Topical Past Paper Questions Workbook

IGCSE Additional Mathematics (0606)
Paper 2

Exam Series: May/June 2012 – Oct/Nov 2022

Format Type B:
Each question is followed by its answer scheme

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Introduction

Each Topical Past Paper Questions Workbook contains a comprehensive collection of hundreds of questions and corresponding answer schemes, presented in worksheet format. The questions are carefully arranged according to their respective chapters and topics, which align with the latest IGCSE or AS/A Level subject content. Here are the key features of these workbooks:

1. The workbook covers a wide range of topics, which are organized according to the latest syllabus content for Cambridge IGCSE or AS/A Level exams.
2. Each topic includes numerous questions, allowing students to practice and reinforce their understanding of key concepts and skills.
3. The questions are accompanied by detailed answer schemes, which provide clear explanations and guidance for students to improve their performance.
4. The workbook's format is user-friendly, with worksheets that are easy to read and navigate.
5. This workbook is an ideal resource for students who want to familiarize themselves with the types of questions that may appear in their exams and to develop their problem-solving and analytical skills.

Overall, Topical Past Paper Questions Workbooks are a valuable tool for students preparing for IGCSE or AS/A Level exams, providing them with the opportunity to practice and refine their knowledge and skills in a structured and comprehensive manner. To provide a clearer description of this book's specifications, here are some key details:

- Title: Cambridge IGCSE Additional Mathematics (0606) Paper 2 Topical Past Paper Questions Workbook
- Subtitle: Exam Practice Worksheets With Answer Scheme
- Examination board: Cambridge Assessment International Education (CAIE)
- Subject code: 0606
- Years covered: May/June 2012 – Oct/Nov 2022
- Paper: 2
- Number of pages: 1460
- Number of questions: 716

Contents

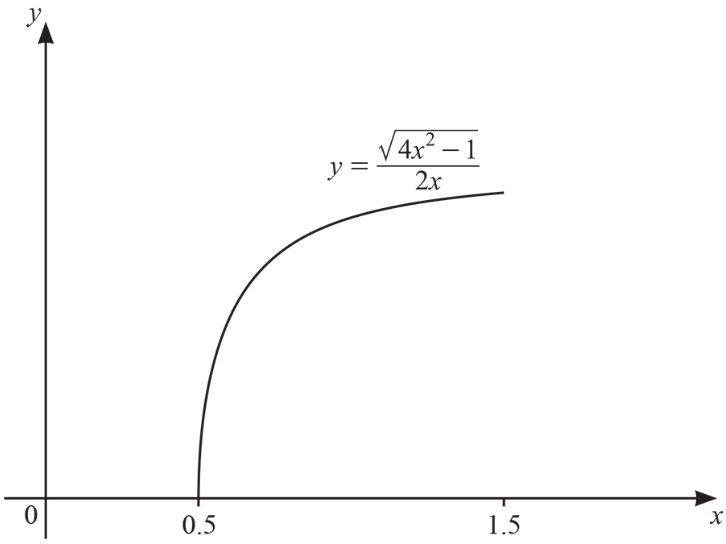
Chapter 1

Functions

1. 0606_m21_qp_22 Q: 10

The function f is defined by $f(x) = \dfrac{\sqrt{4x^2 - 1}}{2x}$ for $0.5 \leqslant x \leqslant 1.5$.

The diagram shows a sketch of $y = f(x)$.



(a) **(i)** It is given that f^{-1} exists. Find the domain and range of f^{-1}. [3]

(ii) Find an expression for $f^{-1}(x)$. [3]

(b) The function g is defined by $g(x) = e^{x^2}$ for all real x. Show that $gf(x) = e^{\left(1-\frac{a}{bx^2}\right)}$, where a and b are integers. [2]

Answer:

(a)(i)	Range f^{-1}: $0.5 \leqslant f^{-1} \leqslant 1.5$	**B1**	
	Domain f^{-1}: $0 \leqslant x \leqslant \frac{2\sqrt{2}}{3}$ oe	**B2**	**B1** for 0 and $\frac{2\sqrt{2}}{3}$ in an incorrect inequality or for $x \geqslant 0$ or $x \leqslant \frac{2\sqrt{2}}{3}$
(a)(ii)	Correctly collects terms ready to factorise e.g. $4x^2 - 4x^2y^2 = 1$ or $4y^2x^2 - 4y^2 = -1$ or simplifies to subject in one term only e.g. $\frac{1}{4y^2} = 1 - x^2$ or $-\frac{1}{4x^2} = y^2 - 1$ oe	**M1**	
	Correctly factorises and/or rearranges at least as far as: $x^2 = \frac{1}{4 - 4y^2}$ or $y^2 = \frac{-1}{4x^2 - 4}$ oe	**M1**	**FT** only if of equivalent difficulty
	$\left[f^{-1}(x) =\right]\sqrt{\frac{1}{4 - 4x^2}}$ or $[y =]\sqrt{\frac{-1}{4x^2 - 4}}$ oe, isw	**A1**	
(b)	Correct order of composition: $gf(x) = e^{\left(\frac{\sqrt{4x^2-1}}{2x}\right)^2}$	**M1**	
	$gf(x) = e^{\left(1 - \frac{1}{4x^2}\right)}$ isw	**A1**	

2. 0606_s21_qp_22 Q: 13

The functions f and g are defined, for $x > 0$, by

$$f(x) = \frac{2x^2 - 1}{3x},$$

$$g(x) = \frac{1}{x}.$$

(a) Find and simplify an expression for $fg(x)$. [2]

(b) **(i)** Given that f^{-1} exists, write down the range of f^{-1}. [1]

(ii) Show that $f^{-1}(x) = \dfrac{px + \sqrt{qx^2 + r}}{4}$, where p, q and r are integers. [4]

Answer:

Question	Answer	Marks	Partial Marks
(a)	$[\mathrm{fg}(x) =] \dfrac{2\left(\frac{1}{x}\right)^2 - 1}{3\left(\frac{1}{x}\right)}$ oe	**M1**	
	$[\mathrm{fg}(x) =] \dfrac{2 - x^2}{3x}$ or $\dfrac{2}{3x} - \dfrac{x}{3}$	**A1**	mark final answer
(b)(i)	$\mathrm{f}^{-1} > 0$	**B1**	
(b)(ii)	$2x^2 - 3xy - 1 = 0$ or $2y^2 - 3xy - 1 = 0$	**B1**	
	Correctly applies quadratic formula: $[x =] \dfrac{-(-3y) \pm \sqrt{(-3y)^2 - 4(2)(-1)}}{2(2)}$ oe or $[y =] \dfrac{-(-3x) \pm \sqrt{(-3x)^2 - 4(2)(-1)}}{2(2)}$ oe	**M1**	**FT** *their* $2x^2 - 3xy - 1 = 0$ or $2y^2 - 3xy - 1 = 0$ with at most one sign error in the equation
	Justifies the positive square root at some point	**B1**	
	$\mathrm{f}^{-1}(x) = \dfrac{3x + \sqrt{9x^2 + 8}}{4}$ cao	**A1**	must be a function of x

3. 0606_s21_qp_23 Q: 9

(a) The function f is defined, for all real x, by $\mathrm{f}(x) = 13 - 4x - 2x^2$.

