Topical Past Paper Questions Workbook

AS & A Level Mathematics (9709) Paper 3 [Pure Mathematics 3]

Exam Series: May 2015 – Nov 2022

Format Type B:
Each question is followed by its answer scheme

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Introduction

Each Topical Past Paper Questions Workbook contains a comprehensive collection of hundreds of questions and corresponding answer schemes, presented in worksheet format. The questions are carefully arranged according to their respective chapters and topics, which align with the latest IGCSE or AS/A Level subject content. Here are the key features of these workbooks:

1. The workbook covers a wide range of topics, which are organized according to the latest syllabus content for Cambridge IGCSE or AS/A Level exams.
2. Each topic includes numerous questions, allowing students to practice and reinforce their understanding of key concepts and skills.
3. The questions are accompanied by detailed answer schemes, which provide clear explanations and guidance for students to improve their performance.
4. The workbook's format is user-friendly, with worksheets that are easy to read and navigate.
5. This workbook is an ideal resource for students who want to familiarize themselves with the types of questions that may appear in their exams and to develop their problem-solving and analytical skills.

Overall, Topical Past Paper Questions Workbooks are a valuable tool for students preparing for IGCSE or AS/A Level exams, providing them with the opportunity to practice and refine their knowledge and skills in a structured and comprehensive manner. To provide a clearer description of this book's specifications, here are some key details:

- Title: AS & A Level Mathematics (9709) Paper 3 Topical Past Paper Questions Workbook
- Subtitle: Exam Practice Worksheets With Answer Scheme
- Examination board: Cambridge Assessment International Education (CAIE)
- Subject code: 9709
- Years covered: May 2015 – Nov 2022
- Paper: 3 [Pure Mathematics 3]
- Number of pages: 1155
- Number of questions: 498

Contents

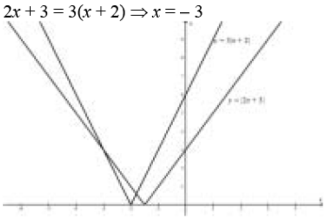
Chapter 1

Algebra

1. 9709_m22_qp_32 Q: 1

Solve the inequality $|2x + 3| > 3|x + 2|$. [4]

Answer:

Question	Answer	Marks	Guidance
	State or imply non-modular inequality $(2x+3)^2 > 3^2(x+2)^2$, or corresponding quadratic equation, or pair of linear equations	**B1**	
	Make a reasonable attempt at solving a 3-term quadratic, or solve two linear equations for x	**M1**	Quadratic formula or $(5x + 9)(x + 3)$
	Obtain critical values $x = -3$ and $x = -\frac{9}{5}$	**A1**	OE
	State final answer $-3 < x < -\frac{9}{5}$ or $x > -3$ and $x < -\frac{9}{5}$	**A1**	[Do not condone $\leqslant$ for $<$ in the final answer.] No ISW
	Alternative method for question 1		
	Obtain critical value $x = -3$ from a graphical method, or by solving a linear equation or linear inequality	**B1**	$2x + 3 = 3(x + 2) \Rightarrow x = -3$ 
	Obtain critical value $x = -\frac{9}{5}$ similarly	**B2**	
	State final answer $-3 < x < -\frac{9}{5}$ or $x > -3$ and $x < -\frac{9}{5}$	**B1**	[Do not condone $\leqslant$ for $<$ in the final answer.] No ISW
		4	

2. 9709_S22_qp_31 Q: 2

(a) Expand $\left(2 - x^2\right)^{-2}$ in ascending powers of x, up to and including the term in x^4, simplifying the coefficients. [4]

(b) State the set of values of x for which the expansion is valid. [1]

Answer:

Question	Answer	Marks	Guidance
(a)	State a correct unsimplified version of the x^2 or the x^4 term of the expansion of $(2-x^2)^{-2}$ or $\left(1-\frac{1}{2}x^2\right)^{-2}$	**M1**	$\frac{1}{4}\left(1+2\frac{x^2}{2}+\frac{-2.-3}{2}\left(\frac{x^2}{2}\right)^2 ...\right)$ Symbolic binomial coefficients are not sufficient for the M1.
	State correct first term $\frac{1}{4}$	**B1**	Accept 2^{-2}.
	Obtain the next two terms $\frac{1}{4}x^2+\frac{3}{16}x^4$	**A1 A1**	A1 for each one correct ISW. Full marks for $\frac{1}{4}\left(1+x^2+\frac{3}{4}x^4\right)$ ISW.
			SC allow **M1 A1 A1** for $\frac{1}{4}$ and $1+x^2+\frac{3}{4}x^4$ SOI. **SC** allow **M1 A1** for $1+x^2+\frac{3}{4}x^4$
		4	
(b)	State answer $\lvert x\rvert<\sqrt{2}$	**B1**	Or $-\sqrt{2}<x<\sqrt{2}$.
		1	

3. 9709_S22_qp_31 Q: 5

The polynomial $ax^3 - 10x^2 + bx + 8$, where a and b are constants, is denoted by $p(x)$. It is given that $(x - 2)$ is a factor of both $p(x)$ and $p'(x)$.

