Topical Past Papers

Edexcel IGCSE Further Pure Mathematics

[(4PM1) | Paper 1]

Exam Series: January 2017 - May 2024

Format Type B:

Each question is followed by its answer scheme

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Introduction

Each Topical Past Paper Questions Workbook contains a comprehensive collection of hundreds of questions and corresponding answer schemes, presented in worksheet format. The questions are carefully arranged according to their respective chapters and topics, which align with the latest Edexcel IGCSE or AS/A Level subject content. Here are the key features of these resources:

1. The workbook covers a wide range of topics, which are organized according to the latest syllabus content for Edexcel IGCSE or A Level exams.
2. Each topic includes numerous questions, allowing students to practice and reinforce their understanding of key concepts and skills.
3. The questions are accompanied by detailed answer schemes, which provide clear explanations and guidance for students to improve their performance.
4. The workbook's format is user-friendly, with worksheets that are easy to read and navigate.
5. This workbook is an ideal resource for students who want to familiarize themselves with the types of questions that may appear in their exams and to develop their problem-solving and analytical skills.

Overall, Topical Past Paper Questions Workbooks are a valuable tool for students preparing for IGCSE or A Level exams, providing them with the opportunity to practice and refine their knowledge and skills in a structured and comprehensive manner. To provide a clearer description of this book's specifications, here are some key details:

- Title: Edexcel International GCSE Further Pure Mathematics (4PM1) Paper 1 Topical Past Papers
- Subtitle: Exam Practice Worksheets With Answer Scheme
- Examination board: Pearson Edexcel
- Subject code: 4PM1
- Years covered: January 2017 - May 2024
- Paper: 1
- Number of pages: 1099
- Number of questions: 258

Contents

Chapter 1

Logarithmic functions and indices

1. 4pm1-01-que-20240522 Q: 9

(a) Find the value of a such that $\log_a 8 = \dfrac{3}{4}$

(2)

(b) Show that

$$3x\log_2 x - 4\log_{16} 8 + 6x\log_4 8 - \log_2 x = \log_2 (8x)^{3x-1}$$

(4)

(c) Hence solve the equation $3x\log_2 x - 4\log_{16} 8 + 6x\log_4 8 - \log_2 x = 0$

(3)

Question 9 continued

Question 9 continued

Question 9 continued

(Total for Question 9 is 9 marks)

Answer:

Question	Scheme	Marks
(a)	$\left(\log_a 8 = \frac{3}{4} \Rightarrow\right) a^{\frac{3}{4}} = 8 \left(\Rightarrow a = \left(\sqrt[3]{8}\right)^4\right) = 16$	M1A1 [2]
(b)	$\left(3x\log_2 x - 4\log_{16} 8 + 6x\log_4 8 - \log_2 x =\right) 3x\log_2 x - \frac{4\log_2 8}{\log_2 16} + \frac{6x\log_2 8}{\log_2 4} - \log_2 x$	M1
	$\Rightarrow 3x\log_2 x - \log_2 8 + 3x\log_2 8 - \log_2 x$	M1
	$= (3x-1)\log_2 x + (3x-1)\log_2 8$ or $3x\log_2 8x - (1)\log_2 8x$	
	$\Rightarrow (3x-1)\log_2 8x$ or $\log_2 (8x)^{3x} + \log_2 (8x)^{-1} = \log_2 (8x)^{3x-1}$ *	M1A1 cso [4]
ALT	$\left(3x\log_2 x - 4\log_{16} 8 + 6x\log_4 8 - \log_2 x =\right) 3x\log_2 x - \frac{4\log_2 8}{\log_2 16} + \frac{6x\log_2 x}{\log_2 4} - \log_2 x$	M1
	$\Rightarrow 3x\log_2 x - \log_2 8 + 3x\log_2 8 - \log_2 x = \log_2 x^{3x} - \log_2 8 + \log_2 8^{3x} - \log_2 x$	M1
	$\left(\Rightarrow \log_2 (8x)^{3x} - \log_2 8x \Rightarrow\right) \log_2 \left(\frac{(8x)^{3x}}{8x}\right)$ or $\log_2 \left((8x)^{3x} \times (8x)^{-1}\right)$	
	or $\log_2 x^{3x-1} 8^{3x-1} = \log_2 (8x)^{3x-1}$ *	M1A1 cso [4]
"Box 3" of part b We will see unanticipated methods once live marking begins. If the answer is correct and **there is no incorrect working**, check the work carefully, to ascertain if they've shown enough steps to demonstrate use of the three main log laws this question tests and award full marks. If in any doubt at all – the response **MUST** be sent to review. Other than this exception, please mark to the following rules. **Also use these rules if students don't gain the 2nd or 3rd M under the main or ALT schemes.** • M1 for any correct change of base to base 2 • M1 for any two correct applications of the power law or for $ax\log_2 8x + b\log_2 8x \Rightarrow \log_2 (8x)^{ax+b}$ or $(ax+b)\log_2 8x \Rightarrow \log_2 (8x)^{ax+b}$ $a, b \neq 0$ • M1 for any two correct applications of the addition or subtraction law In each case – ignore any incorrect working. **Poor or incorrect bracketing may not be recovered in this question**. (general principle of marking is usually that it can).		
(c)	$\left[\log_2 (8x)^{3x-1} = 0 \Rightarrow \log_2 (8x)^{3x-1} = \log_2 8^0 \text{ or } (3x-1)\log_2 (8x) = 0\right]$	
	$\Rightarrow 3x - 1 = 0 \Rightarrow x = \frac{1}{3}$	M1A1
	$\Rightarrow 8x = 1 \Rightarrow x = \frac{1}{8}$	A1 [3]
		Total 9 marks