(i) Write $\mathrm{f}(x)$ in the form $a + b(x+c)^2$, where a, b and c are constants. [3]

(ii) Hence write down the range of f. [1]

(b) The function g is defined, for $x \geqslant 1$, by $\mathrm{g}(x) = \sqrt{x^2 + 2x - 1}$.

(i) Given that $\mathrm{g}^{-1}(x)$ exists, write down the domain and range of g^{-1}. [2]

(ii) Show that $\mathrm{g}^{-1}(x) = -1 + \sqrt{px^2 + q}$, where p and q are integers. [4]

Answer:

(a)(i)	$15 - 2(x+1)^2$ isw	**B3**	**B1** for $(x+1)^2$ **B1** for $a = 15$
(a)(ii)	$f \leqslant 15$	**B1**	**STRICT FT** *their a*
(b)(i)	Domain: $x \geqslant \sqrt{2}$	**B1**	
	Range: $g^{-1} \geqslant 1$	**B1**	
(b)(ii)	$x^2 + 2x + (-1 - y^2) = 0$ or $y^2 + 2y + (-1 - x^2) = 0$	**B1**	
	Correctly applies quadratic formula: $[x =]\dfrac{-2 \pm \sqrt{2^2 - 4(1)(-1-y^2)}}{2}$ or $[y =]\dfrac{-2 \pm \sqrt{2^2 - 4(1)(-1-x^2)}}{2}$	**M1**	**FT** *their* $x^2 + 2x + (-1 - y^2) = 0$ or $y^2 + 2y + (-1 - x^2) = 0$ with at most one sign error in the equation
	Justifies the positive square root at some point	**B1**	
	Correct completion to $g^{-1}(x) = -1 + \sqrt{x^2 + 2}$	**A1**	

4. 0606_w21_qp_21 Q: 9

The following functions are defined for $x > 1$.

$$\mathrm{f}(x) = \frac{x+3}{x-1} \qquad \mathrm{g}(x) = 1 + x^2$$

(a) Find $\mathrm{fg}(x)$. [2]

(b) Find $\mathrm{g}^{-1}(x)$. [2]

(c) DO NOT USE A CALCULATOR IN THIS PART OF THE QUESTION.

Solve the equation $f(x) = g(x)$. [5]

Answer:

(a)	$[\mathrm{fg}(x) =] \dfrac{x^2+4}{x^2}$ oe, final answer	2	**B1** for an attempt at the correct order of composition with at most one error
(b)	Complete, correct method to find the inverse	**M1**	
	$\left[\mathrm{g}^{-1}(x) =\right] \sqrt{x-1}$ final answer	**A1**	
(c)	$x^3 - x^2 - 4 = 0$	**M1**	condone one sign or arithmetic error
	Shows $x - 2$ is a factor or shows that $x = 2$ is a solution	**M1**	
	Uses $x - 2$ is a factor to find $x^2 + x + 2$	**B2**	**B1** for a quadratic factor with 2 terms correct
	Indicates that $x^2 + x + 2$ has no real roots and states $x = 2$ as the only solution	**A1**	dep on all previous marks awarded

5. 0606_s19_qp_22 Q: 12

(a) The functions f and g are defined by

$$f(x) = 5x - 2 \quad \text{for } x > 1,$$
$$g(x) = 4x^2 - 9 \quad \text{for } x > 0.$$

(i) State the range of g. [1]

(ii) Find the domain of gf. [1]

(iii) Showing all your working, find the exact solutions of $gf(x) = 4$. [3]

(b) The function h is defined by $\mathrm{h}(x) = \sqrt{x^2 - 1}$ for $x \leqslant -1$.

(i) State the geometrical relationship between the graphs of $y = \mathrm{h}(x)$ and $y = \mathrm{h}^{-1}(x)$. [1]

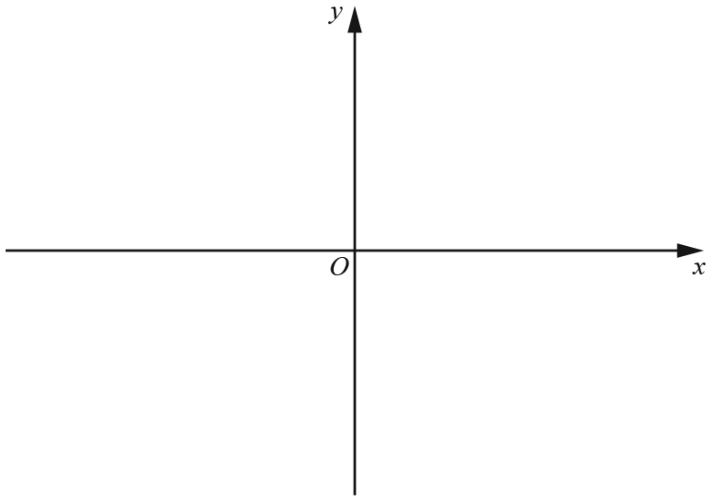
(ii) Find an expression for $\mathrm{h}^{-1}(x)$. [3]

Answer:

(a)(i)	$g > -9$	**B1**	
(a)(ii)	$x > 1$	**B1**	
(a)(iii)	$[\mathrm{gf}(x) =]\ \ 4(5x-2)^2 - 9$	**B1**	
	$100x^2 - 80x - 38 = 0$ or $(5x-2)^2 = \dfrac{45+9}{4}$	**M1**	
	$[x =]\ \dfrac{-(-80) \pm \sqrt{(-80)^2 - 4(100)(-38)}}{2(100)}$ leading to $\dfrac{4+3\sqrt{6}}{10}$ oe only or $\dfrac{1}{5}\left(2+\sqrt{\dfrac{54}{4}}\right)$ or better only	**A1**	
(b)(i)	(They are) **reflection**s (of each other) in (the line) $\boldsymbol{y = x}$ oe	**B1**	
(b)(ii)	$x^2 = y^2 + 1$ or $y^2 = x^2 + 1$	**M1**	
	$x = [\pm]\sqrt{y^2+1}$ or $y = [\pm]\sqrt{x^2+1}$	**A1**	
	$-\sqrt{x^2+1}$ nfww	**A1**	

6. 0606_s18_qp_22 Q: 10

(a) **(i)** On the axes below, sketch the graph of $y = |(x+3)(x-5)|$ showing the coordinates of the points where the curve meets the x-axis. [2]



(ii) Write down a suitable domain for the function $f(x) = |(x+3)(x-5)|$ such that f has an inverse. [1]

