Topical Past Paper Questions Worksheets

AS & A Level Mathematics (9709) Paper 4

[Mechanics]

Exam Series: May/Jun 2015 – Oct/Nov 2022

Format Type B:
Each question is followed by its answer scheme

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Introduction

Each Topical Past Paper Questions Workbook contains a comprehensive collection of hundreds of questions and corresponding answer schemes, presented in worksheet format. The questions are carefully arranged according to their respective chapters and topics, which align with the latest IGCSE or AS/A Level subject content. Here are the key features of these resources:

1. The workbook covers a wide range of topics, which are organized according to the latest syllabus content for Cambridge IGCSE or AS/A Level exams.
2. Each topic includes numerous questions, allowing students to practice and reinforce their understanding of key concepts and skills.
3. The questions are accompanied by detailed answer schemes, which provide clear explanations and guidance for students to improve their performance.
4. The workbook's format is user-friendly, with worksheets that are easy to read and navigate.
5. This workbook is an ideal resource for students who want to familiarize themselves with the types of questions that may appear in their exams and to develop their problem-solving and analytical skills.

Overall, Topical Past Paper Questions Workbooks are a valuable tool for students preparing for IGCSE or AS/A Level exams, providing them with the opportunity to practice and refine their knowledge and skills in a structured and comprehensive manner. To provide a clearer description of this book's specifications, here are some key details:

- Title: AS & A Level Mathematics (9709) Paper 4 Topical Past Paper Questions
- Subtitle: Exam Practice Worksheets With Answer Scheme
- Examination board: Cambridge Assessment International Education (CAIE)
- Subject code: 9709
- Years covered: May/Jun 2015 – Oct/Nov 2022
- Paper: 4 [Mechanics]
- Number of pages: 819
- Number of questions: 337

Contents

1 Forces and equilibrium **7**

2 Kinematics of motion in a straight line **143**

3 Momentum **387**

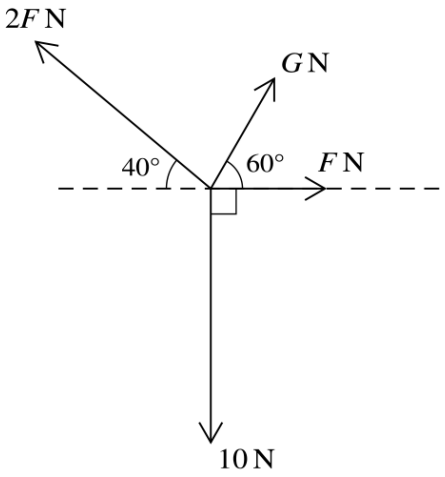
4 Newton's laws of motion **409**

5 Energy, work and power **565**

Chapter 1

Forces and equilibrium

1. 9709_m22_qp_42 Q: 5



Four coplanar forces act at a point. The magnitudes of the forces are 10 N, F N, G N and $2F$ N. The directions of the forces are as shown in the diagram.

(a) Given that the forces are in equilibrium, find the values of F and G. [5]

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(b) Given instead that $F = 3$, find the value of G for which the resultant of the forces is perpendicular to the 10 N force. [2]

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Answer:

Question	Answer	Marks	Guidance
(a)	Attempt to resolve vertically or horizontally	**M1**	Correct number of terms.
	$G \sin 60° + 2F \sin 40° - 10 = 0$	**A1**	Correct resolution vertically.
	$F + G \cos 60° - 2F \cos 40° = 0$	**A1**	Correct resolution horizontally.
	Attempt to solve simultaneously for F or G	**M1**	From equations with 3 relevant terms in each
	$F = 4.53$, $G = 4.82$	**A1**	For both correct.
		5	
(b)	$G \sin 60° + 2 \times 3 \sin 40° - 10 = 0$	**M1**	Resolve forces parallel to the 10 N force and equate this expression to zero, 3 terms.
	$G = 7.09$ to 3 sf	**A1**	
		2	