(a) Find the values of a and b. [5]

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(b) When a and b have these values, factorise $\mathrm{p}(x)$ completely. [3]

Answer:

Question	Answer	Marks	Guidance
(a)	Substitute $x = 2$, equate to zero	**M1**	Or divide by $x - 2$ and equate constant remainder to zero.
	Obtain a correct equation, e.g. $8a - 40 + 2b + 8 = 0$	**A1**	Seen or implied in subsequent work.
	Differentiate $p(x)$, substitute $x = 2$ and equate result to zero	**M1**	Or divide by $x - 2$ and equate constant remainder to zero.
	Obtain $12a - 40 + b = 0$, or equivalent	**A1**	SOI in subsequent work.
	Obtain $a = 3$ and $b = 4$	**A1**	
	Alternative method for question 5(a)		
	State or imply $(x-2)^2$ is a factor	**M1**	
	$p(x)=(x-2)^2(ax+2)$	**A1**	
	Obtain an equation in b	**M1**	
	e.g. by comparing coefficients of x: $b=4a-8$	**A1**	
	Obtain $a = 3$ and $b = 4$	**A1**	
			SC If uses $x = -2$ in both equations allow **M1** and allow **A1** for $a = -3$, $b = -4$.
		5	

Question	Answer	Marks	Guidance
(b)	Attempt division by $(x - 2)$	**M1**	The M1 is earned if division reaches a partial quotient of $ax^2 + kx$, or if inspection has an unknown factor ax^2+ex+f and an equation in e and/or f. Where a has the value found in part **5(a)**.
	Obtain quadratic factor $3x^2 - 4x - 4$	**A1**	
	Obtain factorisation $(3x+2)(x-2)(x-2)$	**A1**	
	Alternative method for question 5(b)		
	State or imply $(x-2)^2$ is a factor	**B1**	
	Attempt division by $(x-2)^2$, reaching a quotient $ax + k$ or use inspection with unknown factor $cx + d$ reaching a value for c or for d	**M1**	
	Obtain factorisation $(3x+2)(x-2)^2$	**A1**	Accept $3\left(x+\frac{2}{3}\right)(x-2)^2$.
		3	

4. 9709_S22_qp_32 Q: 3

The polynomial $ax^3 + x^2 + bx + 3$ is denoted by $p(x)$. It is given that $p(x)$ is divisible by $(2x - 1)$ and that when $p(x)$ is divided by $(x + 2)$ the remainder is 5.

Find the values of a and b. [5]

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Answer:

Question	Answer	Marks	Guidance
	Substitute $x = \frac{1}{2}$, equate result to zero	**M1**	Or divide by $2x–1$ and equate constant remainder to zero.
	Obtain a correct simplified equation	**A1**	e.g. $\frac{1}{8}a+\frac{1}{4}+\frac{1}{2}b+3=0$ or $a+4b=-26$
	Substitute $x = -2$, equate result to 5	**M1**	Or divide by $x+2$ and equate constant remainder to 5.
	Obtain a correct simplified equation	**A1**	e.g. $-8a+4-2b+3=5$ or $8a+2b=2$
	Obtain $a = 2$ and $b = -7$	**A1**	WWW
		5	

5. 9709_S22_qp_33 Q: 1

Find, in terms of a, the set of values of x satisfying the inequality

$$2|3x + a| < |2x + 3a|,$$

where a is a positive constant. [4]

Answer:

Question	Answer	Marks	Guidance
	State or imply non-modular inequality $2^2(3x+a)^2 < (2x+3a)^2$, or corresponding quadratic equation, or pair of linear equations	**B1**	e.g. $(6x+2a)^2 = (2x+3a)^2$ or $32x^2 + 12xa - 5a^2 = 0$ $2(3x + a) = (2x + 3a)$ and $-2(3x + a) = (2x + 3a)$
	Solve 3-term quadratic, or solve two linear equations for x	**M1**	Apply general rules for solving quadratic equation by formula or by factors. Instead of x = {formula}, have {formula} = 0 and try to solve for a then M0
	Obtain critical values $x = \frac{1}{4}a$ and $x = -\frac{5}{8}a$	**A1**	
	State final answer $-\frac{5}{8}a < x < \frac{1}{4}a$ or $-0.625a < x < 0.25a$ or $x > -\frac{5}{8}a$ **and** $x < \frac{1}{4}a$ or $x > -\frac{5}{8}a \cap x < \frac{1}{4}a$	**A1**	Do not condone $\leqslant$ for $<$ in the final answer. Do not ISW. **SC** Set a to value, (say $a = 1$), after initial B1 gained, then $-\frac{5}{8} < x < \frac{1}{4}$ **B1** maximum 2 out of 4.
	Alternative method for question 1		
	Obtain critical value $x = \frac{1}{4}a$ from a graphical method, or by solving a linear equation or linear inequality	**B1**	
	Obtain critical value $x = -\frac{5}{8}a$ similarly	**B2**	
	State final answer $-\frac{5}{8}a < x < \frac{1}{4}a$ or $-0.625a < x < 0.25a$ or $x > -\frac{5}{8}a$ **and** $x < \frac{1}{4}a$ or $x > -\frac{5}{8}a \cap x < \frac{1}{4}a$	**B1**	Do not condone $\leqslant$ for $<$ in the final answer. Do not ISW. **SC** Set a to value, (say $a = 1$), after initial B1 gained, then $-\frac{5}{8} < x < \frac{1}{4}$ **B1** maximum 2 out of 4.
		4	

6. 9709_S22_qp_33 Q: 7

Let $f(x) = \dfrac{5x^2 + 8x - 3}{(x-2)(2x^2+3)}$.

