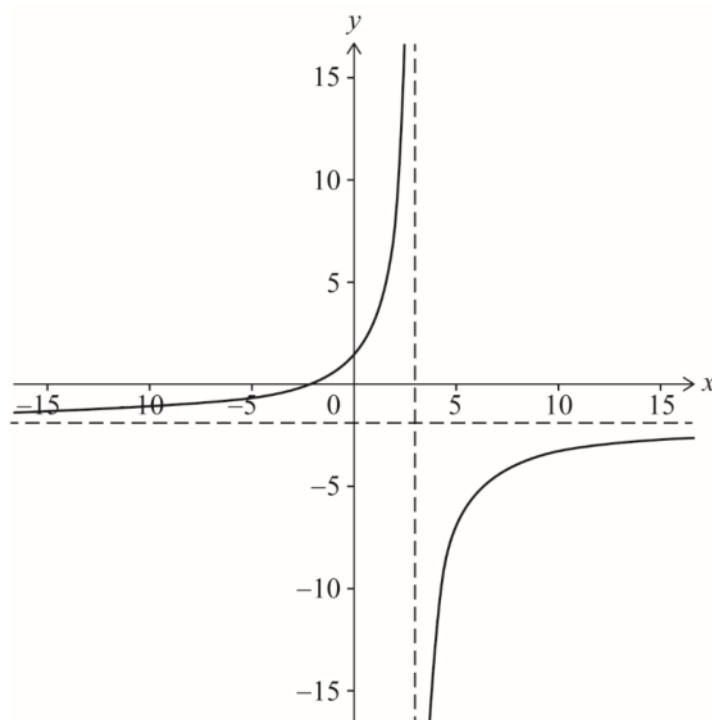
(b) (i) $(-2, 0)$ (accept $x = -2$)

A1

(ii) $\left(0, \frac{4}{3}\right)$ (accept $y = \frac{4}{3}$ and $f(0) = \frac{4}{3}$)

A1**[2 marks]**

(c)

**A1**

Note: Award **A1** for completely correct shape: two branches in correct quadrants with asymptotic behaviour.

[1 mark]

(d) **METHOD 1**

$$(g(x)=)y = \frac{ax+4}{3-x}$$

attempt to find x in terms of y **(M1)****OR** exchange x and y and attempt to find y in terms of x

$$3y - xy = ax + 4$$

A1

$$ax + xy = 3y - 4$$

$$x(a + y) = 3y - 4$$

$$x = \frac{3y-4}{y+a}$$

$$g^{-1}(x) = \frac{3x-4}{x+a}$$

A1
Note: Condone use of $y =$

$$g(x) \equiv g^{-1}(x)$$

$$\frac{ax+4}{3-x} \equiv \frac{3x-4}{x+a}$$

$$\Rightarrow a = -3$$

A1**[4 marks]**

METHOD 2

$$g(x) = \frac{ax+4}{3-x}$$

attempt to find an expression for $g(g(x))$ and equate to x **(M1)**

$$gg(x) = \frac{a\left(\frac{ax+4}{3-x}\right)+4}{3-\left(\frac{ax+4}{3-x}\right)} = x$$
A1

$$\frac{a(ax+4)+4(3-x)}{(9-3x)-(ax+4)} = x$$

$$\frac{a(ax+4)+4(3-x)}{5-(3+a)x} = x$$

$$a(ax+4)+4(3-x) = x(5-(3+a)x)$$
A1

equating coefficients of x^2 (or similar)

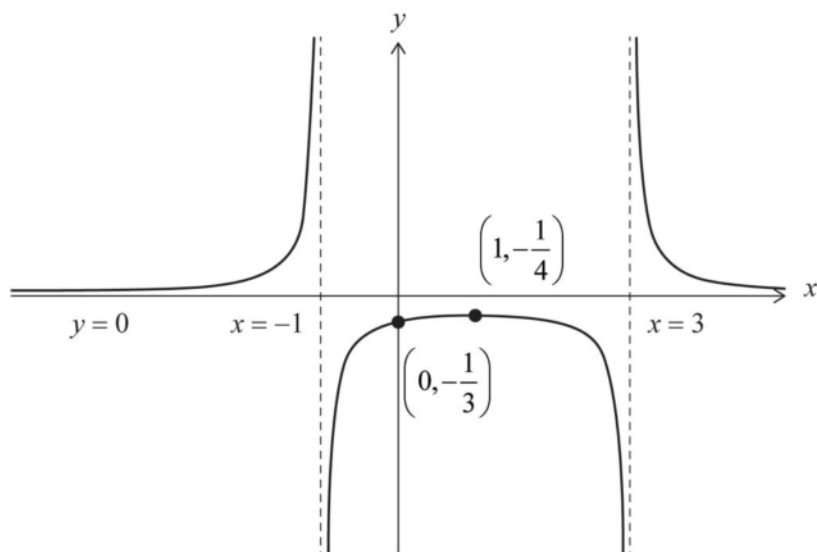
$$a = -3$$
A1

[4 marks]

Total [9 marks]

12 - (MathAA/12_HL_Summer_2022_Q11) - Algebra-functions - Roots

(a)

y-intercept $\left(0, -\frac{1}{3}\right)$

A1

Note: Accept an indication of $-\frac{1}{3}$ on the y -axis.

vertical asymptotes $x = -1$ and $x = 3$

A1

horizontal asymptote $y = 0$

A1

uses a valid method to find the x -coordinate of the local maximum point

(M1)

Note: For example, uses the axis of symmetry or attempts to solve $f'(x) = 0$.

local maximum point $\left(1, -\frac{1}{4}\right)$

A1

Note: Award (M1)A0 for a local maximum point at $x = 1$ and coordinates not given.

three correct branches with correct asymptotic behaviour and the key features in approximately correct relative positions to each other

A1

[6 marks]

(b) (i) $x = \frac{1}{y^2 - 2y - 3}$

M1

Note: Award **M1** for interchanging x and y (this can be done at a later stage).

EITHER

attempts to complete the square

M1

$$y^2 - 2y - 3 = (y-1)^2 - 4$$

A1

$$x = \frac{1}{(y-1)^2 - 4}$$

$$(y-1)^2 - 4 = \frac{1}{x} \quad \left((y-1)^2 = 4 + \frac{1}{x} \right)$$

A1

$$y-1 = \pm \sqrt{4 + \frac{1}{x}} \quad \left(= \pm \sqrt{\frac{4x+1}{x}} \right)$$

ORattempts to solve $xy^2 - 2xy - 3x - 1 = 0$ for y **M1**

$$y = \frac{-(-2x) \pm \sqrt{(-2x)^2 + 4x(3x+1)}}{2x}$$

A1

Note: Award **A1** even if $-$ (in $\pm$) is missing

$$= \frac{2x \pm \sqrt{16x^2 + 4x}}{2x}$$

A1

THEN

$$= 1 \pm \frac{\sqrt{4x^2 + x}}{x}$$

A1

$$y > 3 \text{ and hence } y = 1 - \frac{\sqrt{4x^2 + x}}{x} \text{ is rejected}$$

R1**Note:** Award **R1** for concluding that the expression for y must have the '+' sign.The **R1** may be awarded earlier for using the condition $x > 3$.