2. 4pm1-01r-que-20240522 Q: 7

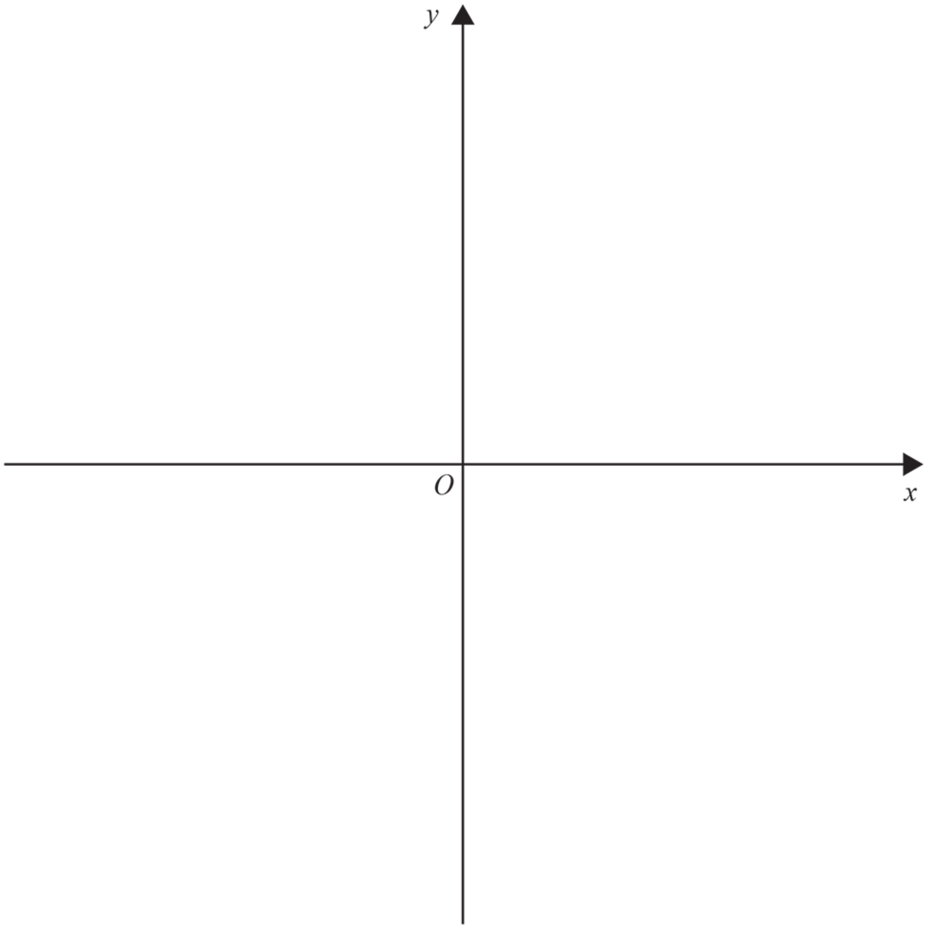
The curve C has equation $y = -\log_4(x + 4)$