(a) Express $f(x)$ in partial fractions. [5]

(b) Hence obtain the expansion of $f(x)$ in ascending powers of x, up to and including the term in x^2. [5]

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Answer:

Question	Answer	Marks	Guidance
(a)	State or imply the form $\dfrac{A}{x-2}+\dfrac{Bx+C}{2x^2+3}$	**B1**	If $1-\dfrac{A}{x-2}+\dfrac{Bx+C}{2x^2+3}$ or $\dfrac{A}{x-2}+\dfrac{C}{2x^2+3}$ B0 then M1 A1 (for $A = 3$) still possible.
	Use a correct method for finding a constant	**M1**	
	Obtain one of $A = 3$, $B = -1$ and $C = 6$	**A1**	Allow all A marks obtained even if method would give errors if equations solved in a different order.
	Obtain a second value	**A1**	
	Obtain the third value	**A1**	
		5	

Question	Answer	Marks	Guidance
(b)	Use correct method to find the first two terms of the expansion of $(x-2)^{-1}$, $\left(1-\frac{1}{2}x\right)^{-1}$, $\left(2x^2+3\right)^{-1}$ or $\left(1+\frac{2}{3}x^2\right)^{-1}$	**M1**	Symbolic binomial coefficients not sufficient for the M1.
	Obtain correct unsimplified expansions, up to the term in x^2, of each partial fraction	**A1 FT** **A1 FT**	The FT is on A, B and C. $-\dfrac{A}{2}\left[1-\left(-\dfrac{x}{2}\right)+\dfrac{(-1)(-2)}{2}\left(-\dfrac{x}{2}\right)^2+\ldots\right]$ $\dfrac{Bx+C}{3}\left[1-\dfrac{2x^2}{3}+\ldots\right]$
	Extract the coefficient 3 correctly from $\left(2x^2+3\right)^{-1}$ with expansion to $1\pm\frac{2}{3}x^2$ then multiply by $Bx + C$ up to the terms in x^2, where $BC \neq 0$	**M1**	$\dfrac{C}{3}+\dfrac{Bx}{3}\pm\dfrac{C}{3}\left(\dfrac{2}{3}\right)x^2$ or $\dfrac{1}{3}\left(C+Bx\pm C\left(\dfrac{2}{3}\right)x^2\right)$ Allow a slip in multiplication for M1. Allow miscopies in B and C from **7(a)**.
	Obtain final answer $\frac{1}{2}-\frac{13}{12}x-\frac{41}{24}x^2$	**A1**	Do not ISW.
		5	

7. 9709_w22_qp_31 Q: 1

(a) Sketch the graph of $y = |2x + 1|$. [1]

(b) Solve the inequality $3x + 5 < |2x + 1|$. [3]

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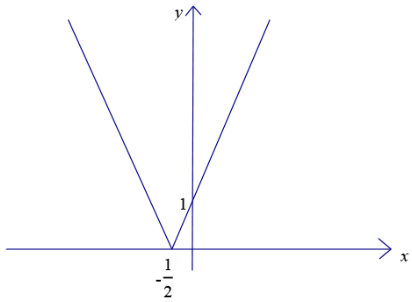
Answer:

Question	Answer	Marks	Guidance
(a)	Show a recognisable sketch graph of $y = \lvert 2x+1 \rvert$	B1	 Ignore $y = 3x + 5$ if also drawn on the sketch.
		1	

Question	Answer	Marks	Guidance
(b)	Find x-coordinate of intersection with $y = 3x + 5$	M1	
	Obtain $x = -\frac{6}{5}$	A1	
	State final answer $x < -\frac{6}{5}$ only	A1	Do not condone $\le$ for $<$ in the final answer.
	Alternative method 1 for question 1(b)		
	Solve the linear inequality $3x+5 < -(2x+1)$, or corresponding equation	M1	Must solve the relevant equation.
	Obtain critical value $x = -\frac{6}{5}$	A1	Ignore -4 if seen.
	State final answer $x < -\frac{6}{5}$ only	A1	
	Alternative method 2 for question 1(b)		
	Solve the quadratic inequality $(3x+5)^2 < (2x+1)^2$, or corresponding equation	M1	$5x^2 + 26x + 24 < 0$
	Obtain critical value $x = -\frac{6}{5}$	A1	Ignore -4 if seen.
	State final answer $x < -\frac{6}{5}$ only	A1	
		3	

8. 9709_w22_qp_31 Q: 10

Let $f(x) = \dfrac{2x^2 + 7x + 8}{(1 + x)(2 + x)^2}$.

(a) Express $f(x)$ in partial fractions. [5]

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(b) Hence obtain the expansion of $f(x)$ in ascending powers of x, up to and including the term in x^2. [5]

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Answer:

Question	Answer	Marks	Guidance
(a)	State or imply the form $\frac{A}{1+x}+\frac{B}{2+x}+\frac{C}{(2+x)^2}$	B1	
	Use a correct method to find a constant	M1	
	Obtain one of $A=3$, $B=-1$ and $C=-2$	A1	SR after B0 can score M1A1 for one correct value
	Obtain a second value	A1	
	Obtain the third value	A1	$\frac{A}{1+x}+\frac{Dx+E}{(2+x)^2}$, where $A=3$, $D=-1$ and $E=-4$, is awarded B1 M1 A1 A1 A1 as above.
		5	
(b)	Use a correct method to find the first two terms of the expansion of $(1+x)^{-1}$, $(2+x)^{-1}$, $\left(1+\frac{1}{2}x\right)^{-1}$, $(2+x)^{-2}$ or $\left(1+\frac{1}{2}x\right)^{-2}$	M1	For the A, D, E form of fractions, award M1 A1FT A1FT for the expanded partial fractions, then if $D \neq 0$, M1 for multiplying out fully, and A1 for the final answer.
	Obtain correct unsimplified expansions up to the term in x^2 of each partial fraction	A3 FT	$3\left(1-x+x^2.....\right)$ $-\frac{1}{2}\left(1-\frac{x}{2}+\frac{x^2}{4}.....\right)$ $-\frac{2}{4}\left(1-x+\frac{3}{4}x^2\right)$
	Obtain final answer $2-\frac{9}{4}x+\frac{5}{2}x^2$	A1	
		5	

9. 9709_w22_qp_32 Q: 2

The polynomial $2x^3 - x^2 + a$, where a is a constant, is denoted by $p(x)$. It is given that $(2x + 3)$ is a factor of $p(x)$.

