AS & A Level Mathematics (9709) Paper 2 [Pure Mathematics 2]

Exam Series: February/March 2017 - October/November 2024

Format Type B: Each question is followed by its answer scheme



Introduction

Each Topical Past Paper Questions Workbook contains a comprehensive collection of hundreds of questions and corresponding answer schemes, presented in worksheet format. The questions are carefully arranged according to their respective chapters and topics, which align with the latest IGCSE or AS/A Level subject content. Here are the key features of these resources:

- 1. The workbook covers a wide range of topics, which are organized according to the latest syllabus content for Cambridge IGCSE or AS/A Level exams.
- 2. Each topic includes numerous questions, allowing students to practice and reinforce their understanding of key concepts and skills.
- 3. The questions are accompanied by detailed answer schemes, which provide clear explanations and guidance for students to improve their performance.
- 4. The workbook's format is user-friendly, with worksheets that are easy to read and navigate.
- 5. This workbook is an ideal resource for students who want to familiarize themselves with the types of questions that may appear in their exams and to develop their problem-solving and analytical skills.

Overall, Topical Past Paper Questions Workbooks are a valuable tool for students preparing for IGCSE or AS/A Level exams, providing them with the opportunity to practice and refine their knowledge and skills in a structured and comprehensive manner. To provide a clearer description of this book's specifications, here are some key details:

- Title: Cambridge AS & A Level Mathematics (9709) Paper 2 Topical Past Papers
- Subtitle: Exam Practice Worksheets With Answer Scheme
- Examination board: Cambridge Assessment International Education (CAIE)
- Subject code: 9709
- Years covered: February/March 2017 October/November 2024
- Paper: 2
- Number of pages: 710
- Number of questions: 287

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Chapter 1

Algebra

$1.9709 s_24 q_2 2$ Q: 1	
Solve the inequality $ 5x+7 > 2x-3 $.	[4]

Question	Answer	Marks	Guidance
	Solve $5x + 7 = 2x - 3$ to obtain $-\frac{10}{3}$	B1	Or inequality.
	Attempt solution of linear equation where $5x$ and $2x$ have different signs	M1	Or inequality.
	Obtain $-\frac{4}{7}$	A1	
	State $x < -\frac{10}{3}, x > -\frac{4}{7}$	A1	A0 if ' and' used.
	Alternative Method for Question 1		
	State or imply non-modulus equation $(5x + 7)^2 = (2x - 3)^2$	(B1)	Or inequality.
	Attempt solution of three-term quadratic equation	(M1)	Or inequality.
	Obtain $-\frac{10}{3}$ and $-\frac{4}{7}$	(A1)	
	State $x < -\frac{10}{3}, x > -\frac{4}{7}$	(A1)	A0 if ' and' used.
		4	

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2.9709 w24 qp 21 Q: 2	
Solve the inequality $ x-7 > 4x+3$.	[4]

Question	Answer	Marks	Guidance
	Attempt solution of equation or inequality, where signs of x and $4x$ are different	M1	
	Obtain $\frac{4}{5}$	A1	OE
	and finally no other value	A1	
	Conclude $x < \frac{4}{5}$	A1	Allow $\left(-\infty, \frac{4}{5}\right)$.
	Alternative Method for Question 2		
	State or imply non-modulus equation $(x-7)^2 = (4x+3)^2$ or inequality	B1	
	Attempt solution of three-term quadratic equation or inequality	M1	
	Obtain finally $\frac{4}{5}$ only	A1	
	Conclude $x < \frac{4}{5}$	A1	Allow $\left(-\infty, \frac{4}{5}\right)$
		4	

3. 9709_m23_qp_22 Q: 3 The polynomial p(x) is defined by

$$\mathbf{p}(x) = ax^3 - ax^2 + ax + b,$$

where *a* and *b* are constants. It is given that (x + 2) is a factor of p(x), and that the remainder is 35 when p(x) is divided by (x - 3).

(a)	Find the values of a and b.	[5]

Hence factorise $p(x)$ and show that the equation $p(x) = 0$ has exactly one real root.