2. 9709_s22_qp_41 Q: 3

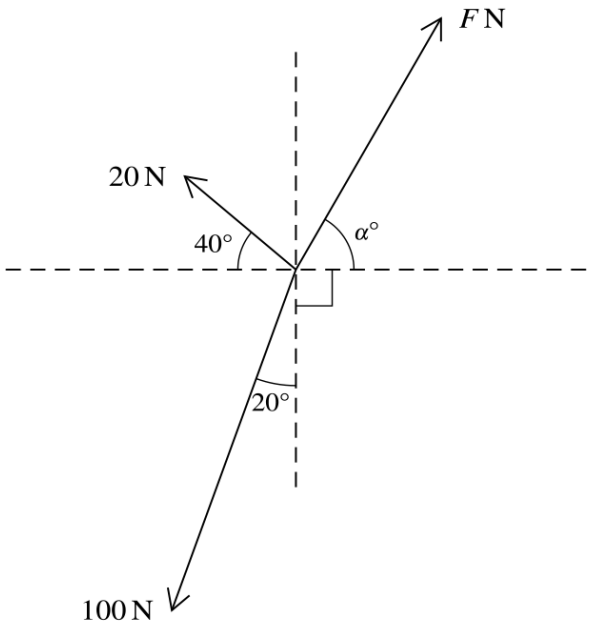
A crate of mass 300 kg is at rest on rough horizontal ground. The coefficient of friction between the crate and the ground is 0.5. A force of magnitude X N, acting at an angle α above the horizontal, is applied to the crate, where $\sin\alpha = 0.28$.

Find the greatest value of X for which the crate remains at rest. [5]

Answer:

Question	Answer	Marks	Guidance
	For attempt at resolving horizontally or vertically	**M1**	Allow sin/cos mix. Allow sign error. Allow g missing. Correct number of terms.
	$R = 300g - 0.28X$ or $R = 300g - X\sin 16.3$	**A1**	$\alpha = 16.26\ldots$
	$0.96X - F = 0$ or $0.96X - 0.5(300g - X\sin\alpha) = 0$ Or $X\cos 16.3 - F = 0$ or $X\cos 16.3 - 0.5(300g - X\sin\alpha) = 0$	**A1**	Or using *their* F
	Use of $F = 0.5R$	**M1**	Use to get an equation in X only. Allow sin/cos mix. Allow sign error. Allow g missing. Must be from 2 term R, which is a linear combination of $300(g)$ and a component of X
	$X = 1360$ [1363.63…]	**A1**	
		5	

3. 9709_s22_qp_41 Q: 4



Three coplanar forces of magnitudes 20 N, 100 N and F N act at a point. The directions of these forces are shown in the diagram.

Given that the three forces are in equilibrium, find F and α. [6]

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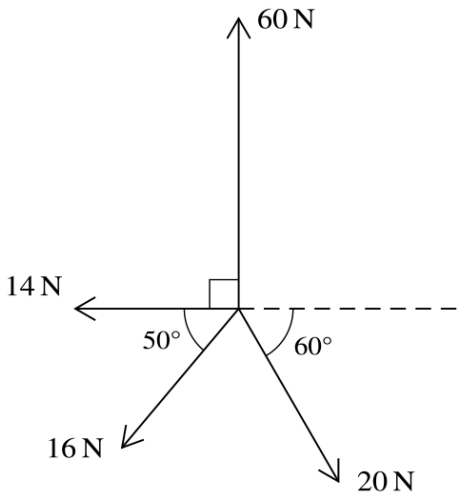
Answer:

Question	Answer	Marks	Guidance
	Attempt to resolve in any direction	**M1**	For resolving. Allow sin/cos mix. Allow sign error. Correct number of terms.
	$F\cos\alpha - 20\cos 40 - 100\sin 20 = 0$ [$F\cos\alpha = 15.320\ldots + 34.202\ldots = 49.5229\ldots$]	**A1**	
	$F\sin\alpha + 20\sin 40 - 100\cos 20 = 0$ [$F\sin\alpha = 93.969\ldots - 12.855\ldots = 81.1135\ldots$]	**A1**	
	$F = \sqrt{(49.5229\ldots)^2 + (81.1135\ldots)^2}$	**M1**	OE; Attempt to solve for F; one term missing in total
	$\alpha = \tan^{-1}\left(\frac{81.1135\ldots}{49.5229\ldots}\right)$	**M1**	OE; Attempt to solve for α; one term missing in total
	$F = 95(.0)$, $\alpha = 58.6$	**A1**	$F = 95.0364\ldots$ and $\alpha = 58.5943\ldots$
	Alternative mark scheme for question 4: For candidates who use cosine and/or sine rule		
	Attempt at cosine rule from triangle of forces	**M1**	Must use lengths 100 and 20 with a suitable angle
	$F^2 = 100^2 + 20^2 - 2\times 100\times 20\cos 70$	**A1**	Correct
	$F = 95[.0]$	**A1**	
	$\frac{95.0364}{\sin 70} = \frac{20}{\sin\beta}$ OR $\frac{95.0364}{\sin 70} = \frac{100}{\sin\gamma}$	**M1**	Attempt at sin rule
		A1	where $\beta = (70-\alpha)$ where $\gamma = (40+\alpha)$
	$\alpha = 58.6$	**A1**	$\alpha = 58.5943\ldots$