(a) Find the value of a. [2]

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(b) When a has this value, solve the inequality $p(x) < 0$. [4]

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Answer:

Question	Answer	Marks	Guidance
(a)	Substitute $x = -\frac{3}{2}$ and equate result to zero	**M1**	Or divide by $2x+3$ and set constant remainder equal to zero. Or state $(2x^3 - x^2 + a) = (2x+3)(x^2 + px + q)$, compare coefficients and solve for p or q.
	Obtain $a = 9$	**A1**	
		2	

Question	Answer	Marks	Guidance
(b)	Commence division by $(2x+3)$ reaching a partial quotient $x^2 + kx$	***M1**	The M1 is earned if inspection reaches an unknown factor: $x^2 + Bx + C$ and an equation in B and/or C, or an unknown factor $Ax^2 + Bx + 3$ and an equation in A and/or B.
	Obtain factorisation $(2x+3)(x^2 - 2x + 3)$	**A1**	Allow if the correct quotient seen. Correct factors seen in **(a)** and quoted or used here scores M1A1.
	Show that $x^2 - 2x + 3$ is always positive, or $2x^3 - x^2 + 9$ only intersects the x-axis once	**DM1**	Must use their quadratic factor. SC If M0, allow B1 if state $x < -\frac{3}{2}$ and no error seen
	State **final** answer $x < -\frac{3}{2}$ from correct work	**A1**	
		4	

10. 9709_w22_qp_33 Q: 2

Expand $\sqrt{\dfrac{1+2x}{1-2x}}$ in ascending powers of x, up to and including the term in x^2, simplifying the coefficients. [5]

Answer:

Question	Answer	Marks	Guidance
	State a correct unsimplified term in x or x^2 of the expansion of either $(1+2x)^{\frac{1}{2}}$ or $(1-2x)^{-\frac{1}{2}}$	**B1**	
	State correct unsimplified expansion of $(1+2x)^{\frac{1}{2}}$ up to the term in x^2	**B1**	
	State correct unsimplified expansion of $(1-2x)^{-\frac{1}{2}}$ up to the term in x^2	**B1**	
	Obtain sufficient terms of the product of the expansions	**M1**	
	Obtain final answer $1+2x+2x^2$	**A1**	
	Alternative method for question 2		
	State that the expression equals $(1+2x)(1-4x^2)^{-\frac{1}{2}}$ and state a term of the expansion	**B1**	
	State correct unsimplified expansion of $(1-4x^2)^{-\frac{1}{2}}$ up to the term in x^2	**B1 + B1**	
	Obtain sufficient terms of the product of $(1 + 2x)$ and the expansion	**M1**	
	Obtain final answer $1+2x+2x^2$	**A1**	
		5	

11. 9709_m21_qp_32 Q: 2

The polynomial $ax^3 + 5x^2 - 4x + b$, where a and b are constants, is denoted by $p(x)$. It is given that $(x + 2)$ is a factor of $p(x)$ and that when $p(x)$ is divided by $(x + 1)$ the remainder is 2.

Find the values of a and b. [5]

Answer:

Question	Answer	Marks	Guidance
	Substitute $x = -2$, equate result to zero and obtain a correct equation, e.g. $-8a + 20 + 8 + b = 0$	**B1**	
	Substitute $x = -1$ and equate result to 2	**M1**	
	Obtain a correct equation, e.g. $-a + 5 + 4 + b = 2$	**A1**	
	Solve for a or for b	**M1**	
	Obtain $a = 3$ and $b = -4$	**A1**	
		5	

12. 9709_s21_qp_31 Q: 1

Solve the inequality $2|3x - 1| < |x + 1|$. [4]

Answer:

Question	Answer	Marks	Guidance
	State or imply non-modular inequality $2^2(3x-1)^2 < (x+1)^2$, or corresponding quadratic equation, or pair of linear equations	**B1**	
	Form and solve a 3-term quadratic, or solve two linear equations for x	**M1**	e.g. $35x^2 - 26x + 3 = 0$
	Obtain critical values $x = \frac{3}{5}$ and $x = \frac{1}{7}$	**A1**	Allow 0.143 or better
	State final answer $\frac{1}{7} < x < \frac{3}{5}$	**A1**	Exact values required. Accept $x > \frac{1}{7}$ **and** $x < \frac{3}{5}$ Do not condone $\leqslant$ for $<$ in the final answer. Fractions need not be in lowest terms.
	Alternative method for Question 1		
	Obtain critical value $x = \frac{3}{5}$ from a graphical method, or by solving a linear equation or linear inequality	**B1**	
	Obtain critical value $x = \frac{1}{7}$ similarly	**B2**	Allow 0.143 or better
	State final answer $\frac{1}{7} < x < \frac{3}{5}$	**B1**	OE. Exact values required. Accept $x > \frac{1}{7}$ **and** $x < \frac{3}{5}$ Do not condone $\leqslant$ for $<$ in the final answer. Fractions need not be in lowest terms.
		4	