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Question	Answer	Marks	Guidance
(a)	Substitute $x = -2$ and equate to zero	*M1	
	Substitute $x=3$ and equate to 35	*M1	
	Obtain $-8a - 4a - 2a + b = 0$ and $27a - 9a + 3a + b = 35$	A1	
	Solve a pair of relevant simultaneous linear equations to find a or b	DM1	Dependent at least one M mark.
	Obtain $a=1$ and $b=14$	A1	
		5	
(b)	Divide by $x+2$ at least as far as the x term	M1	
	Obtain $[(x+2)](x^2-3x+7)$	A1	
	Conclude with reference to -2 , and discriminant is $9-28$ and hence no root	A1	OE
		3	

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4. $9709_s23_{qp}_{21}$ Q: 4 The polynomial p(x) is defined by

$$p(x) = 2x^3 + 3x^2 + kx - 30,$$

where k is a constant. It is given that (x - 3) is a factor of p(x).

(a) Find the value of *k*. [2] (b) Hence find the quotient when p(x) is divided by (x - 3) and factorise p(x) completely. [3] (c) It is given that *a* is one of the roots of the equation p(x) = 0. Given also that the equation |4y - 5| = a is satisfied by two real values of y, find these two values of y. [3]

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Question	Answer	Marks	Guidance
(a)	Substitute $x = 3$, equate to zero and attempt solution	M1	Condone $-\frac{51}{3}$.
	Obtain $k = -17$	A1	
		2	
Question	Answer	Marks	Guidance
(b)	Divide by $x - 3$ at least as far as $2x^2 + mx$	M1	
	Obtain $2x^2 + 9x + 10$	A1	
	Obtain $(x-3)(2x+5)(x+2)$	A1	SC – no attempt at division (or equivalent) and only correct fully factorised form shown: award B1 only.
		3	
(c)	Solve linear equation $4y-5=3$ to obtain $y=2$	B1 FT	following any positive root from <i>their</i> factorised $p(x)$.
	Attempt solution of linear equation $4y-5 = -3$ or equivalent	M1	with RHS of equation being – (<i>their</i> positive root).
	Obtain $y = \frac{1}{2}$	A1	
	Alternative method for Question 4(c)	I	1
	State or imply $(4y-5)^2 = 3^2$	B1 FT	following any positive root from <i>their</i> factorised $p(x)$.
	Attempt solution of 3-term quadratic equation $(4y-5)^2 = 3^2$	M1	with RHS of equation involving their positive root.
	Obtain $\frac{1}{2}$ and 2 and no other solutions	A1	
		3	

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5. $9709_w23_{qp}_{21}$ Q: 5 The polynomial p(x) is defined by

$$p(x) = 6x^3 + ax^2 + bx - 20,$$

where *a* and *b* are constants. It is given that (x + 2) is a factor of p(x) and that the remainder is -11 when p(x) is divided by (x + 1).

(a)	Find the values of a and b.	[5]

(b)	Hence factorise $p(x)$, and determine the exact roots of the equation $p(3x) = 0$.	[4]

Question	Answer	Marks	Guidance
(a)	Substitute $x = -2$ and equate to zero	*M1	
	Substitute $x = -1$ and equate to -11	*M1	
	Obtain $4a-2b-68=0$ and $a-b-26=-11$ or equivalents	A1	
	Solve a pair of relevant simultaneous linear equations to find a or b	DM1	Dependent at least one M1 mark.
	Obtain $a=19$ and $b=4$	A1	
		5	
(b)	Divide by $x+2$ at least as far as the x term	M1	or equivalent (inspection,).
	Obtain $(x+2)^2(6x-5)$	A1	OE
	Replace (or imply replacement of) x by $3x$ in factorised form	M1	
	Obtain $-\frac{2}{3}$ and $\frac{5}{18}$	A1	and no others.
		4	

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6. 9709_w23_qp_22_Q: 1
When the polynomial $ax^3 + 4ax^2 - 7x - 5$
is divided by $(x + 2)$, the remainder is 33.
Find the value of the constant <i>a</i> . [2]

Question	Answer	Marks	Guidance
	Substitute $x = -2$ and equate to 33	M1	OE (long or synthetic division). Note: Long division and synthetic division give a remainder of $8a+14-5$. Allow one sign error for M1.
	Obtain $-8a + 16a + 14 - 5 = 33$ and hence $a = 3$	A1	
		2	

7. 9709_m22_qp_22_Q: 1	
Solve the equation $ 5x - 2 = 4x + 9 $.	[3]