Question	Answer	Marks	Guidance
	Alternative mark scheme for question 4: For candidates who resolve in other directions		
	Attempt to resolve (e.g. parallel or perpendicular to 100 N)	**M1**	For resolving. Allow sin/cos mix. Allow sign error. Correct number of terms.
	$F\sin(\alpha+20) + 20\sin 20 - 100 = 0$ $\left[F\sin(\alpha+20) = 93.159\ldots\right]$	**A1**	
	$F\cos(\alpha+20) - 20\cos 20 = 0$ $\left[F\cos(\alpha+20) = 18.793\ldots\right]$	**A1**	
	$F = \sqrt{93.159\ldots^2 + 18.793^2}$	**M1**	OE; Attempt to solve for F; one term missing in total
	$\alpha = \tan^{-1}\left(\frac{93.159\ldots}{18.793\ldots}\right) - 20$	**M1**	OE; Attempt to solve for α; one term missing in total
	$F = 95[.0]$, $\alpha = 58.6$	**A1**	$F = 95.0364\ldots$ and $\alpha = 58.5943\ldots$
		6	

4. 9709_s22_qp_42 Q: 2



Coplanar forces of magnitudes 60 N, 20 N, 16 N and 14 N act at a point in the directions shown in the diagram.

Find the magnitude and direction of the resultant force. [6]

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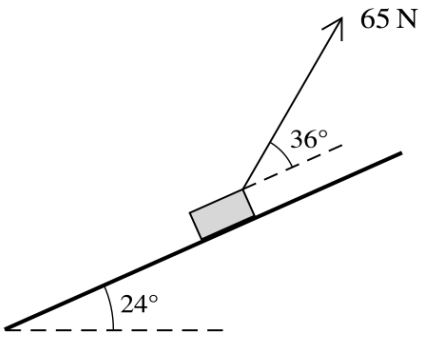
Answer:

Question	Answer	Marks	Guidance
	Resolving either direction	**M1**	3 terms; allow sign errors and allow sin/cos mix
	$(X=)\pm(20\cos 60-14-16\cos 50)\quad[=\mp 14.2846\ldots\ldots]$	**A1**	
	$(Y=)\pm(60-20\sin 60-16\sin 50)\quad[=\pm 30.42278\ldots\ldots]$	**A1**	
	$R=\sqrt{(14.2846\ldots)^2+(30.42278\ldots)^2}$	**M1**	Attempt to solve for R; one missing term in total
	$\theta=\tan^{-1}\left(\frac{30.42278\ldots}{14.2846\ldots}\right)\left[=\tan^{-1}(2.1297\ldots\ldots)\right]$ OR $\alpha=\tan^{-1}\left(\frac{14.2846\ldots}{30.42278\ldots}\right)\left[=\tan^{-1}(0.4596\ldots\ldots)\right]$	**M1**	Attempt to solve for θ or α; one missing term in total
	$R=33.6$ N Direction is 64.8° above the 14 N force or 25.2° above the negative x-axis or 25.2° left of the 60 N force or bearing 335° or 115° anticlockwise from the positive x-axis	**A1**	Both correct. OE; allow 64.9, 25.1 Giving an angle only is insufficient. Direction may be seen on a diagram, with minimum of arrow on resultant. Arrows on both components only is A0 as it doesn't show the direction of the resultant. However the direction is stated, it must be able to be drawn uniquely.
		6	

5. 9709_s22_qp_42 Q: 5



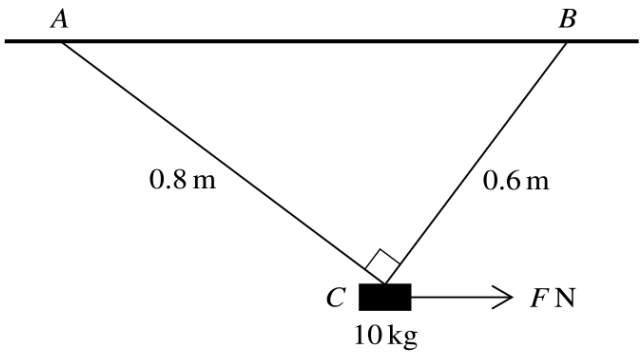
A block of mass 12 kg is placed on a plane which is inclined at an angle of 24° to the horizontal. A light string, making an angle of 36° above a line of greatest slope, is attached to the block. The tension in the string is 65 N (see diagram). The coefficient of friction between the block and plane is μ. The block is in limiting equilibrium and is on the point of sliding up the plane.