13. 9709_s21_qp_32 Q: 1

Solve the inequality $|2x - 1| < 3|x + 1|$. [4]

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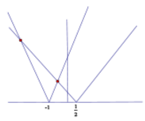
Answer:

Question	Answer	Marks	Guidance
	State or imply non-modular inequality $(2x-1)^2 < 3^2(x+1)^2$, or corresponding quadratic equation	**B1**	e.g. $5x^2+22x+8=0$ Allow recovery from 'invisible brackets' on RHS
	Form and solve a 3-term quadratic in x	**M1**	
	Obtain critical values $x = -4$ and $x = -\frac{2}{5}$	**A1**	
	State final answer $x < -4,\ x > -\frac{2}{5}$	**A1**	Do not condone $\leqslant$ for $<$, or $\geqslant$ for $>$ in the final answer. Allow 'or' but not 'and'. $-\frac{2}{5} < x < -4$ scores A0. Accept equivalent forms using brackets e.g. $x \in (-\infty,-4) \cup (-0.4,\infty)$
	Alternative method for Question 1		
	Obtain critical value $x = -4$ from a graphical method, or by solving a linear equation or linear inequality	**B1**	
	Obtain critical value $x = -\frac{2}{5}$ similarly	**B2**	
	State final answer $x < -4,\ x > -\frac{2}{5}$	**B1**	Do not condone $\leqslant$ for $<$, or $\geqslant$ for $>$ in the final answer. Allow 'or' but not 'and'. $-\frac{2}{5} < x < -4$ scores A0. Accept equivalent forms using brackets e.g. $x \in (-\infty,-4) \cup (-0.4,\infty)$
		4	

14. 9709_s21_qp_32 Q: 9

Let $f(x) = \dfrac{14 - 3x + 2x^2}{(2 + x)(3 + x^2)}$.

(a) Express $f(x)$ in partial fractions. [5]

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(b) Hence obtain the expansion of $f(x)$ in ascending powers of x, up to and including the term in x^2. [5]

Answer:

Question	Answer	Marks	Guidance
(a)	State or imply the form $\dfrac{A}{2+x}+\dfrac{B+Cx}{3+x^2}$	**B1**	
	Use a correct method for finding a constant	**M1**	SOI
	Obtain one of $A=4$, $B=1$ and $C=-2$	**A1**	
	Obtain a second value	**A1**	
	Obtain the third value	**A1**	ISW
		5	
(b)	Use correct method to find the first two terms of the expansion of $(2+x)^{-1}$, $\left(1+\frac{1}{2}x\right)^{-1}$, $(3+x^2)^{-1}$ or $\left(1+\frac{1}{3}x^2\right)^{-1}$	**M1**	Allow unsimplified but not if still including ${}^nC_r$
	Obtain correct unsimplified expansions up to the term in x^2 of each partial fraction	**A1 FT** **A1 FT**	$2\left(1-\frac{1}{2}x+\left(\frac{1}{2}x\right)^2....\right)$ $+\frac{1}{3}(1-2x)\left(1-\frac{1}{3}x^2....\right)$ The FT is on *their* A, B and C
	Multiply out, up to the terms in x^2, by $B + Cx$, where $BC \neq 0$	**M1**	Allow with B and C as implied in part **(b)**
	Obtain final answer $\frac{7}{3}-\frac{5}{3}x+\frac{7}{18}x^2$	**A1**	Or equivalent in form $p+qx+rx^2$. A0 if they multiply through by 18.
		5	

15. 9709_s21_qp_33 Q: 1

Expand $(1 + 3x)^{\frac{2}{3}}$ in ascending powers of x, up to and including the term in x^3, simplifying the coefficients. [4]

Answer:

Question	Answer	Marks	Guidance
	State correct first two terms $1 + 2x$	**B1**	
	State a correct unsimplified version of the x^2 or x^3 term	**M1**	Symbolic binomial coefficients are not sufficient for the M mark.
	Obtain the next term $-x^2$	**A1**	
	Obtain the final term $\frac{4}{3}x^3$	**A1**	
		4	

16. 9709_w21_qp_31 Q: 6

When $(a + bx)\sqrt{1 + 4x}$, where a and b are constants, is expanded in ascending powers of x, the coefficients of x and x^2 are 3 and -6 respectively.

Find the values of a and b. [6]

Answer:

Question	Answer	Marks	Guidance
	State or imply $1 + 2x$ as first terms of the expansion of $\sqrt{1+4x}$	**B1**	Allow for correct unsimplified expression.
	State or imply $-2x^2$ as third term of the expansion of $\sqrt{1+4x}$	**B1**	Allow for correct unsimplified expression.
	Form an expression for the coefficient of x or coefficient of x^2 in the expansion of $(a+bx)\sqrt{1+4x}$ and equate to given coefficient	**M1**	All relevant terms considered.
	Obtain $2a + b = 3$, or equivalent	**A1**	One correct equation.
	Obtain $-2a + 2b = -6$ or equivalent	**A1**	Second correct equation.
	Obtain answer $a = 2$ and $b = -1$	**A1**	
		6	

17. 9709_w21_qp_32 Q: 2

Solve the inequality $|3x - a| > 2|x + 2a|$, where a is a positive constant. [4]

Answer:

Question	Answer	Marks	Guidance
	State or imply non-modular inequality $(3x-a)^2 > 2^2(x+2a)^2$, or corresponding quadratic equation, or pair of linear equations or linear inequalities	**B1**	Need 2^2 seen or implied.
	Make reasonable attempt to solve a 3-term quadratic, or solve two linear equations for x in terms of a	**M1**	$\left(5x^2-22ax-15a^2=0\right)$
	Obtain critical values $x = 5a$ and $x = -\frac{3}{5}a$ and no others	**A1**	OE Accept incorrect inequalities with correct critical values. Must state 2 values i.e. $\frac{a\pm b}{c}$ is not sufficient.
	State final answer $x > 5a$, $x < -\frac{3}{5}a$	**A1**	Do not condone $\geqslant$ for $>$ or $\leqslant$ for $<$ in the final answer. $5a < x < -\frac{3}{5}a$ is **A0**, 'and' is **A0**.
	Alternative method for Question 2		
	Obtain critical value $x = 5a$ from a graphical method, or by solving a linear equation or linear inequality	**B1**	
	Obtain critical value $x = -\frac{3}{5}a$ similarly	**B2**	Maximum 2 marks if more than 2 critical values.
	State final answer $x > 5a$, $x < -\frac{3}{5}a$	**B1**	Do not condone $\geq$ for $>$ or $\leq$ for $<$ in the final answer. $5a < x < -\frac{3}{5}a$ is **B0**, 'and' is **B0**.
		4	

18. 9709_w21_qp_32 Q: 4

Express $\dfrac{4x^2 - 13x + 13}{(2x-1)(x-3)}$ in partial fractions. [5]

Answer:

Question	Answer	Marks	Guidance
	State or imply the form $A + \dfrac{B}{2x-1} + \dfrac{C}{x-3}$	**B1**	$\dfrac{Dx+E}{2x-1}+\dfrac{F}{x-3}$ and $\dfrac{P}{2x-1}+\dfrac{Qx+R}{x-3}$ are also valid.
	Use a correct method for finding a constant	**M1**	
	Obtain one of $A = 2$, $B = -3$ and $C = 2$	**A1**	Allow maximum **M1A1** for one or more 'correct' values after **B0**.
	Obtain a second value	**A1**	
	Obtain the third value	**A1**	
	Alternative method for Question 4		
	Divide numerator by denominator	**M1**	
	Obtain $2\left[+\frac{Px+Q}{(2x-1)(x-3)}\right]$	**A1**	$\left[2+\frac{x+7}{(2x-1)(x-3)}\right]$
	State or imply the form $\dfrac{D}{2x-1} + \dfrac{E}{x-3}$	**B1**	
	Obtain one of $D = -3$ and $E = 2$	**A1**	
	Obtain a second value	**A1**	
		5	

19. 9709_w21_qp_33 Q: 1

Find the quotient and remainder when $2x^4 + 1$ is divided by $x^2 - x + 2$. [3]

Answer:

Question	Answer	Marks	Guidance
	Commence division and reach partial quotient of the form $2x^2 + kx$	M1	
	Obtain quotient $2x^2 + 2x - 2$	A1	
	Obtain remainder $-6x + 5$	A1	
		3	

20. 9709_w21_qp_33 Q: 2

(a) Sketch the graph of $y = |2x - 3|$. [1]

(b) Solve the inequality $|2x - 3| < 3x + 2$. [3]

Answer:

Question	Answer	Marks	Guidance
(a)	Show a recognizable sketch graph of $y=\lvert 2x-3\rvert$	**B1**	
		1	

Question	Answer	Marks	Guidance
(b)	Find x-coordinate of intersection with $y = 3x + 2$	**M1**	
	Obtain $x=\frac{1}{5}$	**A1**	
	State final answer $x>\frac{1}{5}$ only	**A1**	
	Alternative method for Question 2(b)		
	Solve the linear inequality $3-2x<3x+2$, or corresponding equation	**M1**	
	Obtain critical value $x=\frac{1}{5}$	**A1**	
	State final answer $x>\frac{1}{5}$ only	**A1**	
	Alternative method for Question 2(b)		
	Solve the quadratic inequality $(2x-3)^2<(3x+2)^2$, or corresponding equation	**M1**	
	Obtain critical value $x=\frac{1}{5}$	**A1**	
	State final answer $x>\frac{1}{5}$ only	**A1**	
		3	

21. 9709_s20_qp_31 Q: 2

(a) Expand $(2 - 3x)^{-2}$ in ascending powers of x, up to and including the term in x^2, simplifying the coefficients. [4]

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(b) State the set of values of x for which the expansion is valid. [1]

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Answer:

(a)	State a correct unsimplified version of the x or x^2 term of the expansion of $(2-3x)^{-2}$ or $\left(1-\frac{3}{2}x\right)^{-2}$	**M1**
	State correct first term $\frac{1}{4}$	**B1**
	Obtain the next two terms $\frac{3}{4}x+\frac{27}{16}x^2$	**A1 + A1**
		4
(b)	State answer $\lvert x\rvert<\frac{2}{3}$, or equivalent	**B1**
		1

22. 9709_s20_qp_32 Q: 1

Find the quotient and remainder when $6x^4 + x^3 - x^2 + 5x - 6$ is divided by $2x^2 - x + 1$. [3]

Answer:

	Commence division and reach partial quotient $3x^2 + kx$	**M1**
	Obtain quotient $3x^2 + 2x - 1$	**A1**
	Obtain remainder $2x - 5$	**A1**
		4

