Topical Past Paper Questions Worksheets

IGCSE Additional Mathematics (0606) Paper 2

Exam Series: May/June 2012 - Oct/Nov 2023

Format Type A:
Answers to all questions are provided as an appendix

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Introduction

Each Topical Past Paper Questions Compilation contains a comprehensive collection of hundreds of questions and corresponding answer schemes, presented in worksheet format. The questions are carefully arranged according to their respective chapters and topics, which align with the latest IGCSE or AS/A Level subject content. Here are the key features of these resources:

1. The workbook covers a wide range of topics, which are organized according to the latest syllabus content for Cambridge IGCSE or AS/A Level exams.
2. Each topic includes numerous questions, allowing students to practice and reinforce their understanding of key concepts and skills.
3. The questions are accompanied by detailed answer schemes, which provide clear explanations and guidance for students to improve their performance.
4. The workbook's format is user-friendly, with worksheets that are easy to read and navigate.
5. This workbook is an ideal resource for students who want to familiarize themselves with the types of questions that may appear in their exams and to develop their problem-solving and analytical skills.

Overall, Topical Past Paper Questions Workbooks are a valuable tool for students preparing for IGCSE or AS/A level exams, providing them with the opportunity to practice and refine their knowledge and skills in a structured and comprehensive manner. To provide a clearer description of this book's specifications, here are some key details:

- Title: Cambridge IGCSE Additional Mathematics (0606) Paper 2 Topical Past Papers
- Subtitle: Exam Practice Worksheets With Answer Scheme
- Examination board: Cambridge Assessment International Education (CAIE)
- Subject code: 0606
- Years covered: May/June 2012 - Oct/Nov 2023
- Paper: 2
- Number of pages: 1443
- Number of questions: 789

Contents

Chapter 1

Functions

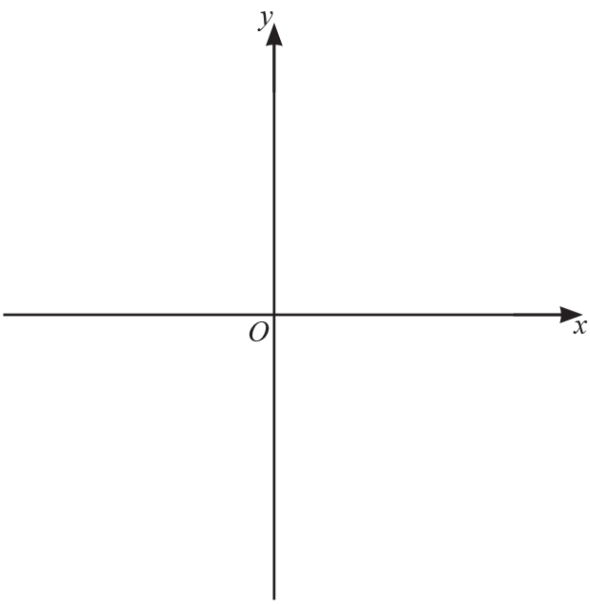
1. 0606_m23_qp_22 Q: 8

The function f is defined for $x \geqslant 0$ by $f(x) = 5 - 2e^{-x}$.

(a) **(i)** Find the domain of f^{-1}. [2]

(ii) Solve $ff^{-1}(x) = \sqrt{5x-4}$. [3]

(iii) On the axes, sketch the graph of $y = f(x)$ and hence sketch the graph of $y = f^{-1}(x)$. Show clearly the positions of any points where your graphs meet the coordinate axes and the positions of any asymptotes. [4]

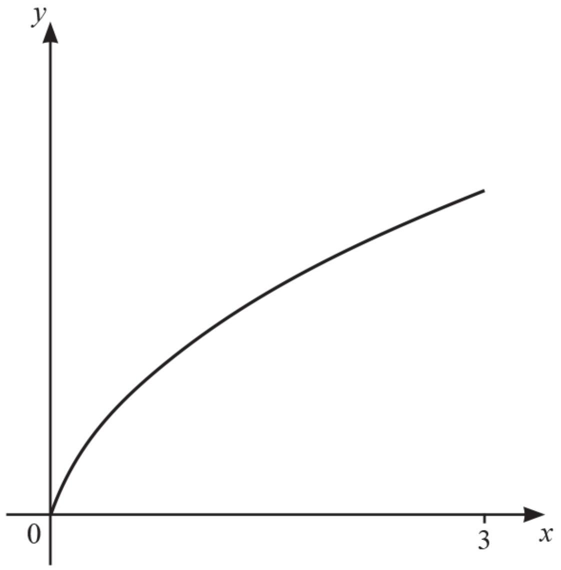


(b) The function g is defined for $0 \leqslant x \leqslant 0.2$ by $g(x) = \dfrac{3}{1-x}$.

Find and simplify an expression for $f^{-1}g(x)$. [4]

2. 0606_s23_qp_21 Q: 8

(a)



The diagram shows the graph of $y = \mathrm{f}(x)$ where f is defined by $\mathrm{f}(x) = \dfrac{3x}{\sqrt{5x+1}}$ for $0 \leqslant x \leqslant 3$.