$$y = 1 + \frac{\sqrt{4x^2 + x}}{x}$$

$$g^{-1}(x) = 1 + \frac{\sqrt{4x^2 + x}}{x}$$

AG(ii) domain of g^{-1} is $x > 0$ **A1****[7 marks]**

(c) attempts to find $(h \circ g)(a)$ **(M1)**

$$(h \circ g)(a) = \arctan\left(\frac{g(a)}{2}\right) \quad \left((h \circ g)(a) = \arctan\left(\frac{1}{2(a^2 - 2a - 3)}\right) \right) \quad \textbf{(A1)}$$

$$\arctan\left(\frac{g(a)}{2}\right) = \frac{\pi}{4} \quad \left(\arctan\left(\frac{1}{2(a^2 - 2a - 3)}\right) = \frac{\pi}{4} \right)$$

attempts to solve for $g(a)$ **M1**

$$\Rightarrow g(a) = 2 \quad \left(\frac{1}{(a^2 - 2a - 3)} = 2 \right)$$

EITHER

$$\Rightarrow a = g^{-1}(2) \quad \textbf{A1}$$

attempts to find their $g^{-1}(2)$ **M1**

$$a = 1 + \frac{\sqrt{4(2)^2 + 2}}{2} \quad \textbf{A1}$$

Note: Award all available marks to this stage if x is used instead of a .

OR

$$\Rightarrow 2a^2 - 4a - 7 = 0 \quad \textbf{A1}$$

attempts to solve their quadratic equation

M1

$$a = \frac{-(-4) \pm \sqrt{(-4)^2 + 4(2)(7)}}{4} \quad \left(= \frac{4 \pm \sqrt{72}}{4} \right) \quad \textbf{A1}$$

Note: Award all available marks to this stage if x is used instead of a .

THEN

$$a = 1 + \frac{3}{2}\sqrt{2} \quad (\text{as } a > 3) \quad \textbf{A1}$$

$$(p = 1, q = 3, r = 2)$$

Note: Award **A1** for $a = 1 + \frac{1}{2}\sqrt{18}$ ($p = 1, q = 1, r = 18$).

[7 marks]

13 - (MathAA/12_HL_Summer_2022_Q9) - Algebra-functions - Roots

METHOD 1 (rearranging the equation)assume there exists some $\alpha \in \mathbb{Z}$ such that $2\alpha^3 + 6\alpha + 1 = 0$ **M1**

Note: Award **M1** for equivalent statements such as ‘assume that α is an integer root of $2\alpha^3 + 6\alpha + 1 = 0$ ’. Condone the use of x throughout the proof.

Award **M1** for an assumption involving $\alpha^3 + 3\alpha + \frac{1}{2} = 0$.

Note: Award **M0** for statements such as “let’s consider the equation has integer roots...” , “let $\alpha \in \mathbb{Z}$ be a root of $2\alpha^3 + 6\alpha + 1 = 0 \dots$ ”

Note: Subsequent marks after this **M1** are independent of this **M1** and can be awarded.

attempts to rearrange their equation into a suitable form

M1**EITHER**

$$2\alpha^3 + 6\alpha = -1$$

A1

$$\alpha \in \mathbb{Z} \Rightarrow 2\alpha^3 + 6\alpha \text{ is even}$$

R1

$$2\alpha^3 + 6\alpha = -1 \text{ which is not even and so } \alpha \text{ cannot be an integer}$$

R1

Note: Accept ‘ $2\alpha^3 + 6\alpha = -1$ which gives a contradiction’.

OR

$$1 = 2(-\alpha^3 - 3\alpha)$$

A1

$$\alpha \in \mathbb{Z} \Rightarrow (-\alpha^3 - 3\alpha) \in \mathbb{Z}$$

R1

$$\Rightarrow 1 \text{ is even which is not true and so } \alpha \text{ cannot be an integer}$$

R1

Note: Accept ‘ $\Rightarrow 1$ is even which gives a contradiction’.

OR

$$\frac{1}{2} = -\alpha^3 - 3\alpha$$

A1

$$\alpha \in \mathbb{Z} \Rightarrow (-\alpha^3 - 3\alpha) \in \mathbb{Z}$$

R1

$-\alpha^3 - 3\alpha$ is not an integer $\left(= \frac{1}{2} \right)$ and so α cannot be an integer

R1

Note: Accept ' $-\alpha^3 - 3\alpha$ is not an integer $\left(= \frac{1}{2} \right)$ which gives a contradiction'.

OR

$$\alpha = -\frac{1}{2(\alpha^2 + 3)}$$

A1

$$\alpha \in \mathbb{Z} \Rightarrow -\frac{1}{2(\alpha^2 + 3)} \in \mathbb{Z}$$

R1

$-\frac{1}{2(\alpha^2 + 3)}$ is not an integer and so α cannot be an integer

R1

Note: Accept $-\frac{1}{2(\alpha^2 + 3)}$ is not an integer which gives a contradiction'.

THEN

so the equation $2x^3 + 6x + 1 = 0$ has no integer roots

AG**[5 marks]**

METHOD 2

assume there exists some $\alpha \in \mathbb{Z}$ such that $2\alpha^3 + 6\alpha + 1 = 0$

M1

Note: Award **M1** for statements such as ‘assume that α is an integer root of $2\alpha^3 + 6\alpha + 1 = 0$ ’. Condone the use of x throughout the proof. Award **M1** for an assumption involving $\alpha^3 + 3\alpha + \frac{1}{2} = 0$ and award subsequent marks based on this.

Note: Award **M0** for statements such as “let’s consider the equation has integer roots...”, “let $\alpha \in \mathbb{Z}$ be a root of $2\alpha^3 + 6\alpha + 1 = 0 \dots$ ”

Note: Subsequent marks after this **M1** are independent of this **M1** and can be awarded.

let $f(x) = 2x^3 + 6x + 1$ (and $f(\alpha) = 0$)

$f'(x) = 6x^2 + 6 > 0$ for all $x \in \mathbb{R} \Rightarrow f$ is a (strictly) increasing function

M1A1

$f(0) = 1$ and $f(-1) = -7$

R1

thus $f(x) = 0$ has only one real root between -1 and 0 , which gives a contradiction

(or therefore, contradicting the assumption that $f(\alpha) = 0$ for some $\alpha \in \mathbb{Z}$),

R1

so the equation $2x^3 + 6x + 1 = 0$ has no integer roots

AG**[5 marks]**

14 - (MathAA/12_HL_Summer_2022_Q6) - Algebra-functions - Roots

- (a) attempts to replace
- x
- with
- $-x$

M1

$$f(-x) = -x\sqrt{1-(-x)^2}$$

$$= -x\sqrt{1-(-x)^2} (= -f(x))$$

A1

Note: Award **M1A1** for an attempt to calculate both $f(-x)$ and $-f(-x)$ independently, showing that they are equal.