23. 9709_s20_qp_33 Q: 1

Solve the inequality $|2x-1| > 3|x+2|$. [4]

Answer:

	State or imply non-modular inequality $(2x-1)^2 > 3^2(x+2)^2$, or corresponding quadratic equation, or pair of linear equations	**B1**
	Make reasonable attempt at solving a 3-term quadratic, or solve two linear equations for x	**M1**
	Obtain critical values $x=-7$ and $x=-1$	**A1**
	State final answer $-7 < x < -1$	**A1**
	Alternative method for question 1	
	Obtain critical value $x=-1$ from a graphical method, or by solving a linear equation or linear inequality	**B1**
	Obtain critical value $x=-7$ similarly	**B2**
	State final answer $-7 < x < -1$ [Do not condone $\leqslant$ for $<$ in the final answer.]	**B1**
		4

24. 9709_w20_qp_31 Q: 1

Solve the inequality $2 - 5x > 2|x - 3|$. [4]

Answer:

	Answer	Mark	Partial Marks
	Make a recognisable sketch graph of $y = 2\lvert x-3\rvert$ and the line $y = 2 - 5x$	**B1**	Need to see correct V at $x = 3$, roughly symmetrical, $x = 3$ stated, domain at least $(-2, 5)$.
	Find x-coordinate of intersection with $y = 2 - 5x$	**M1**	Find point of intersection with $y = 2\lvert x-3\rvert$ or solve $2 - 5x$ with $2(x-3)$ or $-2(x-3)$
	Obtain $x = -\frac{4}{3}$	**A1**	
	State final answer $x < -\frac{4}{3}$	**A1**	Do not accept $x < -1.33$ [Do not condone $\leqslant$ for $<$ in the final answer.]
	Alternative method for question 1		
	State or imply non-modular inequality/equality $(2-5x)^2 >, \geqslant, =, 2^2(x-3)^2$, or corresponding quadratic equation, or pair of linear equations $(2-5x) >, \geqslant, =, \pm 2(x-3)$	**B1**	Two correct linear equations only
	Make reasonable attempt at solving a 3-term quadratic, or solve one linear equation, or linear inequality for x	**M1**	$21x^2 + 4x - 32 = (3x+4)(7x-8) = 0$ $2 - 5x$ or $-(2-5x)$ with $2(x-3)$ or $-2(x-3)$
	Obtain critical value $x = -\frac{4}{3}$	**A1**	
	State final answer $x < -\frac{4}{3}$	**A1**	Do not accept $x < -1.33$ [Do not condone $\leqslant$ for $<$ in the final answer.]
		4	

25. 9709_w20_qp_31 Q: 9

Let $\mathrm{f}(x) = \dfrac{8 + 5x + 12x^2}{(1 - x)(2 + 3x)^2}$.

(a) Express $\mathrm{f}(x)$ in partial fractions. [5]

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(b) Hence obtain the expansion of $f(x)$ in ascending powers of x, up to and including the term in x^2. [5]

Answer:

	Answer	Mark	Partial Marks
(a)	State or imply the form $\frac{A}{1-x}+\frac{B}{2+3x}+\frac{C}{(2+3x)^2}$	**B1**	
	Use a correct method for finding a coefficient	**M1**	
	Obtain one of $A = 1$, $B = -1$, $C = 6$	**A1**	
	Obtain a second value	**A1**	
	Obtain the third value	**A1**	In the form $\frac{A}{1-x}+\frac{Dx+E}{(2+3x)^2}$, where $A = 1$, $D = -3$ and $E = 4$ can score B1 M1 A1 A1 A1 as above.
		5	
(b)	Use a correct method to find the first two terms of the expansion of $(1-x)^{-1}$, $(2+3x)^{-1}$, $\left(1+\frac{3}{2}x\right)^{-1}$, $(2+3x)^{-2}$ or $\left(1+\frac{3}{2}x\right)^{-2}$	**M1**	Symbolic coefficients are not sufficient for the M1 $A\left[\frac{1+(-1)(-x)+(-1)(-2)(-x)^2}{2\ldots}\right]$ $A = 1$ $\frac{B}{2}\left[\frac{1+(-1)\left(\frac{3x}{2}\right)+(-1)(-2)\left(\frac{3x}{2}\right)^2}{2\ldots}\right]$ $B = 1$ $\frac{C}{4}\left[\frac{1+(-2)\left(\frac{3x}{2}\right)+(-2)(-3)\left(\frac{3x}{2}\right)^2}{2\ldots}\right]$ $C = 6$
	Obtain correct un-simplified expansions up to the term in of each partial fraction	**A1 FT** + **A1 FT** + **A1 FT**	$(1+x+x^2)+\left(-\frac{1}{2}+\left(\frac{3}{4}\right)x-\left(\frac{9}{8}\right)x^2\right)$ $+\left(\frac{6}{4}-\left(\frac{18}{4}\right)x+\left(\frac{81}{8}\right)x^2\right)$ [The FT is on A, B, C] $\left(1-\frac{1}{2}+\frac{6}{4}\right)+\left(1+\frac{3}{4}-\frac{18}{4}\right)x+\left(1-\frac{9}{8}+\frac{81}{8}\right)x^2$
	Obtain final answer $2-\frac{11}{4}x+10x^2$, or equivalent	**A1**	Allow unsimplified fractions $\frac{(Dx+E)}{4}\left[\frac{1+(-2)\left(\frac{3x}{2}\right)+(-2)(-3)\left(\frac{3x}{2}\right)^2}{2\ldots}\right]$ $D = -3$, $E = 4$ The FT is on A, D, E.
		5	