(b) The functions g and h are defined by

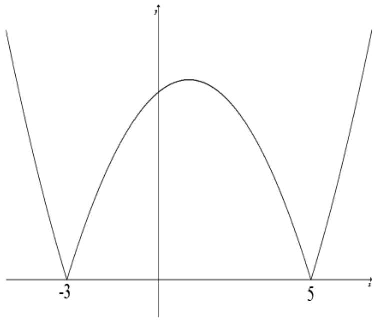
$$g(x) = 3x - 1 \qquad \text{for } x > 1,$$
$$h(x) = \frac{4}{x} \qquad \text{for } x \neq 0.$$

(i) Find $hg(x)$. [1]

(ii) Find $(hg)^{-1}(x)$. [2]

(c) Given that $p(a) = b$ and that the function p has an inverse, write down $p^{-1}(b)$. [1]

Answer:

(a)(i)		**B2**	**B1** for correct shape **B1** for roots marked on the graph or seen nearby provided graph drawn and one root is negative and one is positive
(a)(ii)	Any correct domain	**B1**	
(b)(i)	$\frac{4}{3x-1}$	**B1**	mark final answer
(b)(ii)	Correct method for finding inverse function e.g. swopping variables **<u>and</u>** changing subject or vice versa; or indicates $(\mathrm{hg})^{-1}(x)=\mathrm{g}^{-1}\mathrm{h}^{-1}(x)$ and finds $\mathrm{g}^{-1}(x)=\frac{x+1}{3}$ and $\mathrm{h}^{-1}(x)=\frac{4}{x}$	**M1**	**FT** only if *their*hg(x) of the form $\frac{a}{bx+c}$ where a, b and c are integers
	$\left[(\mathrm{hg})^{-1}(x)=\right]\frac{1}{3}\left(\frac{4}{x}+1\right)$ oe isw or $\left[(\mathrm{hg})^{-1}(x)=\right]\frac{4+x}{3x}$ oe isw	**A1**	**FT** *their* $(\mathrm{hg})^{-1}(x)=\frac{a-cx}{bx}$ oe If **M0** then **SC1** for *their* hg(x) of the form $y=\frac{a}{x}+b$ oe leading to *their* $(\mathrm{hg})^{-1}(x)$ of the form $y=\frac{a}{x-b}$ isw
(c)	a cao	**B1**	

7. 0606_s18_qp_23 Q: 5

The function f is defined by $f(x) = \dfrac{1}{2x-5}$ for $x > 2.5$.

(i) Find an expression for $f^{-1}(x)$. [2]

(ii) State the domain of $f^{-1}(x)$. [1]

(iii) Find an expression for $f^2(x)$, giving your answer in the form $\dfrac{ax+b}{cx+d}$, where a, b, c and d are integers to be found. [3]

Answer:

(i)	Putting $y = \mathrm{f}(x)$, changing subject to x and swopping x and y or vice versa	**M1**	
	$\mathrm{f}^{-1}(x) = \frac{1}{2}\left(\frac{1}{x} + 5\right)$ or $\frac{5x+1}{2x}$ oe isw	**A1**	
(ii)	$x > 0$ oe	**B1**	
(iii)	$\dfrac{1}{2\left(\frac{1}{2x-5}\right) - 5}$	**B1**	
	$\dfrac{1}{\frac{2-5(2x-5)}{2x-5}}$ oe	**M1**	**FT** if expression of equivalent difficulty e.g. $\dfrac{1}{\left(\frac{1}{2x-5}\right) - 5}$
	Completes to $\dfrac{2x-5}{-10x+27}$ oe final answer	**A1**	

8. 0606_w18_qp_22 Q: 11

The functions f and g are defined for real values of $x \geqslant 1$ by

$$f(x) = 4x - 3,$$

$$g(x) = \frac{2x+1}{3x-1}.$$

(i) Find $gf(x)$. [2]

(ii) Find $g^{-1}(x)$. [3]

(iii) Solve $fg(x) = x - 1$. [4]

Answer:

(i)	$\mathrm{gf}(x) = \dfrac{2(4x-3)+1}{3(4x-3)-1}$	**M1**	
	$= \dfrac{8x-5}{12x-10}$	**A1**	
(ii)	$y(3x-1) = 2x+1$ or $x(3y-1) = 2y+1$	**B1**	
	$(3y-2)x = y+1$ or $(3x-2)y = x+1$	**M1**	
	$\mathrm{g}^{-1}(x) = \dfrac{x+1}{3x-2}$	**A1**	
(iii)	$4\left(\dfrac{2x+1}{3x-1}\right) - 3[= x-1]$	**B1**	
	$3x^2 - 3x - 6$ oe	**B1**	
	$3(x+1)(x-2)$	**M1**	
	$x = 2$ only	**A1**	

9. 0606_m17_qp_22 Q: 11

The functions f and g are defined by

$$f(x) = \frac{x^2 - 2}{x} \quad \text{for } x \geqslant 2,$$

$$g(x) = \frac{x^2 - 1}{2} \quad \text{for } x \geqslant 0.$$

(i) State the range of g. [1]

(ii) Explain why fg(1) does not exist. [2]

(iii) Show that $gf(x) = ax^2 + b + \frac{c}{x^2}$, where a, b and c are constants to be found. [3]

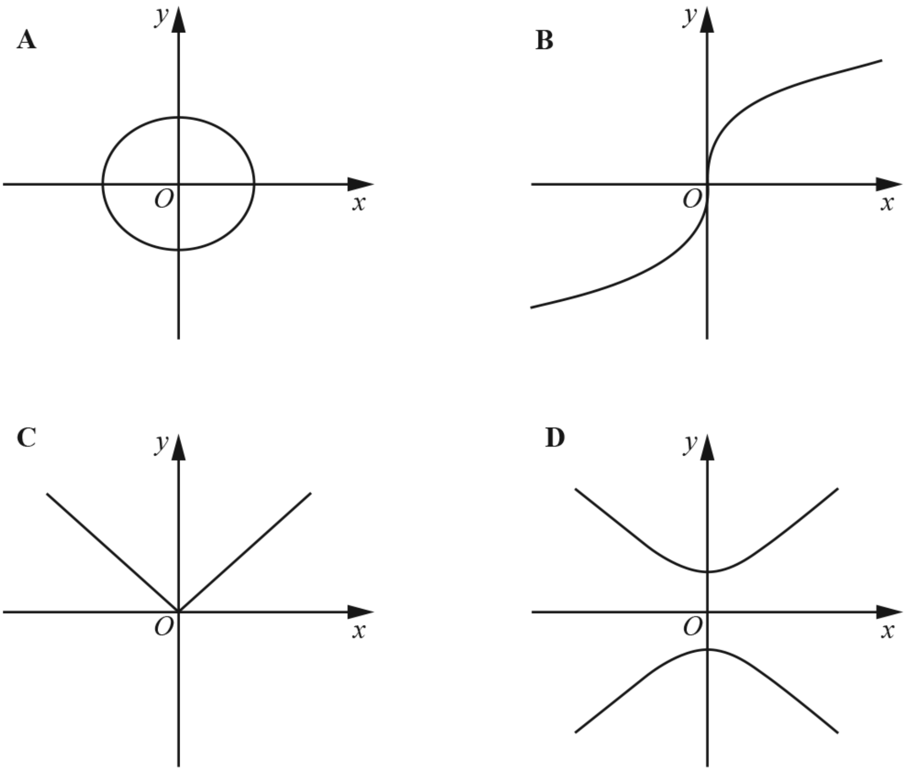
(iv) State the domain of gf. [1]