Find μ. [6]

Answer:

Question	Answer	Marks	Guidance
	Attempt at resolving parallel to the plane	*M1	3 terms. Allow sign errors, sin/cos mix. Allow g missing, otherwise dimensionally correct.
	$65\cos 36 = 12g \times \sin 24 + F$	A1	$F = 3.777707\ldots\ldots$
	Attempt at resolving perpendicular to the plane	*M1	3 terms. Allow sign errors, sin/cos mix. Allow g missing, otherwise dimensionally correct.
	$12g \times \cos 24 = R + 65\sin 36$	A1	$R = 71.419\ldots\ldots$
	Use $F = \mu R$ $\left[\mu = \frac{65\cos 36 - 12g \times \sin 24}{12g \times \cos 24 - 65\sin 36} = \frac{52.586 - 48.808}{109.625 - 38.206} = \frac{3.777\ldots}{71.419\ldots}\right]$	DM1	To get an equation in μ only. Dependent on two previous M marks. Allow g missing
	$\mu = 0.0529$	A1	Allow AWRT 0.053 Do not accept fractional equivalent.
		6	

6. 9709_s22_qp_43 Q: 4



The diagram shows a block of mass 10 kg suspended below a horizontal ceiling by two strings AC and BC, of lengths 0.8 m and 0.6 m respectively, attached to fixed points on the ceiling. Angle $ACB = 90°$. There is a horizontal force of magnitude F N acting on the block. The block is in equilibrium.

(a) In the case where $F = 20$, find the tensions in each of the strings. [5]

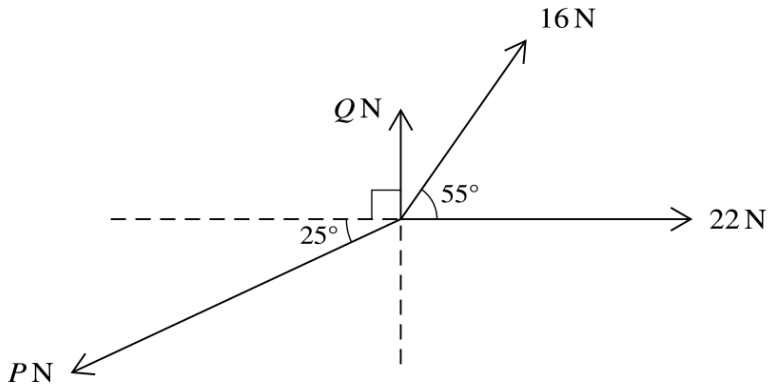
(b) Find the greatest value of F for which the block remains in equilibrium in the position shown. [3]

Answer:

Question	Answer	Marks	Guidance
(a)	$T_A \times 0.8 - T_B \times 0.6 - 20 = 0$ or $T_A \times 0.6 + T_B \times 0.8 - 10g = 0$	**M1**	Resolving horizontally or vertically
	$T_A \times 0.8 - T_B \times 0.6 - 20 = 0$	**A1**	
	$T_A \times 0.6 + T_B \times 0.8 - 10g = 0$	**A1**	
	$0.8T_A - \frac{0.6(10g - 0.6T_A)}{0.8} = 20 \quad \rightarrow \quad T_A = \ldots$	**M1**	Attempt to solve simultaneously
	$T_A = 76$ N, $T_B = 68$ N	**A1**	
		5	

Question	Answer	Marks	Guidance
(b)	$T_A \times 0.6 - 10g = 0 \Rightarrow T_A = \frac{500}{3}$	**B1**	From using $T_B = 0$
	$T_A \times 0.8 - F = 0$	**M1**	
	$F = \frac{400}{3}$	**A1**	Allow $F = 133$ to 3 s.f.
		3	

7. 9709_w22_qp_41 Q: 1



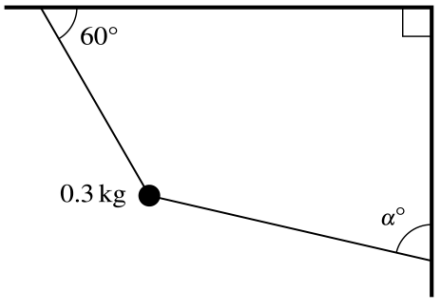
Coplanar forces of magnitudes P N, Q N, 16 N and 22 N act at a point in the directions shown in the diagram. The forces are in equilibrium.