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Question	Answer	Marks	Guidance
	Solve $5x-2=4x+9$ to obtain $x=11$	B1	
	Attempt solution of linear equation where signs of $5x$ and $4x$ are different	M1	
	Obtain final value $x = -\frac{7}{9}$	A1	
	Alternative method for question 1		
	State or imply non-modulus equation $(5x-2)^2 = (4x+9)^2$	B1	
	Attempt solution of 3-term quadratic equation	M1	
	Obtain $x = -\frac{7}{9}$ and $x = 11$	A1	
		3	

 $8. \ 9709_s22_qp_22 \ \ Q: \ 2$

(a) Sketch, on the same diagram, the graphs of y = |2x - 9| and y = 5x - 3. [2]

(b)	Solve the equation $ 2x - 9 = 5x - 3$.	[2]
		,
		,

Question	Answer	Marks	Guidance
(a)	Draw V-shaped graph with vertex on positive x-axis	*B1	
	Draw (more or less) correct graph of $y = 5x - 3$ with greater gradient	DB1	crossing <i>x</i> -axis between origin and vertex of first graph
		2	
(b)	Attempt solution of linear equation where signs of $2x$ and $5x$ are different	M1	
	Solve $-2x + 9 = 5x - 3$ to obtain $\frac{12}{7}$, 1.71 or better	A1	and no second answer
	Alternative method for question 2(b)	1	
	Attempt solution of 3-term quadratic equation $(2x-9)^2 = (5x-3)^2$ to obtain at least one value of x	M1	$7x^2 + 2x - 24 = 0$
	Obtain $\frac{12}{7}$, 1,71 or better	A1	and no second answer
		2	

Solve the inequality 2x - 5 > x. [4]	9. 9709_w22_qp_21 Q: 1	
	Solve the inequality $ 2x - 5 > x$.	[4]

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Answer:

Question	Answer	Marks	Guidance
	Solve $2x-5=x$ to obtain $x=5$	B1	
	Attempt solution of linear equation where signs of $2x$ and x are different	M1	
	Obtain $x = \frac{5}{3}$	A1	
	Conclude $x < \frac{5}{3}, x > 5$	A1	Must be 2 separate inequalities. Allow equivalents $\left(-\infty, \frac{5}{3}\right) \cup \left(5, \infty\right)$.
	Alternative method for question 1		
	State or imply non-modulus equation $(2x-5)^2 = x^2$	B1	
	Attempt solution of 3-term quadratic equation	M1	
	Obtain $\frac{5}{3}$ and 5	A1	
	Conclude $x < \frac{5}{3}, x > 5$	A1	Must be 2 separate inequalities. Allow equivalents $\left(-\infty, \frac{5}{3}\right) \cup \left(5, \infty\right)$.
		4	

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10. $9709 w^{22} qp_{22}$ Q: 2 The solutions of the equation |4x - 1| = |x + 3| are x = p and x = q, where p < q. Find the exact values of p and q, and hence determine the exact value of |p-2| - |q-1|. [5]

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Question	Answer	Marks	Guidance
	Solve $4x-1=x+3$ to obtain $x=\frac{4}{3}$	B1	
	Attempt solution of linear equation where signs of $4x$ and x are different	M1	
	Obtain final value $x = -\frac{2}{5}$	A1	
	Substitute numerical values and apply modulus signs correctly to obtain $\left -\frac{12}{5}\right -\left \frac{1}{3}\right $ or equivalent, retaining exactness and with no subsequent squaring	M1	Allow <i>their</i> p and $q, p < q$.
	Obtain $\frac{31}{15}$	A1	or exact equivalent.
	Alternative method for Question 2		
	State or imply non-modulus equation $(4x-1)^2 = (x+3)^2$	B1	
	Attempt solution of 3-term quadratic equation	M1	
	Obtain final values $-\frac{2}{5}$ and $\frac{4}{3}$	A1	
Question	Answer	Marks	Guidance
	Substitute numerical values and apply modulus signs correctly to obtain $\left -\frac{12}{5}\right -\frac{11}{3}$ or equivalent, retaining exactness and with no subsequent squaring	M1	Allow their $p < q$.
	Obtain $\frac{31}{15}$	A1	or exact equivalent.
		5	