Note: Award **M1A0** for a graphical approach including evidence that **either** the graph is invariant after rotation by 180° about the origin **or** the graph is invariant after a reflection in the y -axis and then in the x -axis (or vice versa).

so f is an odd function**AG****[2 marks]**

- (b) attempts both product rule and chain rule differentiation to find
- $f'(x)$

M1

$$f'(x) = x \times \frac{1}{2} \times (-2x) \times (1-x^2)^{\frac{1}{2}} + (1-x^2)^{\frac{1}{2}} \times 1 \left(= \sqrt{1-x^2} - \frac{x^2}{\sqrt{1-x^2}} \right)$$

A1

$$= \frac{1-2x^2}{\sqrt{1-x^2}}$$

sets their $f'(x) = 0$ **M1**

$$\Rightarrow x = \pm \frac{1}{\sqrt{2}}$$

A1attempts to find at least one of $f\left(\pm \frac{1}{\sqrt{2}}\right)$ **(M1)**

Note: Award **M1** for an attempt to evaluate $f(x)$ at least at one of their $f'(x) = 0$ roots.

$$a = -\frac{1}{2} \text{ and } b = \frac{1}{2}$$

A1

Note: Award **A1** for $-\frac{1}{2} \leq y \leq \frac{1}{2}$.

[6 marks]

15 - (MathAA/12_HL_Summer_2022_Q2) - Algebra-functions - Roots

(a) $(n-1) + n + (n+1)$ (A1)

$= 3n$ A1

which is always divisible by 3 AG

[2 marks]

(b) $(n-1)^2 + n^2 + (n+1)^2$ ($= n^2 - 2n + 1 + n^2 + n^2 + 2n + 1$) A1

attempts to expand either $(n-1)^2$ or $(n+1)^2$ (do not accept $n^2 - 1$ or $n^2 + 1$) (M1)

$= 3n^2 + 2$ A1

demonstrating recognition that 2 is not divisible by 3 or $\frac{2}{3}$ seen after correct

expression divided by 3 R1

$3n^2$ is divisible by 3 and so $3n^2 + 2$ is never divisible by 3

OR the first term is divisible by 3, the second is not

OR $3\left(n^2 + \frac{2}{3}\right)$ OR $\frac{3n^2 + 2}{3} = n^2 + \frac{2}{3}$

hence the sum of the squares is never divisible by 3 AG

[4 marks]

16 - (MathAA/11_HL_Summer_2022_Q8) - Algebra-functions - Roots

Assume that a and b are both odd.**M1****Note:** Award **M0** for statements such as “let a and b be both odd”.**Note:** Subsequent marks after this **M1** are independent of this mark and can be awarded.Then $a = 2m + 1$ and $b = 2n + 1$ **A1**

$$a^2 + b^2 \equiv (2m+1)^2 + (2n+1)^2$$

$$= 4m^2 + 4m + 1 + 4n^2 + 4n + 1$$

A1

$$= 4(m^2 + m + n^2 + n) + 2$$

(A1) $(4(m^2 + m + n^2 + n) + 2)$ is always divisible by 4) but 2 is not divisible by 4. (or equivalent)**R1** $\Rightarrow a^2 + b^2$ is not divisible by 4, a contradiction. (or equivalent)**R1**hence a and b cannot both be odd.**AG****Note:** Award a maximum of **M1A0A0(A0)R1R1** for considering identical or two consecutive odd numbers for a and b .**[6 marks]**

17 - (MathAA/11_HL_Summer_2022_Q3) - Algebra-functions - Roots

$$(a) \quad (f \circ g)(x) = f(2x) \quad (A1)$$

$$f(2x) = \sqrt{3} \sin 2x + \cos 2x \quad A1$$

[2 marks]

$$(b) \quad \sqrt{3} \sin 2x + \cos 2x = 2 \cos 2x$$

$$\sqrt{3} \sin 2x = \cos 2x$$

recognizing to use $\tan$ or $\cot$ M1

$$\tan 2x = \frac{1}{\sqrt{3}} \text{ OR } \cot 2x = \sqrt{3} \text{ (values may be seen in right triangle)} \quad (A1)$$

$$\left(\arctan\left(\frac{1}{\sqrt{3}}\right) = \right) \frac{\pi}{6} \text{ (seen anywhere) (accept degrees)} \quad (A1)$$

$$2x = \frac{\pi}{6}, \frac{7\pi}{6}$$

$$x = \frac{\pi}{12}, \frac{7\pi}{12} \quad A1A1$$

Note: Do not award the final **A1** if any additional solutions are seen.

Award **A1A0** for correct answers in degrees.

Award **A0A0** for correct answers in degrees with additional values.

[5 marks]

18 - (MathAA/10_HL_Winter_2022_Q8) - Trigonometry, Algebra-functions - Roots

(a)

$$a = \frac{\pi}{2}$$

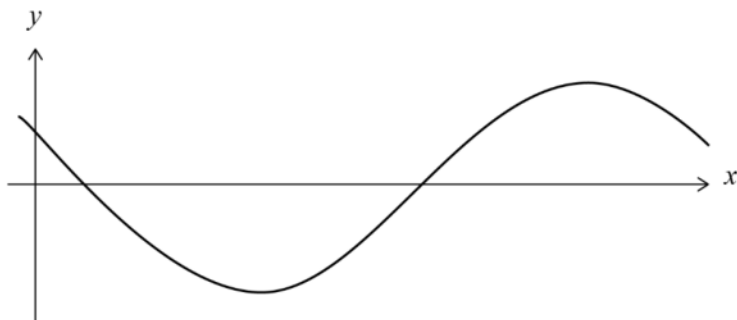
A2

Note: For sinusoidal graph through the origin seen with incorrect a , or use of horizontal line test with incorrect a , award **A1A0**

(b) $a = \pi$

A1

(c)



sketch showing sinusoidal shape decreasing as it crosses the y-axis
(below or above the origin)

(A1)

$$a = k - \pi$$

A1

[5 marks]

19 - (MathAA/10_HL_Winter_2022_Q3) - Algebra-functions - Roots

(a)

Note: Award a maximum of **M1A0A0** if the candidate manipulates both sides of the equation (such as moving terms from one side to the other).

METHOD 1 (working with LHS)

attempting to expand $(a^2 - 1)^2$ (do not accept $a^4 + 1$ or $a^4 - 1$) **(M1)**

$$\text{LHS} = a^2 + \frac{a^4 - 2a^2 + 1}{4} \text{ or } \frac{4a^2 + a^4 - 2a^2 + 1}{4} \quad \textbf{A1}$$

$$= \frac{a^4 + 2a^2 + 1}{4} \quad \textbf{A1}$$

$$= \left(\frac{a^2 + 1}{2} \right)^2 (= \text{RHS}) \quad \textbf{AG}$$

Note: Do not award the final A1 if further working contradicts the AG.

METHOD 2 (working with RHS)

attempting to expand $(a^2 + 1)^2$ **(M1)**

$$\text{RHS} = \frac{a^4 + 2a^2 + 1}{4}$$

$$= \frac{4a^2 + a^4 - 2a^2 + 1}{4} \quad \textbf{A1}$$

$$= a^2 + \frac{a^4 - 2a^2 + 1}{4} \quad \textbf{A1}$$

$$= a^2 + \left(\frac{a^2 - 1}{2} \right)^2 (= \text{LHS}) \quad \textbf{AG}$$

Note: Do not award the final A1 if further working contradicts the AG.