Find the values of P and Q. [5]

Answer:

Question	Answer	Marks	Guidance
	Attempt at resolving horizontally or vertically	**M1**	Allow sign errors, allow sin/cos mix. 3 terms.
	$P\cos 25 = 22 + 16\cos 55$	**A1**	
	$Q + 16\sin 55 = P\sin 25$	**A1**	Allow *their P*.
	Attempt to solve for P or Q	**M1**	No missing/extra terms.
	$P = 34.4 \quad Q = 1.43$	**A1**	$P = 34.40025941,\ Q = 1.431745128$.
		5	

8. 9709_w22_qp_42 Q: 3



A particle of mass 0.3 kg is held at rest by two light inextensible strings. One string is attached at an angle of 60° to a horizontal ceiling. The other string is attached at an angle $\alpha°$ to a vertical wall (see diagram). The tension in the string attached to the ceiling is 4 N.

Find the tension in the string which is attached to the wall and find the value of α. [6]

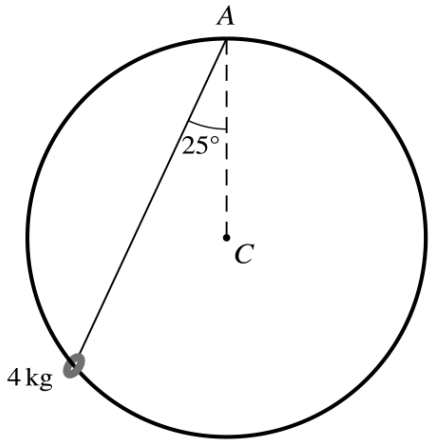
Answer:

Question	Answer	Marks	Guidance
	Attempt to resolve either direction	**M1**	Correct number of terms. Allow sin/cos mix. Allow sign errors. Allow *g* missing.
	$0.3g + T\cos\alpha° - 4\sin 60° = 0$ $(T\cos\alpha° = 0.464\ldots)$	**A1**	OE
	$T\sin\alpha° - 4\cos 60° = 0$ $(T\sin\alpha° = 2)$	**A1**	OE If the two *T*s are different, award maximum A1A0 unless subsequently stated that the two *T*s are the same.
	$\alpha = \tan^{-1}\left(\frac{4\cos 60°}{4\sin 60° - 0.3g}\right) = \tan^{-1}\left(\frac{2}{0.464\ldots}\right)$	**M1**	Attempt to solve for α. No missing/extra terms. Allow *g* missing. Must get to 'α ='.
	$T = \frac{4\cos 60}{\sin(their\alpha)} = \sqrt{(4\cos 60°)^2 + (4\sin 60° - 0.3g)^2} = \sqrt{2^2 + (0.464\ldots)^2}$	**M1**	OE Attempt to solve for *T*. No missing/extra terms. Allow *g* missing. Must get to 'T ='.
	Tension = 2.05 N $\alpha = 76.9$	**A1**	For both AWRT 2.05, 76.9 (Tension = 2.05314… N $\alpha = 76.9356\ldots$)
	Alternative method for Q3 using triangle of forces		
	Attempt at cosine rule from triangle of forces	**M1**	Must use lengths 4 and 0.3*g* with a suitable angle. Allow *g* missing.
	$T^2 = 4^2 + (0.3g)^2 - 2\times 4\times(0.3g)\times\cos 30$	**A1**	
	Tension = 2.05	**A1**	Tension = 2.05314… AWRT 2.05
	Attempt at sin rule	**M1**	Must have angle 30° and another angle in terms of α with correct numerators, but allow *g* missing.
	$\frac{Their\,T}{\sin 30} = \frac{4}{\sin(180-\alpha)}$ or $\frac{Their\,T}{\sin 30} = \frac{0.3g}{\sin(\alpha - 30)}$	**A1**	Correct. Allow $\sin\alpha$ instead of $\sin(180-\alpha)$.
	$\alpha = 76.9$	**A1**	$\alpha = 76.9356\ldots$ AWRT 76.9