(a) Using the axes below, sketch the graph of C.
Label the coordinates of the points of intersection of C with the coordinate axes and the equation of any asymptote to C.

(4)

(b) Solve the equation $\log_{(x+4)} 256 - \log_4(x + 4) = 0$

(5)



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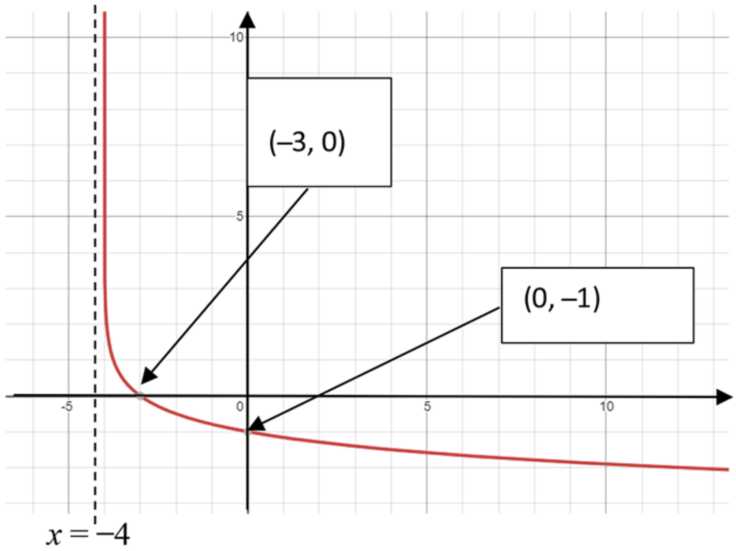
Question 7 continued

(Total for Question 7 is 9 marks)

Answer:

<table>
<tr><th>Question</th><th>Scheme</th><th>Marks</th></tr>
<tr><td>(a)</td><td>
</td><td>B1
Shape and position

B1
Intersection with x-axis

B1
Intersection with y-axis

B1
Asymptote
[4]</td></tr>
<tr><td>(b)</td><td>$\log_{(x+4)} 256 - \log_4(x+4) = 0 \Rightarrow \dfrac{\log_4 256}{\log_4(x+4)} - \log_4(x+4) = 0$

$\Rightarrow 4 - \left(\log_4(x+4)\right)^2 = 0 \Rightarrow \left(\log_4(x+4)\right)^2 = 4$

$\Rightarrow \log_4(x+4) = \pm 2$

$x + 4 = 4^2 \Rightarrow x = 12$

$x + 4 = 4^{-2} \Rightarrow x = -\dfrac{63}{16}$ or -3.9375</td><td>M1

M1

M1
A1

A1
[5]</td></tr>
<tr><td colspan="3" align="right">Total 9 marks</td></tr>
</table>

3. 4pm1-01-que-20230527 Q: 6

Solve the equation

$$\log_2 x^3 + \log_4 x^2 - 3\log_x 2 = 0$$

giving your answers to 3 significant figures.

(8)

Question 6 continued

(Total for Question 6 is 8 marks)

Answer:

Question	Scheme	Marks
	$\log_2 x^3 + \log_4 x^2 - 3\log_x 2 = \log_2 x^3 + \dfrac{\log_2 x^2}{\log_2 4} - \dfrac{3\log_2 2}{\log_2 x} = 0$	M1
	$= 3\log_2 x + \dfrac{2\log_2 x}{\log_2 4} - \dfrac{\log_2 2^3}{\log_2 x} = 0$	M1
	$3\log_2 x + \dfrac{2\log_2 x}{2} - \dfrac{3}{\log_2 x} = 0$	B1
	$\Rightarrow 3(\log_2 x)^2 + (\log_2 x^2)^2 - 3 = 0$	M1
	$\Rightarrow 4(\log_2 x)^2 = 3 \Rightarrow (\log_2 x)^2 = \dfrac{3}{4} \Rightarrow \log_2 x = \pm\sqrt{\dfrac{3}{4}}$	M1
	$\Rightarrow x = 2^{\sqrt{\frac{3}{4}}} \approx 1.82$ or $x = 2^{-\sqrt{\frac{3}{4}}} \approx 0.549$	M1A1A1
		[8]
		Total 8 marks

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4. 4PM1_01_que_20220112 Q: 7

(a) Write down the value of $\log_2 16$

(1)

Given that $4 + 2\log_4 x = \log_2 y$

(b) show that $y = 16x$

(4)

(c) Hence solve the equation $4 + 2\log_4 x = \log_2(4x + 5)$

(3)

Question 7 continued

(Total for Question 7 is 8 marks)

Answer:

Question	Scheme	Marks
(a)	4	B1 [1]
(b)	**Working in $\log_2$** $2\log_4 x = \frac{2\log_2 x}{\log_2 4} = \frac{2\log_2 x}{2} = [\log_2 x]$ $\log_2 16 + \frac{2\log_2 x}{2} = \log_2 y$ $\log_2 16x = \log_2 y$ OR $\log_2\left(\frac{x}{y}\right) = -4 \Rightarrow \frac{x}{y} = 2^{-4}$ $y = 16x$*	M1 M1 M1 A1 cso [4]
	ALT	
	Working in $\log_4$ $\log_2 y = \frac{\log_4 y}{\log_4 2} = \frac{\log_4 y}{\frac{1}{2}} = 2\log_4 y = [\log_4 y^2]$ $\log_4 256 + \log_4 x^2 = \log_4 y^2$ $\log_4\left(256x^2\right) = \log_4 y^2$ OR $2\log_4\left(\frac{y}{x}\right) = 4 \Rightarrow \frac{y}{x} = 4^2$ $256x^2 = y^2 \Rightarrow y = 16x$*	[M1 M1 M1 A1]
(c)	$16x = 4x + 5$ $16x = 4x + 5 \Rightarrow 12x = 5 \Rightarrow x = ...$ $x = \frac{5}{12}$	B1 M1 A1 [3]
		Total 8 marks

5. 4pm1-01r-que-20220527 Q: 7

A curve C has equation $y = \log_{10}(x + 2)$

(a) Using the axes below, sketch the graph of C.
Label the coordinates of the points of intersection of C with the coordinate axes.

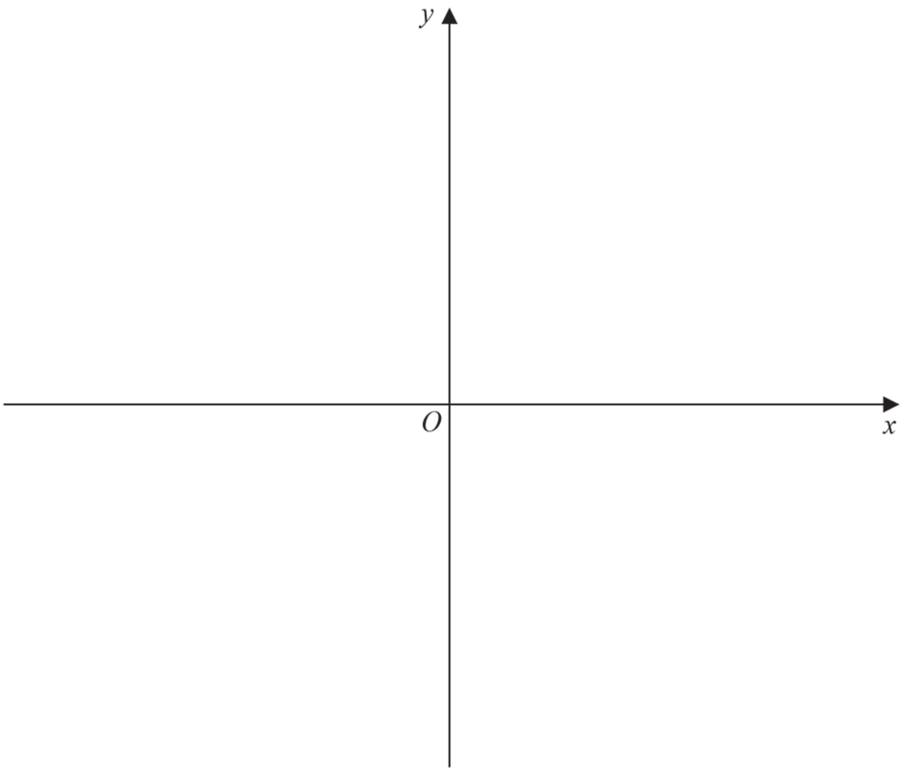
(2)