(v) Show that $f^{-1}(x) = \dfrac{x + \sqrt{x^2 + 8}}{2}$. [4]

Answer:

(i)	$g \geqslant -\frac{1}{2}$	**B1**	
(ii)	g(1) = 0 valid comment e.g. domain of f is $x \geqslant 2$	**B1** **B1**	**B1** for either
(iii)	$\dfrac{\left(\dfrac{x^2-2}{x}\right)^2 - 1}{2}$	**M1**	or $\dfrac{\left(x-\dfrac{2}{x}\right)^2 - 1}{2}$
	$\left(\dfrac{x^2-2}{x}\right)^2 = \dfrac{x^4-4x^2+4}{x^2}$ soi	**B1**	or $\left(x-\dfrac{2}{x}\right)^2 = x^2-4+\dfrac{4}{x^2}$
	$\dfrac{1}{2}x^2 - \dfrac{5}{2} + \dfrac{2}{x^2}$	**A1**	or correct 3 term equivalent or $a = 0.5$, $b = -2.5$, $c = 2$
(iv)	$x \geqslant 2$	**B1**	
(v)	$x^2 - yx - 2 = 0$	**B1**	or $y^2 - xy - 2 = 0$
	$[x =]\dfrac{-(-y) \pm \sqrt{(-y)^2 - 4(1)(-2)}}{2}$	**M1**	or $[y =]\dfrac{-(-x) \pm \sqrt{(-x)^2 - 4(1)(-2)}}{2}$
	Explains why negative square root should be discarded	**B1**	at some point
	$f^{-1}(x) = \dfrac{x+\sqrt{x^2+8}}{2}$	**A1**	allow $y = \dfrac{x+\sqrt{x^2+8}}{2}$ If zero scored, allow **SC2** for showing correctly that the inverse of the given f^{-1} is f.

10. 0606_s17_qp_23 Q: 2



The four graphs above are labelled **A**, **B**, **C** and **D**.

(i) Write down the letter of each graph that represents a function, giving a reason for your choice. [2]

(ii) Write down the letter of each graph that represents a function which has an inverse, giving a reason for your choice. [2]

Answer:

(i)	*B* and *C* with valid reason	**B2**	**B1** for one graph and valid reason or both graphs and no reason
(ii)	*B* only with valid reason	**B2**	**B1** for graph *B* or valid reason

11. 0606_w17_qp_23 Q: 6

The functions f and g are defined for real values of x by

$$f(x) = (x+2)^2 + 1,$$

$$g(x) = \frac{x-2}{2x-1},\ x \neq \frac{1}{2}.$$

(i) Find $f^2(-3)$. [2]

(ii) Show that $g^{-1}(x) = g(x)$. [3]

(iii) Solve $gf(x) = \frac{8}{19}$. [4]

Answer:

(i)	$f^2 = f(f)$ used algebraic $([(x+2)^2+1]+2)^2+1$	**M1**	numerical or algebraic
	17	**A1**	
(ii)	$x = \dfrac{y-2}{2y-1}$	**M1**	change x and y
	$2xy - x = y - 2 \rightarrow y(2x-1) = x-2$	**M1**	**M1dep** multiply, collect y terms, factorise
	$y = \dfrac{x-2}{2x-1} \quad [= g(x)]$	**A1**	correct completion
(iii)	$gf(x) = \dfrac{[(x+2)^2+1]-2}{2[(x+2)^2+1]-1}$ oe	**B1**	
	$\dfrac{(x+2)^2-1}{2(x+2)^2+1} = \dfrac{8}{19}$ $3(x+2)^2 = 27$ oe $\quad 3x^2 + 12x - 15 = 0$	**M1**	*their* gf $= \dfrac{8}{19}$ and simplify to quadratic equation
	solve quadratic	**M1**	**M1dep** Must be of equivalent form
	$x = 1 \quad x = -5$	**A1**	

12. 0606_s14_qp_21 Q: 12

The functions f and g are defined by

$$f(x) = \frac{2x}{x+1} \text{ for } x > 0,$$

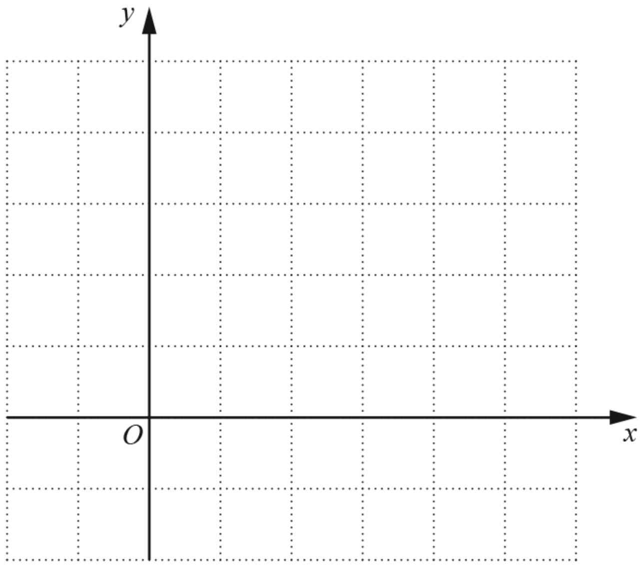
$$g(x) = \sqrt{x+1} \text{ for } x > -1.$$

(i) Find $fg(8)$. [2]

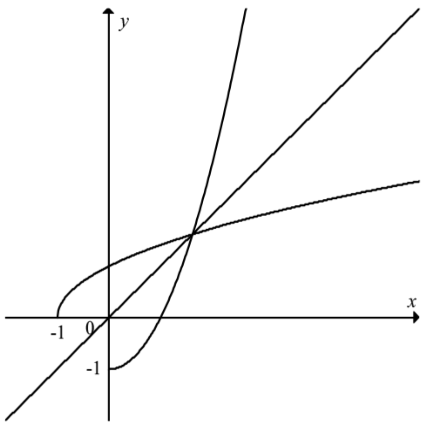
(ii) Find an expression for $f^2(x)$, giving your answer in the form $\frac{ax}{bx+c}$, where a, b and c are integers to be found. [3]

(iii) Find an expression for $g^{-1}(x)$, stating its domain and range. [4]

(iv) On the same axes, sketch the graphs of $y = \mathrm{g}(x)$ and $y = \mathrm{g}^{-1}(x)$, indicating the geometrical relationship between the graphs. [3]



Answer:

(i)	f(3) $\frac{6}{4}$ oe	**M1** **A1**	or $\text{fg}(x)=\frac{2\sqrt{(x+1)}}{\sqrt{(x+1)}+1}$
(ii)	$\frac{2\left(\frac{2x}{x+1}\right)}{\frac{2x}{x+1}+1}$	**M1**	allow omission of 2(…..) in numerator or (…..) + 1 in denominator, but not both.
	A correct and valid step in simplification	**dM1**	e.g. multiplying numerator and denominator by $x+1$, , or simplifying $\frac{2x}{x+1}+1$ to $\frac{2x+x+1}{x+1}$
	Correctly simplified to $\frac{4x}{3x+1}$	**A1**	
(iii)	Putting $y = \text{g}(x)$, changing subject to x and swopping x and y or vice versa	**M1**	condone $x = y^2 -1$; reasonable attempt at correct method
	$\text{g}^{-1}(x) = x^2 -1$	**A1**	condone $y =$….. , $\text{f}^{-1} =$……
	(Domain) $x > 0$ (Range) $\text{g}^{-1}(x) > -1$	**B1** **B1**	condone $y > -1$ $\text{f}^{-1} > -1$
(iv)		**B1 + B1** **B1**	correct graphs; −1 need not be labelled but could be implied by 'one square' idea of reflection or symmetry in line $y = x$ must be stated.

13. 0606_s14_qp_23 Q: 12

The function f is such that $f(x) = 2 + \sqrt{x-3}$ for $4 \leqslant x \leqslant 28$.

(i) Find the range of f. [2]

(ii) Find $f^2(12)$. [2]

(iii) Find an expression for $f^{-1}(x)$. [2]

The function g is defined by $g(x) = \dfrac{120}{x}$ for $x \geqslant 0$.

(iv) Find the value of x for which $gf(x) = 20$. [3]

Answer:

(i)	$3 < f < 7$	**B1,B1**	If **B0** then **SC1** for $3 \leq f \leq 7$
(ii)	$f(12) = 5$	**B1**	$f^2(x)\ \sqrt{\left(\sqrt{(x-3)}+2-3\right)}+2$ earns **B1**
	$(f(5) =)\ 2+\sqrt{2}$	**B1**	
(iii)	Clear indication of method $f^{-1}(x) = (x-2)^2 + 3$	**M1** **A1**	condone $y = (x-2)^2 + 3$
(iv)	$gf(x) = \dfrac{120}{\sqrt{(x-3)}+2}$	**B1**	
	Attempt to solve *their* $gf(x) = 20$	**M1**	
	$x = 19$	**A1**	

14. 0606_w14_qp_21 Q: 4

The functions f and g are defined for real values of x by

$$\mathrm{f}(x) = \sqrt{x-1} - 3 \qquad \text{for } x > 1,$$

$$\mathrm{g}(x) = \frac{x-2}{2x-3} \qquad \text{for } x > 2.$$

(i) Find gf(37). [2]

(ii) Find an expression for $\mathrm{f}^{-1}(x)$. [2]

(iii) Find an expression for $\mathrm{g}^{-1}(x)$. [2]

Answer:

(i)	$f(37)=3$ or $gf(x)=\dfrac{\sqrt{x-1}-3-2}{2\left(\sqrt{x-1}-3\right)-3}$	**B1**	
	$gf(37)=\dfrac{3-2}{6-3}=\dfrac{1}{3}$	**B1**	
(ii)	$y=\sqrt{x-1}-3 \rightarrow (y+3)^2=x-1$	**M1**	Rearrange and square in any order
	$(x+3)^2+1=f^{-1}(x)$ oe isw	**A1**	Interchange x and y and complete
(iii)	$y=\dfrac{x-2}{2x-3}$		
	$2xy-3y=x-2 \rightarrow 2xy-x=3y-2$	**M1**	Multiply and collect like terms
	$\dfrac{3x-2}{2x-1}=g^{-1}(x)$ oe	**A1**	Interchange and complete Mark final answer

15. 0606_s13_qp_21 Q: 11

A one-one function f is defined by $f(x) = (x-1)^2 - 5$ for $x \geqslant k$.

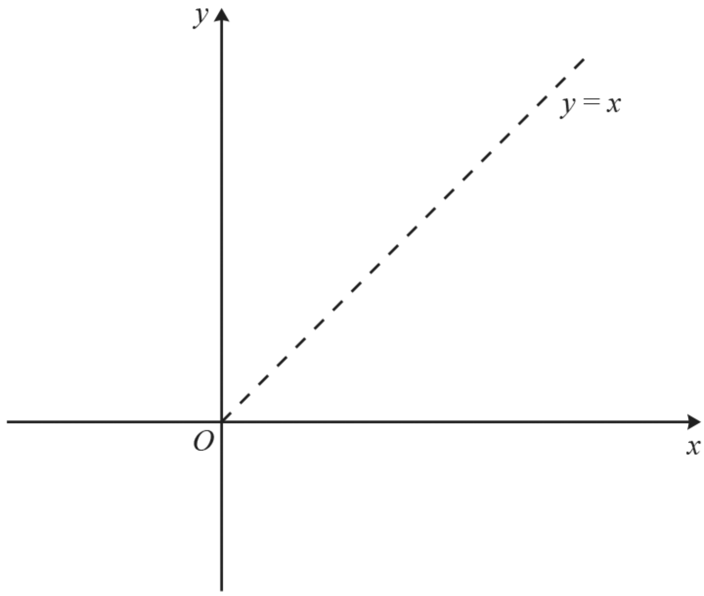
(i) State the least value that k can take. [1]

For this least value of k

(ii) write down the range of f, [1]

(iii) find $f^{-1}(x)$, [2]

(iv) sketch and label, on the axes below, the graph of $y = \mathrm{f}(x)$ and of $y = \mathrm{f}^{-1}(x)$, [2]



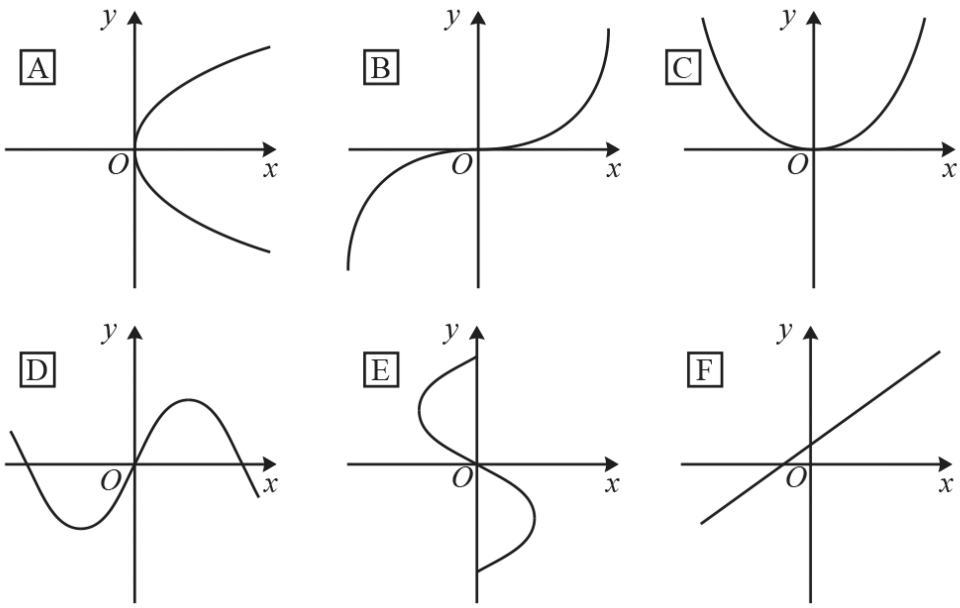
(v) find the value of x for which $\mathrm{f}(x) = \mathrm{f}^{-1}(x)$. [2]