- $11. \ 9709 _ m21 _ qp _ 22 \ \ Q: \ 1$
- (a) Sketch, on the same diagram, the graphs of y = |3x 5| and y = x + 2. [2]

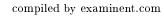
(b)	Solve the equation $ 3x - 5 = x + 2$.	[3]

Question	Answer	Marks	Guidance
(a)	Draw V-shaped graph with vertex on positive <i>x</i> -axis	B1	
	Draw correct graph of $y = x + 2$ with smaller positive gradient	B1	Crossing <i>y</i> -axis between 0 and <i>y</i> -intercept of first graph.
		2	
(b)	Solve $3x-5=x+2$ to obtain $x=\frac{7}{2}$	B1	
	Attempt solution of linear equation where signs of $3x$ and x are different.	M1	
	Obtain $x = \frac{3}{4}$	A1	
	Alternative method for question 1(b)		
	State or imply non-modulus equation $(3x-5)^2 = (x+2)^2$	B1	
	Attempt solution of 3-term quadratic equation	M1	
	Obtain $\frac{3}{4}$ and $\frac{7}{2}$	A1	
		3	

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12. 9709_s21_qp_21 Q: 1	
Solve the inequality $ 3x - 7 < 4x + 5 $.	[4]
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	••••••
	••••••
	••••••
	•••••

Question	Answer	Marks	Guidance
	State or imply non-modulus inequality $(3x-7)^2 < (4x+5)^2$ or corresponding equation or pair of linear equations	B1	
	Attempt solution of 3-term quadratic equation/inequality or of two linear equations	M1	
	Obtain critical values -12 and $\frac{2}{7}$	A1	May be seen in a number line.
	State answer $x < -12$, $x > \frac{2}{7}$ or $(-\infty, -12) \cup \left(\frac{2}{7}, \infty\right)$ or $(-\infty, -12)$, $\left(\frac{2}{7}, \infty\right)$	A1	OE $-12 > x > \frac{2}{7}$ or similar would get A0 Mark the final answer.
		4	



13. 9709_s21_qp_22_Q: 2						
The solutions of the equation $5 x = 5 - 2x$ are $x = a$ and $x = b$, where $a < b$.						
Find the value of $ 3a - 1 + 7b - 1 $. [5]						

Question	Answer	Marks	Guidance		
	Solve $5x = 5 - 2x$ to obtain $x = \frac{5}{7}$	B1	Allow AWRT 0.714		
	Attempt solution of linear equation where signs of $5x$ and $2x$ are the same	M1			
	Obtain $x = -\frac{5}{3}$	A1	Allow AWRT –1.67		
	Substitute <i>their</i> values correctly	M1	Substitution must be seen unless implied by a correct answer. <i>Their</i> values must come from consideration of $5 x = 5 - 2x$		
	Obtain $ -6 + 4 $ and hence 10	A1			
	Alternative method for Question 2				
	State or imply non-modulus equation $25x^2 = (5-2x)^2$	B1			
	Attempt solution of 3-term quadratic equation	M1			
	Obtain $-\frac{5}{3}$ and $\frac{5}{7}$	A1	Allow AWRT 0.714 and AWRT -1.67		
	Substitute <i>their</i> values correctly	M1	Substitution must be seen unless implied by a correct answer. <i>Their</i> values must come from consideration of $5 x = 5 - 2x$		
	Obtain $ -6 + 4 $ and hence 10	A1			
		5			

- 14. 9709_w21_qp_21 Q: 2
- (a) Sketch, on the same diagram, the graphs of y = 3x and y = |x 3|. [2]

[3]
[1]

Answer:

Question	Answer	Marks	Guidance
(a)	Draw V-shaped graph with vertex on positive x-axis	B1	Must be straight lines.
	Draw straight line through origin with positive gradient greater than gradient of first graph, together with a V shaped graph	B1	Must have the first B1 .
		2	
(b)	Solve linear equation with signs of 3x and x different or solve non-modulus equation $(3x)^2 = (x-3)^2$	M1	
	Obtain $x = \frac{3}{4}$	A1	
	Obtain $y = \frac{9}{4}$	A1	And no other point.
		3	
(c)	State $x < \frac{3}{4}$	B1 FT	Following <i>their</i> (single) <i>x</i> -coordinate from part (b).
		1	