(b) Solve the equation $2(\log_a 4 + \log_a 16) = 1$

(3)

(c) Solve the equation $5\log_q 16 + 4\log_2 q = 24$

(6)



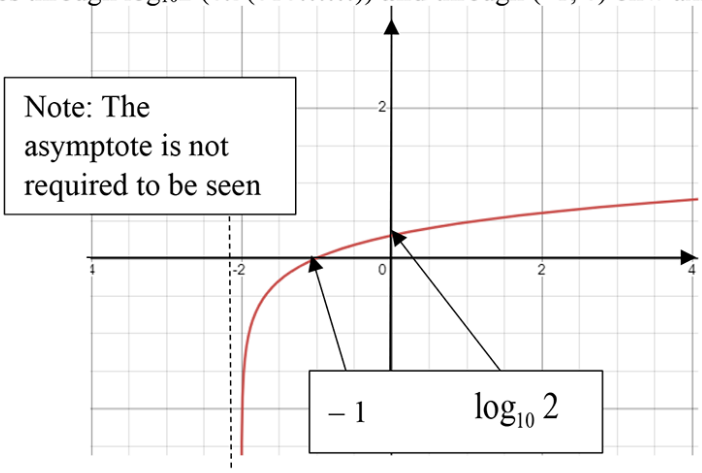
Question 7 continued

Question 7 continued

Question 7 continued

(Total for Question 7 is 11 marks)

Answer:

Question number	Scheme		Marks
(a)	Shape of the curve, **crossing** negative x-axis and positive y-axis with the asymptotic nature of the curve shown.		B1
	passes through $\log_{10}2$ (0.3(010……)) **and** through (–1, 0) on x-axis		B1 [2]
			
(b)	$\left(2\log_a 4 + 2\log_a 4^2\right)$ $2\log_a 4 + 4\log_a 4$	or $(2\log_a 2^2 + 2\log_a 2^4)$ $4\log_a 2 + 8\log_a 2$	M1
	$\log_a 4 = \frac{1}{6}$	or $\log_a 2 = \frac{1}{12}$ oe	dM1
	$a^{\frac{1}{6}} = 4$	or $a^{\frac{1}{12}} = 2$	
	$a = 4096$		A1
	ALT		
	$(2)\log_a 64 = 1 \Rightarrow \log_a 64 = \frac{1}{2}$		[M1
	$\Rightarrow a^{\frac{1}{2}} = 64$		dM1
	$\Rightarrow a = 4096$		A1] [3]
(c)	$\log_q 16 \to \frac{\log_2 16}{\log_2 q}$	$\log_2 q \to \frac{\log_q q}{\log_q 2}$ or $\frac{1}{\log_q 2}$	M1
	$\left(5\frac{\log_2 16}{\log_2 q} + 4\log_2 q = 24\right)$	$\left(5\log_q 16 + 4\frac{\log_q q}{\log_q 2} = 24\right)$	
	$20 + 4(\log_2 q)^2 = 24\log_2 q$	$20(\log_q 2)^2 + 4 = 24\log_q 2$	M1 A1
	$(\log_2 q - 5)(\log_2 q - 1)(=0)$	$(5\log_q 2 - 1)(\log_q 2 - 1)(=0)$	dM1
	$(\log_2 q = 5) \to q = 32$ $(\log_2 q = 1) \to q = 2$	$\left(\log_q 2 = \frac{1}{5}\right) \to q = 32$ $(\log_q 2 = 1) \to q = 2$	M1 A1 [6]
			Total 11 marks

6. 4PM1_01_que_20210427 Q: 8

Given that n satisfies the equation

$$\log_a n = \log_a 3 + \log_a(2n - 1)$$

(a) find the value of n.