[3 marks]

- (b) recognise base and height as a and $\left(\frac{a^2-1}{2}\right)$ (may be seen in diagram) (M1)

correct substitution into triangle area formula A1

$$\text{Area} = \frac{a}{2} \left(\frac{a^2-1}{2} \right) \text{ (or equivalent) } \left(= \frac{a(a^2-1)}{4} = \frac{a^3-a}{4} \right)$$

[2 marks]

20 - (MathAA/11_HL_Summer_2023_Q1) - Algebra-functions - Roots

- (a) recognizing $f(x) = 0$ (M1)

$x = -1$ A1

[2 marks]

- (b) (i) $x = 2$ (must be an equation with x) A1

(ii) $y = \frac{7}{2}$ (must be an equation with y) A1

[2 marks]

- (c) EITHER

interchanging x and y (M1)

$$2xy - 4x = 7y + 7$$

correct working with y terms on the same side: $2xy - 7y = 4x + 7$ (A1)

OR

$$2yx - 4y = 7x + 7$$

correct working with x terms on the same side: $2yx - 7x = 4y + 7$ (A1)

interchanging x and y OR making x the subject $x = \frac{4y+7}{2y-7}$ (M1)

THEN

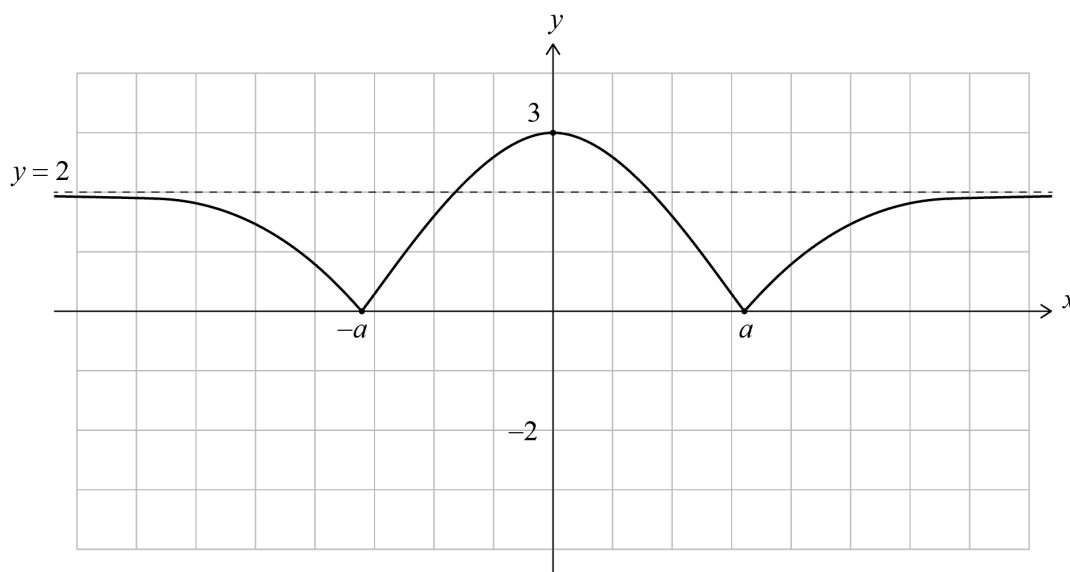
$$f^{-1}(x) = \frac{4x+7}{2x-7} \text{ (or equivalent) } \left(x \neq \frac{7}{2} \right) \quad \text{A1}$$

[3 marks]

21 - (MathAA/11_HL_Summer_2023_Q8) - Algebra-functions - Roots

(a) attempt to reflect f in the x OR y axis

(M1)



A1A1A1

Note: For a curve with an approximately correct shaped right-hand branch, award:

A1 for correct asymptotic behaviour at $y = 2$ (either side)

A1 for correctly reflected RHS of the graph in the y -axis with smooth maximum at $(0, 3)$.

A1 for labelled x -intercept at $(-a, 0)$ and labelled asymptote at $y = 2$ with sharp points (cusps) at the x -intercepts.

[4 marks]

(b) $k = 0$

A1

 $4 \leq k < 9$

A2

Note: If final answer incorrect, award **A1** for critical values 4 and 9 seen anywhere.

Exception to FT:

Award a maximum of **A0A2FT** if their graph from (a) is not symmetric about the y -axis.

[3 marks]

22 - (MathAA/12_HL_Summer_2023_Q2) - Algebra-functions - Roots

(a) (i) $x = 2$

A1

(ii) $y = 1$

A1

[2 marks]

(b) (i) $\left(0, \frac{3}{2}\right)$

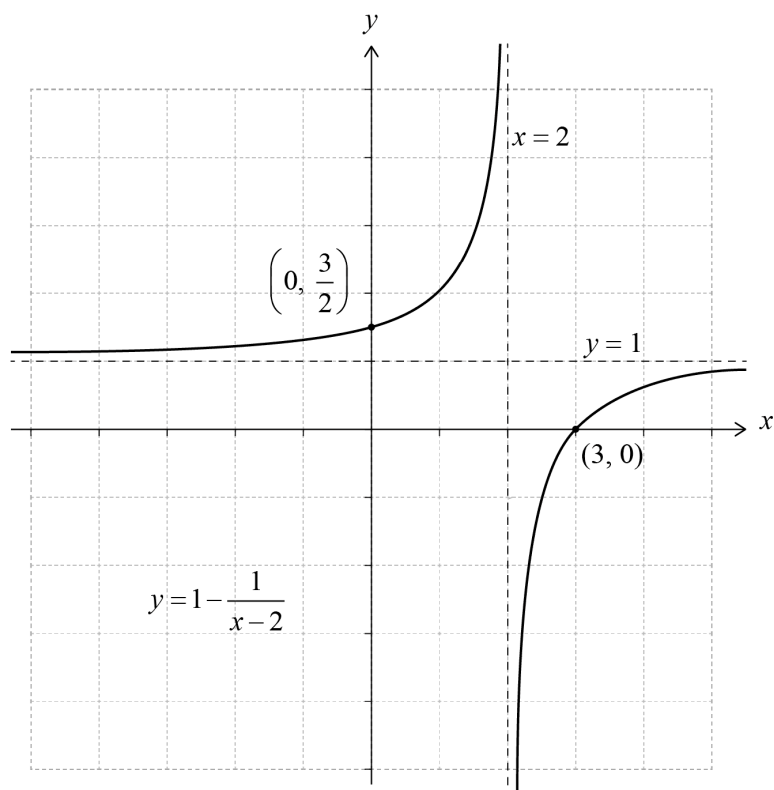
A1

(ii) $(3, 0)$

A1

[2 marks]

(c)



two correct branches with correct asymptotic behaviour and intercepts clearly shown

A1

[1 mark]

23 - (MathAA/12_HL_Summer_2023_Q5) - Algebra-functions - Roots

attempts to form $(g \circ f)(x)$ (M1)

$$[f(x)]^2 + f(x) + 3 \quad \text{OR} \quad (ax + b)^2 + ax + b + 3$$

$$a^2x^2 + 2abx + b^2 + ax + b + 3 (= 4x^2 - 14x + 15) \quad \text{(A1)}$$

equates their corresponding terms to form at least one equation (M1)

$$a^2x^2 = 4x^2 \quad \text{OR} \quad a^2 = 4 \quad \text{OR} \quad 2abx + ax = -14x \quad \text{OR} \quad 2ab + a = -14 \quad \text{OR} \quad b^2 + b + 3 = 15$$

$$a = \pm 2 \text{ (seen anywhere)} \quad \text{A1}$$

attempt to use $2ab + a = -14$ to pair the correct values (seen anywhere) (M1)

$$f(x) = 2x - 4 \text{ (accept } a = 2 \text{ with } b = -4), f(x) = -2x + 3 \text{ (accept } a = -2 \text{ with } b = 3) \quad \text{A1A1}$$