Answer:

(i)	1	**B1**	Not a range for k, but condone $x = 1$ and $x \geqslant 1$
(ii)	$f \geqslant -5$	**B1**	Not x, but condone y
(iii)	Method of inverse	**M1**	Do not reward poor algebra but allow slips
	$1+\sqrt{x+5}$	**A1**	Must be f^{-1} = ...or y = ...
(iv)	f: Positive quadratic curve correct range and domain	**B1**	Must cross x-axis
	f^{-1}: Reflection of f in $y = x$	**B1√**	√*their* f*(x)* sketch Condone slight inaccuracies unless clear contradiction.
(v)	Arrange f $(x) = x$ or f^{-1} $(x) = x$ to 3 term quadratic = 0	**M1**	
	4 only www	**A1**	Allow $x = 4$ with no working. Condone (4, 4). Do not allow final **A** mark if –1 also given in answer

16. 0606_s13_qp_22 Q: 3

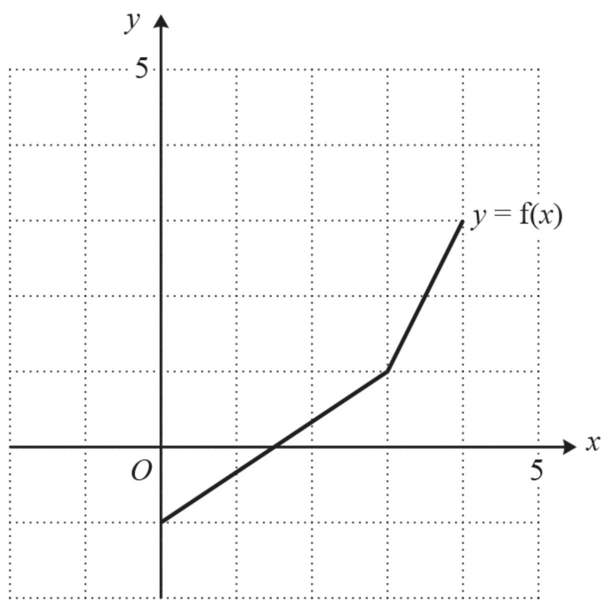
(a)



(i) Write down the letter of each graph which does **not** represent a function. [2]

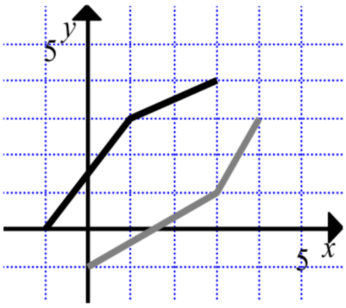
(ii) Write down the letter of each graph which represents a function that does **not** have an inverse. [2]

(b)



The diagram shows the graph of a function $y = \mathrm{f}(x)$. On the same axes sketch the graph of $y = \mathrm{f}^{-1}(x)$. [2]

Answer:

(a) (i)	A and E	**B2**	1 mark for each **B1** for 1 extra, **B0** if 2 or more extras
(ii)	C and D	**B2**	1 mark for each **B1** if 1 extra, **B0** if 2 or more extras
(b)		**B2**	(–1, 0), (1, 3), (3, 4) or **B1** for two points correct and joined or for three points correct but clearly not joined

Chapter 2

Quadratic functions

17. 0606_m22_qp_22 Q: 4

Find the x-coordinates of the points of intersection of the curves $\frac{x^2}{4}+\frac{y^2}{9}=1$ and $y=\frac{3}{2x}$.
Give your answers correct to 3 decimal places. [5]

Answer:

Question	Answer	Marks	Partial Marks
	Eliminates one unknown e.g. $\frac{x^2}{4}+\frac{1}{9}\left(\frac{3}{2x}\right)^2=1$	M1	
	Rearranges to solvable form e.g. $x^4-4x^2+1=0$	A1	
	Solves : $\frac{-(-4)\pm\sqrt{(-4)^2-4(1)(1)}}{2}$	M1	**dep** on attempt to eliminate one unknown and having a 3-term quadratic in x^2
	$x^2=2\pm\sqrt{3}$ oe isw or 3.7320[5...] and 0.2679[4...]	A1	
	$x=\pm 1.932$ or $x=\pm 0.518$	A1	

18. 0606_s22_qp_22 Q: 3

Find the possible values of k for which the equation $kx^2+(k+5)x-4=0$ has real roots. [5]

Answer:

Question	Answer	Marks	Partial Marks
	Uses $b^2 - 4ac$ oe: $(k+5)^2 - 4k(-4)$ [* 0, where * could be = or any inequality sign]	**M1**	
	Forms a correct 3-term expression: $k^2 + 26k + 25$	**A1**	
	Factorises $k^2 + 26k + 25$ or solves $k^2 + 26k + 25 = 0$ oe	**M1**	**dep** on first **M1**, **FT** *their* 3-term quadratic in k
	Correct critical values -1, -25 soi	**A1**	
	$k \leqslant -25$, $k \geqslant -1$	**A1**	mark final answer

19. 0606_w22_qp_21 Q: 2

DO NOT USE A CALCULATOR IN THIS QUESTION.

Find the x-coordinates of the points where the line $y = 3x - 8$ cuts the curve $y = 2x^3 + 3x^2 - 26x + 22$. [5]

Answer:

Question	Answer	Marks	Guidance
	$2x^3+3x^2-29x+30[=0]$	**B1**	
	Uses a correct factor $x-2$ or $x+5$ to find a quadratic factor with at least 2 terms correct	**M1**	
	$(x-2)\to(2x^2+7x-15)\quad[=0]$ or $(x+5)\to(2x^2-7x+6)\quad[=0]$	**A1**	
	Factorises or solves *their* 3-term quadratic: $(x+5)(2x-3)$ [= 0] or $(x-2)(2x-3)$ [= 0] or $[x=]\dfrac{-7\pm\sqrt{7^2-4(2)(-15)}}{2(2)}$ or $\dfrac{7\pm\sqrt{(-7)^2-4(2)(6)}}{2(2)}$	**M1**	**dep** on previous **M1**
	$x=2, -5, 1.5$	**A1**	

20. 0606_w22_qp_21 Q: 6

(a) Write $3x^2+15x-20$ in the form $a(x+b)^2+c$ where a, b and c are rational numbers. [4]