(3)

Given that $\log_p x = 3$ and $\log_p y - 3\log_p 2 = 4$

(b) (i) express x in terms of p,

(1)

(ii) express xy in terms of p.

(4)

Question 8 continued

Question 8 continued

Question 8 continued

(Total for Question 8 is 8 marks)

Answer:

Question number	Scheme	Marks
(a)	$\log_a n = \log_a 3(2n-1)$	M1
	$n = 3(2n-1)$	M1
	$n = \dfrac{3}{5}$	A1 (3)
(b)(i)	$x = p^3$	B1 (1)
(b)(ii)	$\log_p y - \log_p 2^3 = 4 \Rightarrow \log_p\left(\dfrac{y}{2^3}\right) = 4$ or $\log_p\left(\dfrac{y}{8}\right) = 4$	M1
	$\dfrac{y}{2^3} = p^4 \Rightarrow \left(y = 2^3 p^4 \text{ or } 8p^4\right)$	M1
	$xy = 8p^7$	M1A1 (4)
	ALT (b)(ii)	
	$\log_p x + \log_p y - 3\log_p 2 = 4 + 3 \Rightarrow \log_p\left(\dfrac{xy}{2^3}\right) = 7$	{M1}
	$\dfrac{xy}{2^3} = p^7$	{M1}
	$xy = 8p^7$	{M1A1} (4)
		Total 8 marks

7. 4PM1_01_que_20211207 Q: 5

Solve the equation

$$\log_3 \sqrt{x-5} + \log_9 (x+3) - 1 = 0$$

Show clear algebraic working.

(7)

Question 5 continued

(Total for Question 5 is 7 marks)

Answer:

Question number	Scheme	Marks
	$\log_3 \sqrt{x-5} + \log_9 (x+3) - 1 = 0$ $\frac{1}{2}\log_3(x-5) + \frac{\log_3(x+3)}{\log_3 9} = 1 \Rightarrow \left(\frac{1}{2}\log_3(x-5) + \frac{\log_3(x+3)}{2} = 1\right)$ $\log_3(x-5) + \log_3(x+3) = 2 \Rightarrow \log_3\left[(x-5)(x+3)\right] = 2$ $\Rightarrow (x-5)(x+3) = 3^2 \Rightarrow x^2 - 2x - 24 = 0$ $(x+4)(x-6) = 0 \Rightarrow x = 6$ (reject $x = -4$)	M1M1 M1 M1A1 dM1A1 [7]
Total 7 marks		

8. 4PM1_01_que_20200305 Q: 7

Solve the equation

$$\log_7(8x^2 - 6x + 3) - \log_{49} x^2 = 3\log_7 2$$

(5)

Question 7 continued

(Total for Question 7 is 5 marks)

Answer:

Question number	Scheme	Marks
	$\frac{\log_7 x^2}{\log_7 49}$	B1
	$\log_7\left(\frac{8x^2-6x+3}{x}\right)$, $\log_7 2^3$	M1 A1
	$\frac{8x^2-6x+3}{x} = 2^3$	
	$8x^2 - 14x + 3 = 0$	M1
	$(4x-1)(2x-3) = 0$	
	$x = \frac{1}{4}, \frac{3}{2}$	A1
		[5]

9. 4PM1_01R_que_20200305 Q: 5

(a) Show that $\log_4 32 = \frac{5}{2}$

(2)

(b) Hence, or otherwise, find the exact solutions of the equation

$$\log_2 x - \log_4 32 + \frac{1}{4}\log_x 16 = 0$$

(7)

Question 5 continued

(Total for Question 5 is 9 marks)

Answer:

Question number	Scheme	Marks
(a)	$\log_4 32 = \frac{\log_2 32}{\log_2 4} = \frac{5}{2}$* or $\log_4 32 = \log_4 4^{\frac{5}{2}} = \frac{5}{2}$* or $\log_4 32 = \log_{2^2} 2^5 = \frac{5}{2}$* **ALT** $\log_4 32 = a \quad \Rightarrow 4^a = 32 \quad \Rightarrow a = \frac{5}{2}$ *	M1A1cso [2] {M1}{A1} cso [2]
(b)	$\log_2 x - \log_4 32 + \frac{1}{4}\log_x 16 = 0$ Let $\log_2 x = y$ $y - \frac{5}{2} + \frac{1}{4}\left(\frac{\log_2 16}{\log_2 x}\right) = 0$ or $y - \frac{5}{2} + \frac{1}{\log_2 x} = 0$ $\Rightarrow y - \frac{5}{2} + \frac{1}{y} = 0$ $\Rightarrow 2y^2 - 5y + 2 = 0$ $\Rightarrow (2y-1)(y-2) = 0$ $\Rightarrow y = \log_2 x = \frac{1}{2}$ or 2 $\Rightarrow x = 2^{\frac{1}{2}} = \sqrt{2}$ and $x = 2^2 = 4$	M1 M1A1 M1 M1 M1A1 [7]
		Total 9 marks
(a)		
M1	For $\log_4 32 = \frac{\log_2 32}{\log_2 4}$ or $\log_4 32 = \log_4 4^{\frac{5}{2}}$ or $\log_4 32 = \log_{2^2} 2^5$	
ALT		
M1	For $4^a = 32$	
A1 cso	Obtains the **given** answer with no errors in the working	
(b)		
M1	Use of $\log_a x = \frac{\log_b x}{\log_b a}$ or $\log_a b = \frac{1}{\log_b a}$	
M1	Forming a 3TQ	
A1	$2y^2 - 5y + 2 = 0$	
M1	Solving the 3TQ	
M1	For $y = \log_2 x = \frac{1}{2}$ or 2	
M1	Either $x = 2^{\frac{1}{2}} = \sqrt{2}$ or $x = 2^2 = 4$	
A1	Both $x = 2^{\frac{1}{2}} = \sqrt{2}$ and $x = 2^2 = 4$	

10. 4PM1_01R_que_20201120 Q: 3

The nth term of an arithmetic series is u_n such that

$$u_n = \ln a + (n - 1)\ln b$$

where a and b are positive integers.

Given that $u_2 = \ln 12$ and that $u_5 = \ln 768$

find the value of a and the value of b.

(7)

Question 3 continued

(Total for Question 3 is 7 marks)

Answer:

Question number	Scheme	Marks
	$\ln 12 = \ln a + (2-1)\ln b$ oe $ab = 12$ oe $\ln 768 = \ln a + (5-1)\ln b$ oe $ab^4 = 768$ oe $\frac{768}{12} = \frac{ab^4}{ab}$ $(b^3 = 64)$ $b = 4 \quad a = 3$	M1 A1 M1 A1 ddM1 A1 A1
	ALT 1 $\ln a + (2-1)\ln b = \ln 12$ oe $\ln a + (5-1)\ln b = \ln 768$ oe $3\ln b = \ln b^3 \quad \ln 768 - \ln 12 = \ln 64$ $b^3 = 64$ $b = 4 \quad a = 3$	M1 A1 M1 A1 ddM1 A1 A1
	ALT 2 $d = \ln 12 - \ln a$ $(d = \ln b = \ln 12 - \ln a \Rightarrow \ln b = \ln\left(\frac{12}{a}\right)) \Rightarrow b = \frac{12}{a}$ $\ln 768 = \ln a + \ln\left(\frac{12}{a}\right)^4$ $\ln 768 = \ln\left(\frac{12^4}{a^3}\right)$ $a^3 = \frac{20736}{768}$ $b = 4 \quad a = 3$	M1 A1 M1 A1 ddM1 A1 A1
	ALT 3 $(u_2 =) \quad u_1 + d = \ln 12$ $(u_5 =) \quad u_1 + 4d = \ln 768$ $3d = \ln 768 - \ln 12$ $d = \ln 4$ $u_1 = \ln 12 - \ln 4 = \ln 3 \ (= \ln a)$ $b = 4 \quad a = 3$	M1 A1 M1 A1 ddM1 A1 A1 **[7]**