Question	Answer	Marks	Guidance
	Alternative method for Q3 using Lami's theorem		
	Attempt at Lami's theorem	**M1**	Must have numerators correct and at least one angle correct. Allow *g* missing.
	$\frac{4}{\sin\alpha} = \frac{0.3g}{\sin(210-\alpha)} = \frac{T}{\sin(150)}$	**A1 A1**	A1 for two parts second A1 for all three.
	$\alpha = \tan^{-1}\left(\frac{4\sin 210}{0.3g + 4\cos 210}\right)$	**M1**	For solving for α using compound angle formula. Must be correct for their angles. Allow *g* missing.
	$T = \frac{4\sin(150)}{\sin\alpha}$ or $T = \frac{0.3g\sin(150)}{\sin(210-\alpha)}$	**M1**	For solving for *T* using their α. Allow *g* missing.
	Tension = 2.05 N $\alpha = 76.9$	**A1**	For both AWRT 2.05, 76.9
		6	
	SC: Tension and the 4N force considered in the wrong directions		
	Attempt to resolve either direction	**M1**	Correct number of terms. Allow sin/cos mix. Allow sign errors. Allow *g* missing.
	$T\cos 60° - 4\sin\alpha° = 0$ And: $T\sin 60° - 4\cos\alpha° - 0.3g = 0$	**A1**	For both OE If the two *T*s are different, they get **SC A0** unless they subsequently state that the two *T*s are the same.
	$\left(\frac{T\cos 60°}{4}\right)^2 + \left(\frac{T\sin 60° - 0.3g}{4}\right)^2 = 1 \Rightarrow \frac{1}{4}T^2 + \frac{3}{4}T^2 - 3\sqrt{3}T + 9 = 16$ $\Rightarrow T^2 - 3\sqrt{3}T - 7 = 0 \Rightarrow T = 6.31 (\text{or} -1.11)$ OR: $4\sqrt{3}\sin\alpha - 4\cos\alpha = 3 \Rightarrow 8\sin(\alpha - 30) = 3 \Rightarrow \alpha = \sin^{-1}\frac{3}{8} + 30$	**M1**	OE Attempt to solve for *T or* α. No missing/extra terms. Allow *g* missing. Must get to 'T =' or 'α='.

Question	Answer	Marks	Guidance
	$T = 6.31\text{N}$ $\alpha = 52.0$	**A1**	(T = 6.30617…, α=52.0243…)
		6	

9. 9709_w22_qp_43 Q: 3



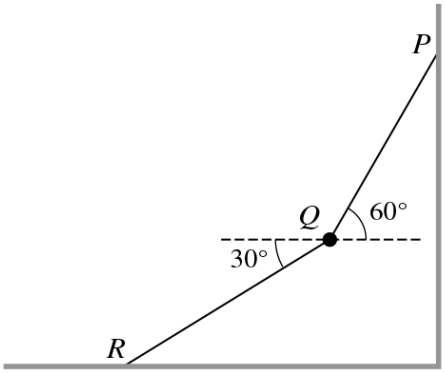
A ring of mass 4 kg is threaded on a smooth circular rigid wire with centre C. The wire is fixed in a vertical plane and the ring is kept at rest by a light string connected to A, the highest point of the circle. The string makes an angle of 25° to the vertical (see diagram).

Find the tension in the string and the magnitude of the normal reaction of the wire on the ring. [6]

Answer:

Question	Answer	Marks	Guidance
	$T \cos 25 = 40 + R \cos 50$	**M1**	Resolving in any direction e.g. horizontal, vertical, along radius or tangent.
	$R \sin 50 = T \sin 25$	**M1**	Resolving in a second direction.
	Radially: $T \cos 25 = R + 40 \cos 50$ Tangentially: $T \sin 25 = 40 \sin 50$ Parallel to T: $T = R \cos 25 + 40 \cos 25$ Perpendicular to T: $R \sin 25 = 40 \sin 25$ Vertically: $T \cos 25 = 40 + R \cos 50$ Horizontally: $R \sin 50 = T \sin 25$	**A1**	Two correct equations.
	Solving equation(s) to find either T or R	**M1**	
	$T = 72.5$ N	**A1**	From 72.504….
	$R = 40$ N	**A1**	
		6	