[7 marks]

24 - (MathAA/11_HL_Winter_2023_Q1) - Algebra-functions - Roots

(a) attempt to form $(g \circ f)(x)$ (M1)

$$((g \circ f)(x)) = (x - 3)^2 + k^2 \quad (= x^2 - 6x + 9 + k^2) \quad \text{A1}$$

[2 marks]

(b) substituting $x = 2$ into their $(g \circ f)(x)$ and setting their expression $= 10$ (M1)

$$(2 - 3)^2 + k^2 = 10 \quad \text{OR} \quad 2^2 - 6(2) + 9 + k^2 = 10$$

$$k^2 = 9 \quad \text{(A1)}$$

$$k = \pm 3 \quad \text{A1}$$

[3 marks]

25 - (MathAA/11_HL_Winter_2023_Q9) - Algebra-functions - Roots

$$(a) \quad f(-x) = \frac{\sin^2(-kx)}{(-x)^2}$$

M1

$$= \frac{(-\sin(kx))^2}{(-x)^2}$$

A1

$$= \frac{\sin^2(kx)}{x^2} (= f(x))$$

hence $f(x)$ is even**AG****[2 marks]**

(b) **METHOD 1**

Noting that $\lim_{x \rightarrow 0} (f(x)) = \frac{0}{0}$ **(M1)**

attempt to differentiate numerator and denominator: **M1**

$$\lim_{x \rightarrow 0} (f(x)) = \lim_{x \rightarrow 0} \left(\frac{2k \sin kx \cos kx}{2x} \right) \left(= \lim_{x \rightarrow 0} \left(\frac{k \sin 2kx}{2x} \right) \right) \quad \text{A1}$$

(evaluates to $\frac{0}{0}$) and attempts to differentiate a second time: **M1**

$$= \lim_{x \rightarrow 0} \left(\frac{2k^2 (\cos^2 kx - \sin^2 kx)}{2} \right) \left(= \lim_{x \rightarrow 0} \left(\frac{2k^2 \cos 2kx}{2} \right) \right) = k^2 \quad \text{A1}$$

$$(k^2 = 16 \Rightarrow) k = 4 \quad \text{A1}$$

Note: Award relevant marks, even if 'lim' is not explicitly seen.

METHOD 2

attempt to express $\sin(kx)$ as a Maclaurin series **M1**

$$\sin(kx) = kx(+\dots)$$

$$\sin^2(kx) = k^2 x^2(+\dots) \quad \text{A1}$$

$$\lim_{x \rightarrow 0} \left(\frac{\sin^2(kx)}{x^2} \right) = \lim_{x \rightarrow 0} \left(\frac{k^2 x^2(+\dots)}{x^2} \right) \quad \text{M1}$$

$$= \lim_{x \rightarrow 0} (k^2 + \text{termsin } x) \quad \text{R1}$$

Note: This **R1** is awarded independently of any other marks.

$$= k^2 \quad \text{A1}$$

$$(k^2 = 16 \Rightarrow) k = 4 \quad \text{A1}$$

Note: Award relevant marks, even if 'lim' is not explicitly seen.

METHOD 3

splitting function into $\left(\frac{\sin kx}{x}\right)\left(\frac{\sin kx}{x}\right)$ and using limit of product = product of

limits

(M1)

$$\lim_{x \rightarrow 0} \left(\frac{\sin^2(kx)}{x^2} \right) = \lim_{x \rightarrow 0} \left(\frac{\sin kx}{x} \right) \times \lim_{x \rightarrow 0} \left(\frac{\sin kx}{x} \right)$$

A1

EITHER

using L' Hôpital's rule

(M1)

$$\lim_{x \rightarrow 0} \left(\frac{\sin kx}{x} \right) = \lim_{x \rightarrow 0} \left(\frac{k \cos kx}{1} \right) = k$$

(A1)

OR

using Maclaurin expansion for $\sin kx$

(M1)

$$\sin(kx) = kx(+\dots)$$

$$\lim_{x \rightarrow 0} \left(\frac{\sin kx}{x} \right) = \lim_{x \rightarrow 0} \left(\frac{kx + \dots}{x} \right) = \lim_{x \rightarrow 0} (k + \text{terms in } x) = k$$

(A1)

THEN

$$\text{hence } \lim_{x \rightarrow 0} \left(\frac{\sin^2(kx)}{x^2} \right) = k \times k = k^2$$

A1

$$k^2 = 16 \Rightarrow k = 4 (k > 0)$$

A1

Note: Award relevant marks, even if 'lim' is not explicitly seen.

[6 marks]

26 - (MathAA/12_HL_Winter_2023_Q1) - Algebra-functions - Roots

(a) attempt to form $(g \circ f)(x)$

(M1)

$$((g \circ f)(x)) = (x-3)^2 + k^2 \quad (= x^2 - 6x + 9 + k^2)$$

A1

[2 marks]

(b) substituting $x = 2$ into their $(g \circ f)(x)$ and setting their expression = 10

(M1)

$$(2-3)^2 + k^2 = 10 \quad \text{OR} \quad 2^2 - 6(2) + 9 + k^2 = 10$$

$$k^2 = 9$$

(A1)

$$k = \pm 3$$

A1

[3 marks]

27 - (MathAA/12_HL_Winter_2023_Q9) - Algebra-functions - Roots

$$(a) \quad f(-x) = \frac{\sin^2(-kx)}{(-x)^2}$$

M1

$$= \frac{(-\sin(kx))^2}{(-x)^2}$$

A1

$$= \frac{\sin^2(kx)}{x^2} (= f(x))$$

hence $f(x)$ is even

AG

[2 marks]

(b) **METHOD 1**

Noting that $\lim_{x \rightarrow 0} (f(x)) = \frac{0}{0}$ **(M1)**

attempt to differentiate numerator and denominator: **M1**

$$\lim_{x \rightarrow 0} (f(x)) = \lim_{x \rightarrow 0} \left(\frac{2k \sin kx \cos kx}{2x} \right) \left(= \lim_{x \rightarrow 0} \left(\frac{k \sin 2kx}{2x} \right) \right) \quad \text{A1}$$

(evaluates to $\frac{0}{0}$) and attempts to differentiate a second time: **M1**

$$= \lim_{x \rightarrow 0} \left(\frac{2k^2 (\cos^2 kx - \sin^2 kx)}{2} \right) \left(= \lim_{x \rightarrow 0} \left(\frac{2k^2 \cos 2kx}{2} \right) \right) = k^2 \quad \text{A1}$$

$$(k^2 = 16 \Rightarrow) k = 4 \quad \text{A1}$$

Note: Award relevant marks, even if 'lim' is not explicitly seen.