(b) State the minimum value of $3x^2+15x-20$ and the value of x at which it occurs. [2]

(c) Use your answer to **part (a)** to solve the equation $3y^{\frac{2}{3}}+15y^{\frac{1}{3}}-20=0$, giving your answers correct to three significant figures. [3]

Answer:

Question	Answer	Marks	Guidance
(a)	$3\left(x+\frac{5}{2}\right)^2-\frac{155}{4}$	B4	**B2** for $3\left(x+\frac{5}{2}\right)^2$ or $3(x+2.5)^2$ or **B1** for $\left(x+\frac{5}{2}\right)^2$ or $(x+2.5)^2$ **B2** for $c=-\frac{155}{4}$ or -38.75 or **B1** for $-\frac{25}{4}\times 3-20$ oe
(b)	Min value $-\frac{155}{4}$ when x is $-\frac{5}{2}$	B2	**FT** *their c* from part **a** and –*their b* from **(a)** **B1** for either without contradiction
Question	**Answer**	**Marks**	**Guidance**
(c)	$3\left(y^{\frac{1}{3}}+\frac{5}{2}\right)^2=\frac{155}{4}$ soi	M1	**FT** an expression of correct form from **(a)**
	Rearranges as far as: $y^{\frac{1}{3}}=-\frac{5}{2}\pm\sqrt{\frac{155}{12}}$ soi	A1	
	$y = 1.31$ or -226	A1	

21. 0606_m21_qp_22 Q: 2

Find the values of the constant k for which the equation $kx^2 - 3(k+1)x + 25 = 0$ has equal roots.

[4]

Answer:

	Uses $b^2 - 4ac$ with at most one error in substitution: $(-3(k+1))^2 - 4(k)(25) * 0$	**M1**	
	$9k^2 - 82k + 9 * 0$	**A1**	
	Factorises or solves *their* 3-term quadratic	**M1**	
	$k = \frac{1}{9}$ or 9; mark final answer	**A1**	

22. 0606_s21_qp_21 Q: 1

(a) Write the expression $x^2 - 6x + 1$ in the form $(x+a)^2 + b$, where a and b are constants. [2]

(b) Hence write down the coordinates of the minimum point on the curve $y = x^2 - 6x + 1$. [1]

Answer:

Question	Answer	Marks	Partial Marks
(a)	$(x-3)^2 - 8$	**B2**	**B1** for $(x-3)^2 + k$ where $k \neq -8$ or $a = -3$ or $(x+m)^2 - 8$ where $m \neq -3$ or $b = -8$
(b)	$(3, -8)$	**B1**	**strict FT** *their* a and b

23. 0606_s21_qp_21 Q: 7

Find the exact values of the constant k for which the line $y = 2x + 1$ is a tangent to the curve $y = 4x^2 + kx + k - 2$. [6]

Answer:

Question	Answer	Marks	Partial Marks
	$4x^2 + kx + k - 2 = 2x + 1$	**M1**	
	$4x^2 + (k-2)x + k - 3[*0]$ soi	**A1**	* can be $<$, $>$, $=$, $\leq$, $\geq$
	$(k-2)^2 - 4(4)(k-3)$	**M1**	
	$k^2 - 20k + 52 * 0$	**A1**	
	$k = \dfrac{-(-20) \pm \sqrt{(-20)^2 - 4(1)(52)}}{2}$	**M1**	
	$k = 10 \pm \sqrt{48}$ oe isw	**A1**	
	Alternative (using calculus) $2 = 8x + k$ oe	**(M1)**	
	$y = 4x^2 + (2-8x)x + 2 - 8x - 2$ or $y = -4x^2 - 6x$	**(M1)**	
	$0 = 4x^2 + 8x + 1$	**(A1)**	
	$x = \dfrac{-8 \pm \sqrt{8^2 - 4(4)(1)}}{8}$	**(M1)**	
	$x = -1 \pm \dfrac{\sqrt{48}}{8}$ oe	**(A1)**	
	for $k = 10 \pm \sqrt{48}$ oe	**(A1)**	

24. 0606_s21_qp_22 Q: 3

Find the values of the constant k for which $(2k-1)x^2+6x+k+1=0$ has real roots. [5]

Answer:

	Uses $b^2 - 4ac$: $6^2 - 4(2k - 1)(k + 1)$	**M1**	
	$-8k^2 - 4k + 40 * 0$ oe	**M1**	**dep** on first **M1** where * is = or any inequality sign condone one sign or arithmetic slip in simplification
	Factorises or solves *their* 3-term quadratic expression or equation for CVs e.g. $(5 + 2k)(8 - 4k)$ oe	**M1**	
	Finds correct CVs: -2.5 oe, 2	**A1**	
	$-2.5 \leqslant k \leqslant 2$	**A1**	mark final answer

25. 0606_w21_qp_21 Q: 6

Find the values of m for which the line $y = mx - 2$ does not touch or cut the curve $y = (m+1)x^2 + 8x + 1$. [6]

Answer:

Question	Answer	Marks	Partial Marks
6	$(m+1)x^2+(8-m)x+3=0$ oe, soi	B1	
	$(8-m)^2-4(m+1)(3)$	M1	
	$m^2-28m+52$ [*0] oe	M1	dep on previous **M1**; condone one sign error where * is = or any inequality sign
	Factorises or solves *their* 3-term quadratic expression or equation for CVs	M1	dep on use of b^2-4ac
	Finds correct CVs: 2, 26	A1	
	$2<m<26$	A1	Mark final answer

26. 0606_m20_qp_22 Q: 7

Find the coordinates of the points of intersection of the curves $x^2=5y-1$ and $y=x^2-2x+1$. [5]

Answer:

	Eliminates one variable e.g. $x^2 = 5(x^2 - 2x + 1) - 1$ or $y = 5y - 1 - 2\sqrt{5y-1} + 1$	**M1**	
	Collects terms ready to solve e.g. $4x^2 - 10x + 4 = 0$ or $4y^2 - 5y + 1 = 0$	**A1**	
	Factorises, applies the formula or completes the square e.g. $2(2x-1)(x-2)$ or $(4y-1)(y-1)$	**M1**	
	Both $(0.5,\ 0.25)$ and $(2, 1)$	**A2**	**A1** for either $(0.5, 0.25)$ or $(2, 1)$ provided nfww or $x = 0.5, 2$ or $y = 0.25, 1$

27. 0606_s20_qp_21 Q: 2

(a) Write $9x^2 - 12x + 5$ in the form $p(x-q)^2 + r$, where p, q and r are constants. [3]

(b) Hence write down the coordinates of the minimum point of the curve $y = 9x^2 - 12x + 5$. [1]

Answer:

(a)	$9\left(x-\frac{2}{3}\right)^2+1$ oe	**B3**	**B1** for each of p, q, r correct in correctly formatted expression; allow correct equivalent values If **B0** then **SC2** for $9\left(x-\frac{2}{3}\right)+1$ or **SC1** for correct values but other incorrect format
(b)	*their* $\left(\frac{2}{3}, 1\right)$ oe	**B1**	**FT** *their* (a)