26. 9709_w20_qp_32 Q: 2

(a) Expand $\sqrt[3]{1+6x}$ in ascending powers of x, up to and including the term in x^3, simplifying the coefficients. [4]

(b) State the set of values of x for which the expansion is valid. [1]

Answer:

	Answer	Mark	Partial Marks
(a)	State a correct unsimplified version of the x or x^2 or x^3 term	**M1**	For the given expression
	State correct first two terms $1 + 2x$	**A1**	
	Obtain the next two terms $-4x^2 + \frac{40}{3}x^3$	**A1 + A1**	One mark for each correct term. ISW Accept $13\frac{1}{3}$ The question asks for simplified coefficients, so candidates should cancel fractions.
		4	
(b)	State answer $\lvert x \rvert < \frac{1}{6}$	**B1**	OE. Strict inequality
		1	

27. 9709_m19_qp_32 Q: 8

Let $f(x) = \dfrac{12 + 12x - 4x^2}{(2 + x)(3 - 2x)}$.

(i) Express $f(x)$ in partial fractions. [5]

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(ii) Hence obtain the expansion of $\mathrm{f}(x)$ in ascending powers of x, up to and including the term in x^2. [5]

Answer:

	Answer	Mark	Partial Marks
(i)	State or imply the form $A+\dfrac{B}{2+x}+\dfrac{C}{3-2x}$	B1	
	Use a correct method for finding a constant	M1	
	Obtain one of $A = 2$, $B = -4$ and $C = 6$	A1	
	Obtain a second value	A1	
	Obtain the third value	A1	
		5	
(ii)	Use correct method to find the first two terms of the expansion of $(2+x)^{-1}$ or $(3-2x)^{-1}$, or equivalent	M1	
	Obtain correct unsimplified expansions up to the term in x^2 of each partial fraction	A1ft +A1ft	The ft is on B and C
	Add the value of A to the sum of the expansions	M1	
	Obtain final answer $2+\dfrac{7}{3}x+\dfrac{7}{18}x^2$	A1	
		5	

28. 9709_s19_qp_31 Q: 8

Let $\mathrm{f}(x) = \dfrac{16 - 17x}{(2 + x)(3 - x)^2}$.

(i) Express $\mathrm{f}(x)$ in partial fractions. [5]

(ii) Hence obtain the expansion of $f(x)$ in ascending powers of x, up to and including the term in x^2. [5]

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Answer:

	Answer	Mark	Partial Marks
(i)	State or imply the form $\frac{A}{2+x}+\frac{B}{3-x}+\frac{C}{(3-x)^2}$	B1	
	Use a correct method to obtain a constant	M1	
	Obtain one of $A=2$, $B=2$, $C=-7$	A1	
	Obtain a second value	A1	
	Obtain the third value	A1	[Mark the form $\frac{A}{2+x}+\frac{Dx+E}{(3-x)^2}$, where $A=2$, $D=-2$ and $E=-1$, B1M1A1A1A1.]
		5	
(ii)	Use a correct method to find the first two terms of the expansion of $(2+x)^{-1}$, $(3-x)^{-1}$ or $(3-x)^{-2}$, or equivalent, e.g. $\left(1+\frac{1}{2}x\right)^{-1}$	M1	
	Obtain correct unsimplified expansions up to the term in x^2 of each partial fraction	A1 A1 A1	**FT** on A, B and C $1-\frac{x}{2}+\frac{x^2}{4}$ $\frac{2}{3}\left(1+\frac{x}{3}+\frac{x^2}{9}\right)$ $-\frac{7}{9}\left(1+\frac{2x}{3}+\frac{3x^2}{9}\right)$
	Obtain final answer $\frac{8}{9}-\frac{43}{54}x+\frac{7}{108}x^2$	A1	
			For the A, D, E form of fractions give M1A1**ft**A1**ft** for the expanded partial fractions, then, if $D\neq 0$, M1 for multiplying out fully, and A1 for the final answer.
		5	

29. 9709_s19_qp_32 Q: 1

Find the coefficient of x^3 in the expansion of $(3 - x)(1 + 3x)^{\frac{1}{3}}$ in ascending powers of x. [4]

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Answer:

	Answer	Mark	Partial Marks
	State unsimplified term in x^2, or its coefficient in the expansion of $(1+3x)^{\frac{1}{3}}$ $\left(\dfrac{\frac{1}{3}\times\frac{-2}{3}}{2}(3x)^2\right)$	B1	Symbolic binomial coefficients are not sufficient for the B marks
	State unsimplified term in x^3, or its coefficient in the expansion of $(1+3x)^{\frac{1}{3}}$ $\left(\dfrac{\frac{1}{3}\times\frac{-2}{3}\times\frac{-5}{3}}{6}(3x)^3\right)$	B1	
	Multiply by $(3-x)$ to give 2 terms in x^3, or their coefficients	M1	$\left(3\times\frac{10}{6}+1\right)$ Ignore errors in terms other than x^3 $3\times x^3\text{coeff} - x^2\text{coeff}$ and no other term in x^3
	Obtain answer 6	A1	Not $6x^3$
		4	

30. 9709_s19_qp_33 Q: 9

Let $\mathrm{f}(x) = \dfrac{2x(5-x)}{(3+x)(1-x)^2}$.

(i) Express $\mathrm{f}(x)$ in partial fractions. [5]

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