METHOD 2

attempt to express $\sin(kx)$ as a Maclaurin series **M1**

$$\sin(kx) = kx (+\dots)$$

$$\sin^2(kx) = k^2 x^2 (+\dots) \quad \text{A1}$$

$$\lim_{x \rightarrow 0} \left(\frac{\sin^2(kx)}{x^2} \right) = \lim_{x \rightarrow 0} \left(\frac{k^2 x^2 (+\dots)}{x^2} \right) \quad \text{M1}$$

$$= \lim_{x \rightarrow 0} (k^2 + \text{termsin } x) \quad \text{R1}$$

Note: This **R1** is awarded independently of any other marks.

$$= k^2 \quad \text{A1}$$

$$(k^2 = 16 \Rightarrow) k = 4 \quad \text{A1}$$

Note: Award relevant marks, even if 'lim' is not explicitly seen.

METHOD 3

splitting function into $\left(\frac{\sin kx}{x}\right)\left(\frac{\sin kx}{x}\right)$ and using limit of product = product of

limits

(M1)

$$\lim_{x \rightarrow 0} \left(\frac{\sin^2(kx)}{x^2} \right) = \lim_{x \rightarrow 0} \left(\frac{\sin kx}{x} \right) \times \lim_{x \rightarrow 0} \left(\frac{\sin kx}{x} \right)$$

A1

EITHER

using L' Hôpital's rule

(M1)

$$\lim_{x \rightarrow 0} \left(\frac{\sin kx}{x} \right) = \lim_{x \rightarrow 0} \left(\frac{k \cos kx}{1} \right) = k$$

(A1)

OR

using Maclaurin expansion for $\sin kx$

(M1)

$$\sin(kx) = kx(+\dots)$$

$$\lim_{x \rightarrow 0} \left(\frac{\sin kx}{x} \right) = \lim_{x \rightarrow 0} \left(\frac{kx + \dots}{x} \right) = \lim_{x \rightarrow 0} (k + \text{terms in } x) = k$$

(A1)

THEN

$$\text{hence } \lim_{x \rightarrow 0} \left(\frac{\sin^2(kx)}{x^2} \right) = k \times k = k^2$$

A1

$$k^2 = 16 \Rightarrow k = 4 (k > 0)$$

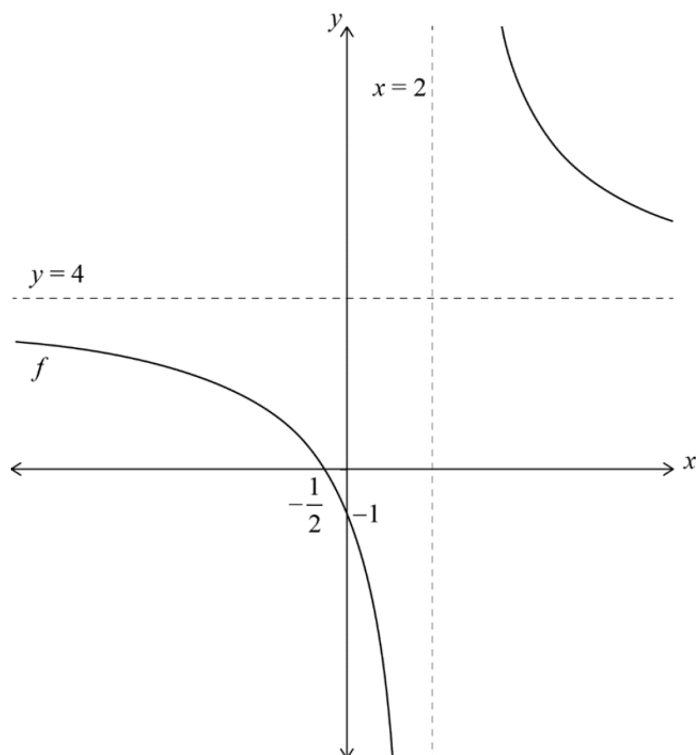
A1

Note: Award relevant marks, even if 'lim' is not explicitly seen.

[6 marks]

28 - (MathAA/11_HL_Summer_2024_Q10) - Algebra-functions - Roots, Graphs

(a)



vertical asymptote $x = 2$ sketched and labelled with correct equation

A1

horizontal asymptote $y = 4$ sketched and labelled with correct equation

A1

For an approximate rational function shape:

labelled intercepts $-\frac{1}{2}$ on x -axis, -1 on y -axis

A1A1

two branches in correct opposite quadrants with correct asymptotic behaviour

A1

Note: These marks may be awarded independently.

[5 marks]

(b) $y \neq 4$ (or equivalent)**A1****[1 mark]**

$$(c) \quad 2 + \frac{5}{2} \text{ OR } \left(-\frac{1}{2}\right) + 2 \times \frac{5}{2} \text{ OR } \frac{-\frac{1}{2} + p}{2} = 2 \text{ OR } -4 = -p + \frac{1}{2}$$

A1

$$p = \frac{9}{2}$$

AG**[1 mark]**(d) **METHOD 1**

attempt to substitute both roots to form a quadratic

(M1)**EITHER**

$$\left(x + \frac{1}{2}\right)\left(x - \frac{9}{2}\right) \text{ OR } x^2 - \left(-\frac{1}{2} + \frac{9}{2}\right)x + \left(-\frac{1}{2} \times \frac{9}{2}\right)$$

$$= x^2 - 4x - \frac{9}{4}$$

A1A1

$$\left(b = -4, c = -\frac{9}{4}\right)$$

Note: Award **A1** for each correct value. They may be embedded or stated explicitly.

OR

$$(2x + 1)(2x - 9) = 4\left(x^2 - 4x - \frac{9}{4}\right)$$

$$b = -4, c = -\frac{9}{4}$$

A1A1

Note: Award **A1** for each correct value. They must be stated explicitly.

METHOD 2

$$-\frac{b}{2} = 2 \text{ OR } 4 + b = 0 \Rightarrow b = -4$$

A1attempt to form a valid equation to find c using their b **(M1)**

$$\left(-\frac{1}{2}\right)^2 + -4\left(-\frac{1}{2}\right) + c = 0 \text{ OR } \left(\frac{9}{2}\right)^2 + -4\left(\frac{9}{2}\right) + c = 0$$

$$c = -\frac{9}{4}$$

A1**METHOD 3**attempt to form two valid equations in b and c **(M1)**

$$\left(-\frac{1}{2}\right)^2 + b\left(-\frac{1}{2}\right) + c = 0, \left(\frac{9}{2}\right)^2 + b\left(\frac{9}{2}\right) + c = 0$$

$$b = -4, c = -\frac{9}{4}$$

A1A1**METHOD 4**attempt to write $g(x)$ in the form $(x-h)^2 + k$ and substitute for x, h and $g(x)$ **(M1)**

$$\left(-\frac{1}{2} - 2\right)^2 + k = 0 \Rightarrow k = -\frac{25}{4}$$

$$(x-2)^2 - \frac{25}{4}$$

$$= x^2 - 4x - \frac{9}{4}$$

A1A1

$$\left(b = -4, c = -\frac{9}{4}\right)$$

Note: Award **A1** for each correct value. They may be embedded or stated explicitly.