28. 0606_s20_qp_22 Q: 3

Find the values of k for which the line $y = x - 3$ intersects the curve $y = k^2x^2 + 5kx + 1$ at two distinct points. [6]

Answer:

	$x-3=k^2x^2+5kx+1$	**M1**	
	$k^2x^2+(5k-1)x+4=0$ soi	**A1**	
	$(5k-1)^2-4(k^2)(4)$	**M1**	
	$9k^2-10k+1*0$	**M1**	
	Critical values: $\frac{1}{9}$ and 1 soi	**A1**	
	$k<\frac{1}{9}$ or $k>1$	**A1**	

29. 0606_s20_qp_23 Q: 2

Find the set of values of k for which $4x^2-4kx+2k+3=0$ has no real roots. [5]

Answer:

	Uses $b^2 - 4ac$ $(-4k)^2 - 4(4)(2k+3)$ soi	**M1**	
	Correctly simplifies $16k^2 - 32k - 48$	**A1**	**FT** provided of equivalent difficulty
	$16(k+1)(k-3)$ oe	**M1**	
	CV $-1, 3$	**A1**	
	$-1 < k < 3$	**A1**	**FT** *their* lower CV $< k <$ *their* upper CV

30. 0606_w20_qp_21 Q: 2

Find the coordinates of the points of intersection of the curve $x^2 + xy = 9$ and the line $y = \frac{2}{3}x - 2$. [5]

Answer:

	$x^2 + x\left(\frac{2}{3}x - 2\right) = 9$	**M1**	Eliminate *y*
	$5x^2 - 6x - 27 = 0$	**A1**	
	$(x-3)(5x+9) = 0$	**M1**	Factorise or formula
	(3, 0)	**A1**	Or both *x* values
	$\left(-\frac{9}{5}, -\frac{16}{5}\right)$	**A1**	

31. 0606_w20_qp_23 Q: 3

Find the values of k for which the equation $x^2 + (k+9)x + 9 = 0$ has two distinct real roots. [4]

Answer:

	$(k+9)^2 - 4\times 9$ (>0)	**M1**	use $b^2 - 4ac$
	$k^2 + 18k + 45$ (>0)	**A1**	
	$k = -15$ $k = -3$	**A1**	
	$k < -15$ or $k > -3$ no isw mark final answer	**A1**	not 'and' A0 if combined as one statement

32. 0606_m19_qp_22 Q: 4

(a) Find the values of x for which $(2x+1)^2 \leqslant 3x+4$. [3]

(b) Show that, whatever the value of k, the equation $\frac{x^2}{4}+kx+k^2+1=0$ has no real roots. [3]

Answer:

4(a)	Expands, rearranges to form a 3-term quadratic on one side $4x^2+x-3[*0]$	**M1**	
	Critical values $\frac{3}{4}$ and -1	**A1**	
	$-1 \leqslant x \leqslant \frac{3}{4}$ final answer	**A1**	**FT** *their* critical values
4(b)	$k^2-4\left(\frac{1}{4}\right)(k^2+1)$	**M1**	
	-1	**A1**	
	discriminant independent of k and negative oe	**A1**	**FT** *their* -1

33. 0606_s19_qp_21 Q: 1

Find the values of x for which $x(6x+7) \geqslant 20$. [3]

Answer:

	$6x^2+7x-20\,[*0]$	**M1**	where * may be any inequality sign or =
	Critical values $\frac{4}{3}$, $-\frac{5}{2}$	**A1**	
	$x \leqslant -\frac{5}{2}$ or $x \geqslant \frac{4}{3}$ final answer	**A1**	**FT** *their* critical values using outside regions

34. 0606_s19_qp_22 Q: 2

Find the values of k for which the equation $(k-1)x^2 + kx - k = 0$ has real and distinct roots. [4]

Answer:

	$k^2 - 4(k-1)(-k)$ oe	**B1**	
	$k(5k-4)$	**M1**	
	Correct critical values 0, 0.8 oe	**A1**	
	$k < 0, k > 0.8$ oe	**A1**	**FT** *their* critical values provided *their* $ak^2 + bk + c > 0$ has positive a and there are 2 values; mark final answer If **B1 M0** allow **SC1** for a final answer of $k > 0.8$ oe

35. 0606_s19_qp_22 Q: 5

(i) Express $5x^2 - 15x + 1$ in the form $p(x+q)^2 + r$, where p, q and r are constants. [3]

(ii) Hence state the least value of $x^2 - 3x + 0.2$ and the value of x at which this occurs. [2]

Answer:

(i)	$5(x-1.5)^2-10.25$ isw	B3	**B1** for each of p, q, r correct in correctly formatted expression; allow correct equivalent values If **B0** then **SC2** for $5(x-1.5)-10.25$ or **SC1** for correct values but other incorrect format
(ii)	$\dfrac{their\ -10.25}{5}$ is least value when $x = their 1.5$	B2	**STRICT FT** *their* part (i); **B1 STRICT FT** for each

36. 0606_w19_qp_22 Q: 4

Find the values of k for which the line $y = kx + 3$ does not meet the curve $y = x^2 + 5x + 12$. [5]

Answer:

	$kx + 3 = x^2 + 5x + 12$ $\rightarrow x^2 + (5 - k)x + 9(= 0)$	**M1**	Equate and attempt to simplify to all terms on one side.
	Use discriminant of *their* quadratic.	**M1**	**dep**
	$(5 - k)^2 - 36$ oe	**A1**	Unsimplified
	$k = -1$ and 11	**A1**	Both boundary values
	$-1 < k < 11$	**A1**	Must be in terms of k.
	OR $2x + 5 \sim k$	**M1**	Connect gradients of line and curve
	$y = (2x + 5)x + 3 \rightarrow$ $2x^2 + 5x + 3 = x^2 + 5x + 12$	**M1**	Eliminate k and y.
	$x^2 = 9 \rightarrow x = \pm 3$	**A1**	
	$k = 11$ or $k = -1$	**A1**	
	$-1 < k < 11$	**A1**	

37. 0606_w19_qp_22 Q: 11

Do not use a calculator in this question.

Solve the quadratic equation $(\sqrt{5}-3)x^2+3x+(\sqrt{5}+3)=0$, giving your answers in the form $a+b\sqrt{5}$, where a and b are constants. [6]

Answer:

	$(\sqrt{5}-3)(\sqrt{5}+3)=-4$	**B1**	Seen anywhere
	Attempt formula	**M1**	
	$x=\dfrac{-3\pm 5}{2(\sqrt{5}-3)}$	**A1**	
	Multiply by *their* $(\sqrt{5}+3)$	**M1**	Attempt must be seen with a further line of working. oe
	$x=\sqrt{5}+3$	**A1**	oe Mark final answer
	$x=\dfrac{-1(\sqrt{5}+3)}{4}$	**A1**	oe Mark final answer