10. 9709_m21_qp_42 Q: 3



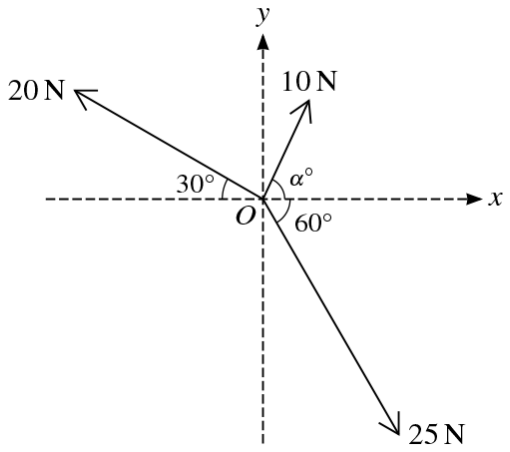
A particle Q of mass 0.2 kg is held in equilibrium by two light inextensible strings PQ and QR. P is a fixed point on a vertical wall and R is a fixed point on a horizontal floor. The angles which strings PQ and QR make with the horizontal are 60° and 30° respectively (see diagram).

Find the tensions in the two strings. [5]

Answer:

Question	Answer	Marks	Guidance
	For attempting to resolve forces in either direction.	**M1**	Correct number of relevant terms.
	$T_P \cos 60 = T_R \cos 30$	**A1**	
	$T_P \sin 60 = T_R \sin 30 + 0.2g$	**A1**	
	Attempt to solve simultaneously for either tension.	**M1**	From 2 equations, with correct number of relevant terms.
	$T_P = 3.46$ N and $T_R = 2$ N	**A1**	Both correct. Allow $T_P = 2\sqrt{3}$ N.
	Alternative method for question 3		
	$\frac{T_P}{\sin 60} = \frac{T_R}{\sin 150} = \frac{0.2g}{\sin 150}$	**M1**	Attempt one pair of Lami's equations. Correct angles.
	One pair correct	**A1**	
	Equations all correct	**A1**	
	Solve for T_P or T_R	**M1**	From equations of the correct form.
	$T_P = 3.46$ N and $T_R = 2$ N	**A1**	Both correct. Allow $T_P = 2\sqrt{3}$ N
		5	

11. 9709_s21_qp_41 Q: 6



Three coplanar forces of magnitudes 10 N, 25 N and 20 N act at a point O in the directions shown in the diagram.

(a) Given that the component of the resultant force in the x-direction is zero, find α, and hence find the magnitude of the resultant force. [4]

(b) Given instead that $\alpha = 45$, find the magnitude and direction of the resultant of the three forces. [5]

Answer:

Question	Answer	Marks	Guidance
(a)	$20\cos 30 = 25\cos 60 + 10\cos\alpha$ $[17.32 = 12.5 + 10\cos\alpha,\ \rightarrow \cos\alpha = 0.4821]$	**M1**	For resolving forces horizontally, all relevant terms included
	$\alpha = 61.2$	**A1**	From $\alpha = 61.18$
	Resultant $= 20\sin 30 + 10\sin 61.2 - 25\sin 60$ $[= 10 + 8.761 - 21.651]$	**M1**	For resolving forces vertically, all relevant terms included
	Magnitude of resultant force = 2.89 N	**A1**	A0 for –2.89 N or for ±2.89 N. Allow 2.89 N downwards
		4	
(b)	$X = 25\cos 60 + 10\cos 45 - 20\cos 30$ $= 12.5 + 7.07107 - 17.32051 = 2.25056$ $Y = 20\sin 30 + 10\sin 45 - 25\sin 60$ $= 10 + 7.07107 - 21.65064 = -4.57957$	**M1**	For either horizontal or vertical component, correct number of relevant terms. Allow $\pm X$ and/or $\pm Y$
		A1	For both correct, allow unsimplified
	$R = \sqrt{X^2 + Y^2}$	**M1**	OE. Using a method to find the resultant force, using expressions for X and Y with at least 5 relevant terms.
	$\alpha = \tan^{-1}\dfrac{Y}{X}$	**M1**	OE. A method to find the direction, using expressions for X and Y with at least 5 relevant terms.
	Resultant = 5.10 N, Direction = 63.8° below positive x-axis	**A1**	For both correct, angle clearly explained. May use a diagram with a correct arrow and arc for angle. Allow angle 296° (measured anticlockwise from +ve x-axis)
		5	