[3 marks]

(e) attempt to substitute $x = 2$ into their $g(x)$ OR

complete the square on their $g(x)$ (may be seen in part (d))

(M1)

$$y = -\frac{25}{4}$$

A1

[2 marks]

(f) $\frac{4x+2}{x-2} = \left(x + \frac{1}{2}\right)\left(x - \frac{9}{2}\right)$ OR $\frac{4x+2}{x-2} = x^2 - 4x - \frac{9}{4}$

attempt to form a cubic equation

(M1)

EITHER

$$4x+2 = (x-2)\left(x + \frac{1}{2}\right)\left(x - \frac{9}{2}\right) \text{ OR } 4x+2 = \left(x^2 - 4x - \frac{9}{4}\right)(x-2) \text{ OR}$$

$$(x-2)\left(x + \frac{1}{2}\right)\left(x - \frac{9}{2}\right) - 4x - 2 \text{ OR } (x-2)\left(x^2 - 4x - \frac{9}{4}\right) - 4x - 2$$

$$x^3 + \dots + \frac{5}{2} (=0) \text{ OR } 4x^3 + \dots + 10 (=0)$$

(A1)(A1)

Note: Award **(A1)** for each of the terms x^3 and $\frac{5}{2}$ or $4x^3$ and 10. Ignore extra terms.

$$\text{product of roots} = \left(\frac{(-1)^3 \times \frac{5}{2}}{1}\right) \text{ OR } \left(\frac{(-1)^3 \times 10}{4}\right)$$

$$= -\frac{5}{2}$$

A1

OR

$$4\left(x + \frac{1}{2}\right) = (x-2)\left(x + \frac{1}{2}\right)\left(x - \frac{9}{2}\right)$$

$$x = -\frac{1}{2}$$

(A1)

$$\text{or } 4 = x^2 + \dots + 9 \Rightarrow x^2 + \dots + 5 = 0$$

product of roots of quadratic is 5

(A1)

$$\text{product is therefore } -\frac{1}{2} \times 5$$

$$= -\frac{5}{2}$$

A1

[4 marks]

Total [16 marks]

29 - (MathAA/12_HL_Summer_2024_Q5) - Algebra-functions - Roots, Graphs

(a) $y = \frac{2}{3}$ (must be written as equation with $y =$)

A1

[1 mark]

(b) (i) 2

A1

(ii) EITHER

$$\frac{2(x+3)}{3(x+2)} = mx + 1$$

attempt to expand to obtain a quadratic equation

(M1)

$$2x + 6 = 3mx^2 + 6mx + 3x + 6$$

$$3mx^2 + (6m+1)x = 0 \quad \text{OR} \quad 3mx^2 + 6mx + x = 0$$

A1

recognition that discriminant $\Delta = 0$ for one solution

(M1)

$$(6m+1)^2 = 0$$

OR

$$\frac{2(x+3)}{3(x+2)} = mx + 1$$

attempt to expand to obtain a quadratic equation

(M1)

$$2x + 6 = 3mx^2 + 6mx + 3x + 6$$

$$3mx^2 + (6m+1)x = 0 \quad \text{OR} \quad 3mx^2 + 6mx + x = 0$$

A1attempt to solve their quadratic for x and equating their solutions**(M1)**

$$x(3mx + 6m + 1) = 0$$

$$x = 0 \quad \text{OR} \quad x = -\frac{6m+1}{3m} (= 0)$$

$$-\frac{6m+1}{3m} = 0$$

ORattempt to find $f'(x)$ using the quotient rule**(M1)**

$$f'(x) = \frac{2}{3} \left(\frac{(x+2) - (x+3)}{(x+2)^2} \right) = \left(\frac{-2}{3(x+2)^2} \right) \quad \text{OR} \quad \frac{2(3x+6) - 3(2x+6)}{(3x+6)^2} \quad \text{or}$$

equivalent

A1recognition that m is the derivative of $f(x)$ at $x = 0$ **(M1)****THEN**

$$\Rightarrow m = -\frac{1}{6}$$

A1

(iii)

Note: In this part, FT may be awarded only for values of m between -1 and 0 .

$$-\frac{1}{6} < m < 0$$

A2

Note: Award **A1** for only $m > -\frac{1}{6}$. Award **A1** for only $m < 0$.

[7 marks]**Total [8 marks]**

30 - (MathAA/12_HL_Summer_2024_Q6) - Algebra-functions - Roots

(a) 2.5(%)

A1

[1 mark]

(b) $P(\text{weight of cooking apples} > 140) = 0.5 (50\%)$ (seen anywhere)

(A1)

recognition of conditional probability in context

(M1)

$$P(\text{eating apple} | \text{weight} > 140) = \frac{P(\text{eating apple and weight of eating apple} > 140)}{P(\text{weight of apple} > 140)}$$

OR

$$\frac{P(\text{eating apple} \cap \text{weight of eating apple} > 140)}{P(\text{eating apple} \cap \text{weight of eating apple} > 140) + P(\text{cooking apple} \cap \text{weight of cooking apple} > 140)}$$

OR

$$\frac{P(\text{eating apple})P(\text{weight of eating apple} > 140)}{P(\text{eating apple})P(\text{weight of eating apple} > 140) + P(\text{cooking apple})P(\text{weight of cooking apple} > 140)}$$

$$= \frac{0.8 \times 0.025}{0.8 \times 0.025 + 0.2 \times 0.5} \left(= \frac{80 \times 2.5}{80 \times 2.5 + 20 \times 50} \right) \quad (\text{A1})$$

$$= \frac{200}{1200} \left(= \frac{1}{6} \right) \quad \text{A1}$$

Note: Accept any equivalent exact answer written as a fraction.

[4 marks]

Total [5 marks]

31 - (MathAA/12_HL_Summer_2024_Q7) - Algebra-functions - Roots

(a) $\alpha + \beta + \gamma = \frac{7}{2}$

A1

[1 mark]

(b) $p - 3i$ is also a root (seen anywhere)

A1

recognition of 5 roots and attempt to sum these roots

(M1)

$$p + 3i + p - 3i + \frac{7}{2}$$

$$p + 3i + p - 3i + \frac{7}{2} = \frac{11}{2}$$

A1

$$p = 1$$

AG

[3 marks]

(c) (i) attempt to find product of 5 roots and equate to ± 10

(M1)

$$(1 + 3i)(1 - 3i)\frac{1}{2}\alpha\beta = 10$$

$$\alpha\beta = 2$$

A1

(ii) $\alpha = 1$ and $\beta = 2$

A1

[3 marks]

Total [7 marks]

32 - (MathAA/1_HL_Winter_2024_Q3) - Algebra-functions - Roots

METHOD 1

attempt to expand $(3n+2)^2 - (3n-2)^2$ **M1**

Note: Award **M0** for invalid attempts such as $(3n+2)^2 = 9n^2 + 4$.

$= 9n^2 + 12n + 4 - (9n^2 - 12n + 4)$ or equivalent **A1**

$= 24n$ OR $12n + 12n$ **A1**

$= 12(2n)$ OR $\frac{24}{12} = 2$ OR $\frac{24n}{2} = 12n$ OR $12n + 12n = 12(n+n)$ (or equivalent) **R1**

so is a multiple of 12 **AG**

Note: Do not award the **R1** unless both **A** marks have been awarded.