11. 4PM1_01R_que_20201120 Q: 9

Showing your working clearly, use algebra to solve the equations

$$\frac{16^x}{8^y} = \frac{1}{4}$$

$$4^x 2^y = 16$$

(7)

Question 9 continued

(Total for Question 9 is 7 marks)

Answer:

Question number	Scheme	Marks
	$\frac{2^{4x}}{2^{3y}} = \frac{1}{2^2}$	M1
	$2^{4x-2y} = 2^{-2}$ $(\rightarrow\ 4x - 3y = -2)$	dM1
	$2^{2x}2^{y} = 2^4$	M1
	$2^{2x+y} = 2^4$ $\rightarrow (2x + y = 4)$	dM1
	A fully correct method using for solving simultaneously leading to either $10x = 10$ **or** $5y = 10$ $4x - 3y = -2 \Rightarrow 10x = 10$ or $4x - 3y = -2 \Rightarrow 5y = 10$ $6x + 3y = 12$ $4x + 2y = 8$	ddddM1
	$y = 2$ $x = 1$	A1 A1 **[7]**
	Alternative Method $4^x = \frac{16}{2^y}$	M1
	$\frac{4^{2x}}{8^y} = \frac{1}{4}$	M1
	$\left(\frac{16}{2^y}\right)^2 \times \frac{1}{8^y} = \frac{1}{4}$	ddM1
	$8^y \times 2^{2y} = 4 \times 16^2$	dddM1
	$2^{3y} \times 2^{2y} = 2^2 \times 2^8$	ddddM1
	$(2^{5y} = 2^{10})$ $y = 2$	A1
	$(4^x \times 4 = 16)$ $x = 1$	A1

12. 4PM1_01_que_20190618 Q: 9

(a) Solve the equation $2\log_p 9 + 3\log_3 p = 8$

(6)

Given that $\log_2 3 = \log_4 3^k$

(b) find the value of k

(2)

(c) Show that

$$6x\log_4 x - 3x\log_2 3 - 5\log_4 x + 10\log_2 3 = \log_4\left(\frac{x^{6x-5}}{3^{6x-20}}\right)$$

(4)

Question 9 continued

Question 9 continued

Question 9 continued

(Total for Question 9 is 12 marks)

Answer:

Question	Scheme	Marks
(a)	$2\log_p 9 + 3\log_3 p = 8$	
	$2\dfrac{\log_3 9}{\log_3 p} + 3\log_3 p = 8$	M1
	$2\log_3 9 + 3(\log_3 p)^2 = 8\log_3 p$	M1
	$3(\log_3 p)^2 - 8\log_3 p + 4 = 0$	A1
	$(3\log_3 p - 2)(\log_3 p - 2) = 0$	M1
	$\log_3 p = \dfrac{2}{3} \quad p = 3^{\frac{2}{3}} = \sqrt[3]{9} \quad (= 2.08)$	A1
	$\log_3 p = 2 \quad p = 3^2 = 9$	A1 [6]
(b)	$\log_2 3 = \dfrac{\log_4 3}{\log_4 2} = \dfrac{\log_4 3}{\frac{1}{2}} = 2\log_4 3 = \log_4 3^2 \Rightarrow k = 2$	M1A1 [2]
(c)	$6x\log_4 x - 3x\log_2 3 - 5\log_4 x + 10\log_2 3$	
	$= 6x\log_4 x - 5\log_4 x - 3x\log_4 3^2 + 10\log_4 3^2$	M1
	$= \log_4 x^{6x} - \log_4 x^5 - \log_4 3^{6x} + \log_4 3^{20}$	M1
	$= \log_4 \dfrac{x^{6x} \times 3^{20}}{x^5 \times 3^{6x}}$	M1
	$= \log_4 \dfrac{x^{6x-5}}{3^{6x-20}}$ *	A1 [4]
		Total 12 marks