METHOD 2

use of $a^2 - b^2 = (a+b)(a-b)$ where $a = 3n+2$, $b = 3n-2$ **M1**

$= (3n+2+3n-2)(3n+2-3n+2)$

$= 6n \times 4$ **A1**

$= 24n$ **A1**

$= 12(2n)$ OR $\frac{24n}{12} = 2n$ OR $\frac{24n}{2} = 12n$ OR $12n + 12n = 12(n+n)$ (or equivalent)

R1

so is a multiple of 12 **AG**

Note: Do not award the **R1** unless both **A** marks have been awarded.

Question 3 continued.

METHOD 3

base case $n = 1$: $(3(1) + 2)^2 - (3(1) - 2)^2 = 25 - 1 = 24$

so true for $n = 1$

A1

assume true for $n = k$ i.e. $(3k + 2)^2 - (3k - 2)^2$ is a multiple of 12

M1

consider $n = k + 1$:

$$(3(k + 1) + 2)^2 - (3(k + 1) - 2)^2$$

$$((3k + 2) + 3)^2 - ((3k - 2) + 3)^2$$

$$(3k + 2)^2 + 6(3k + 2) + 9 - ((3k - 2)^2 + 6(3k - 2) + 9)$$

$$(3k + 2)^2 - (3k - 2)^2 + 24$$

using the assumption $(3k + 2)^2 - (3k - 2)^2 = 12M$

$$12M + 24$$

$$12(M + 2)$$

which is a multiple of 12, hence true for $n = k + 1$

A1

since true for $n = 1$, and true for $n = k$ implies true for $n = k + 1$

therefore, true for all $n \in \mathbb{Z}^+$

R1

Note: Do not award the **R1** unless both **A** marks have been awarded.

[4 marks]

33 - (MathAA/1_HL_Winter_2024_Q6) - Algebra-functions - Roots

(a) attempt to find critical values

(M1)

$$x = \frac{3}{2}, x = 6$$

(A1)

$$\frac{3}{2} < x < 6$$

A1

Note: Allow equivalent, and/or interval notation.

[3 marks]

(b) $k = \frac{3}{2}$

A1

since we require $2x^2 - 15x + 18 \geq 0$ (and f must be one to one)

R1

OR

Does not obey horizontal line test for $x \geq \frac{3}{2}$

R1

[2 marks]

Total [5 marks]

34 - (MathAA/11_HL_Summer_2025_Q1) - Differentiation, Algebra-functions - Roots, Graphs

$$f'(x) = 4x^2 - 16$$

A1

sets their derivative equal to zero

(M1)

$$4x^2 - 16 = 0, (x = \pm 2)$$

$$p = 2 \text{ (accept } x = 2 \text{)}$$

A1

substitutes their **positive** p into $f(x)$

(M1)

$$y = \frac{4(2^3)}{3} - 16(2) \left(= \frac{32}{3} - 32 = -\frac{64}{3} \right)$$

$$q = -\frac{64}{3} \text{ (accept } y = \frac{-64}{3} \text{)}$$

A1

Total [5 marks]

35 - (MathAA/11_HL_Summer_2025_Q10) - Quadratics, Algebra-functions - Roots, Exponentials - Logarithms

(a) METHOD 1

$$a = 5 \quad (A1)$$

attempt to use roots and symmetry to find h (M1)

$$h = \frac{(-1) + (-3)}{2} \text{ OR half the distance between the roots } \frac{(-1) - (-3)}{2} = 1 \text{ (may be seen on a diagram)}$$

$$h = -2 \text{ (accept } x = -2) \quad (A1)$$

$$f(x) = 5(x - (-2))^2 - 5 (= 5(x + 2)^2 - 5) \quad A1$$

$$(a = 5, h = -2, k = -5)$$

METHOD 2

$$a = 5 \quad (A1)$$

attempt to expand

$$(x + 1)(x + 3) = x^2 + 4x + 3 \text{ OR } 5(x + 1)(x + 3) = 5x^2 + 20x + 15$$

EITHER

uses their expansion to attempt to complete the square to the form (M1)

$$p(x + q)^2 + r, \text{ where } q \text{ is half the coefficient of their } x \text{ term}$$

$$= (x + 2)^2 - 2^2 + 3 (= (x + 2)^2 - 1) \text{ OR } 5[(x + 2)^2 - 2^2 + 3] (= 5(x + 2)^2 - 5) \quad (A1)$$

OR

uses their expansion to attempt to differentiate and sets equal to zero (M1)

$$\frac{dy}{dx} = 2x + 4 = 0 \text{ OR } \frac{dy}{dx} = 10x + 20 = 0$$

$$h = -2 \text{ (accept } x = -2) \quad (A1)$$

OR

uses their expansion to attempt to find axis of symmetry using $h = \frac{-b}{2a}$ (M1)

$$h = \frac{-4}{2} \text{ OR } h = \frac{-20}{10}$$

$$h = -2 \text{ (accept } x = -2) \quad (A1)$$

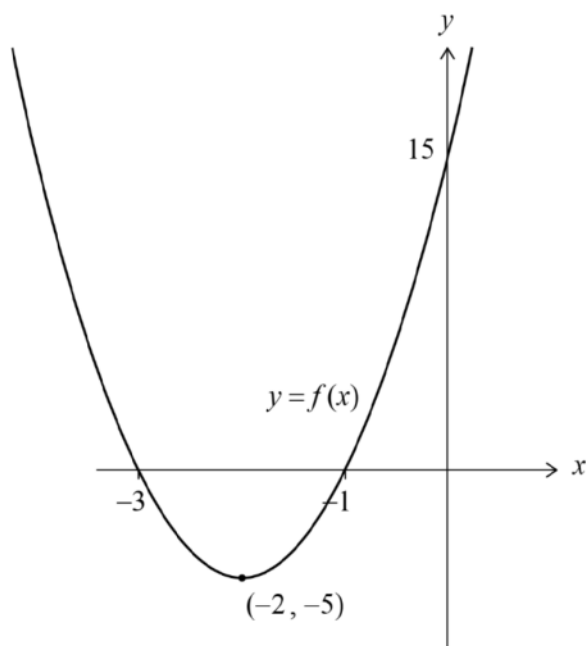
THEN

$$f(x) = 5(x - (-2))^2 - 5 (= 5(x + 2)^2 - 5) \quad A1$$

$$(a = 5, h = -2, k = -5)$$

[4 marks]

(b)

**M1A1A1A1**award **M1** for a roughly symmetric curve which is concave upaward **A1** for x intercepts at -3 and -1 award **A1** for y intercept at 15 award **A1** for vertex at $(-2, -5)$ **[4 marks]**(c) $5(x+2)^2 - 5 \leq 40$ OR $5(x+1)(x+3) \leq 40$ OR $(x+1)(x+3) \leq 8$ leading to $(x+2)^2 \leq 9$ OR $5x^2 + 20x - 25 \leq 0$ OR $x^2 + 4x - 5 \leq 0$ **(A1)**valid attempt to find the critical values for their quadratic inequality **(M1)** $x+2 = \pm 3$ OR $(x+5)(x-1) = 0$ $x = -5, x = 1$ **(A1)** $-5 \leq x \leq 1$ **A1**

Note: Accept $(x \in)[-5, 1]$ or equivalent.

[4 marks]

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