(i) Given that f is a one-one function, find the domain and range of f^{-1}. [3]

(ii) Solve the equation $\mathrm{f}(x) = x$. [2]

(iii) On the diagram above, sketch the graph of $y = \mathrm{f}^{-1}(x)$. [2]

(b) The functions g and h are defined by

$$g(x) = \sqrt[3]{8x^3 + 3} \quad \text{for} \quad x \geqslant 1,$$

$$h(x) = e^{4x} \quad \text{for} \quad x \geqslant k.$$

(i) Find an expression for $g^{-1}(x)$. [2]

(ii) State the least value of the constant k such that $gh(x)$ can be formed. [1]

(iii) Find and simplify an expression for $gh(x)$. [1]

3. 0606_s23_qp_23 Q: 8

(a) The functions f and g are defined by

$$f(x) = \sec x \qquad \text{for } \frac{\pi}{2} < x < \frac{3\pi}{2}$$

$$g(x) = 3(x^2 - 1) \qquad \text{for all real } x.$$

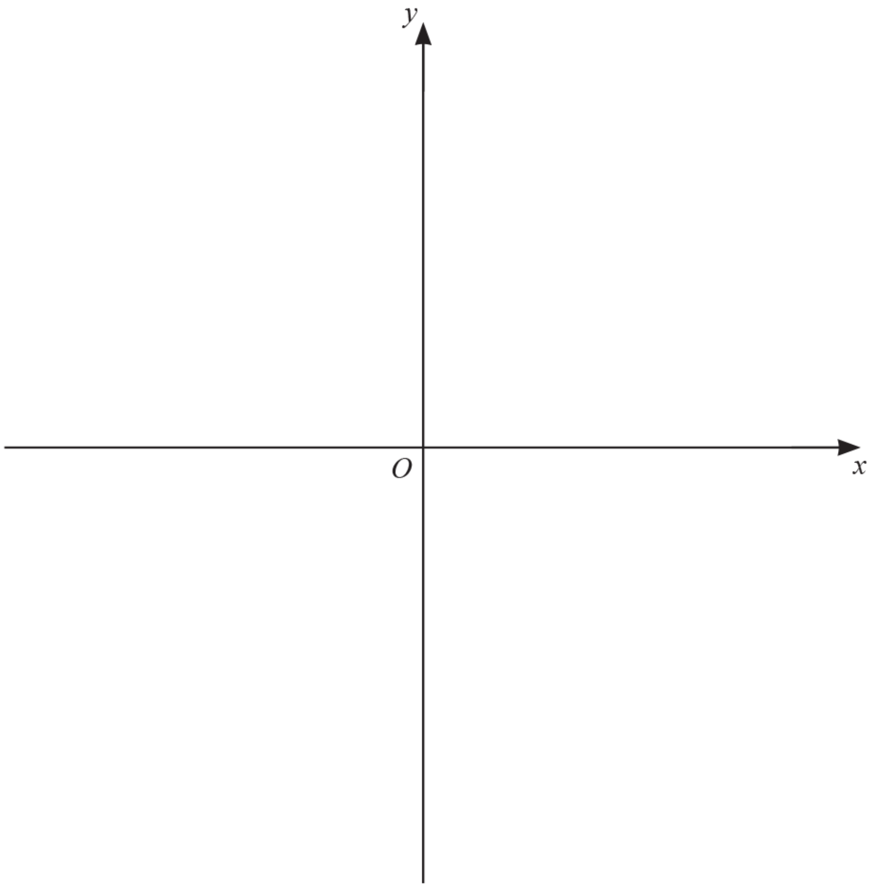
(i) Find the range of f. [1]

(ii) Solve the equation $f^{-1}(x) = \frac{2\pi}{3}$. [3]

(iii) Given that gf exists, state the domain of gf. [1]

(iv) Solve the equation $gf(x) = 1$. [5]

(b) The function h is defined by $\mathrm{h}(x)=\ln(4-x)$ for $x<4$. Sketch the graph of $y=\mathrm{h}(x)$ and hence sketch the graph of $y=\mathrm{h}^{-1}(x)$. Show the position of any asymptotes and any points of intersection with the coordinate axes. [4]



4. 0606_w23_qp_21 Q: 9

The functions f and g are defined as follows, for all real values of x.

$$f(x) = 2x^2 - 1$$
$$g(x) = e^x + 1$$

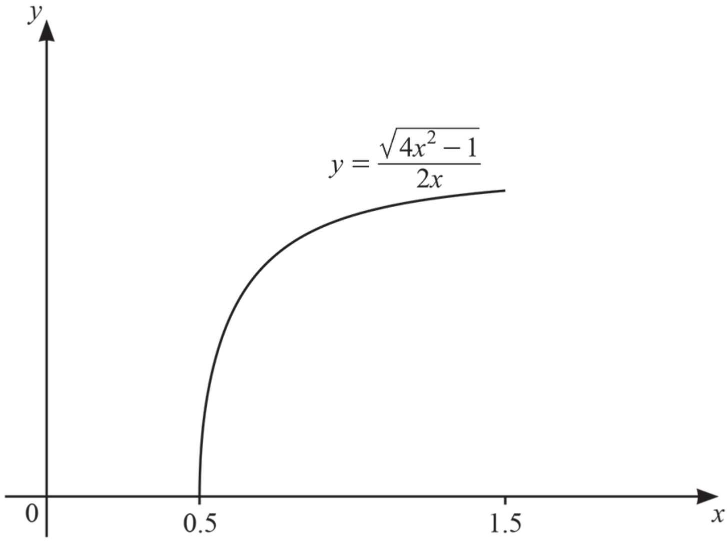
(a) Solve the equation $fg(x) = 8$. [3]

(b) For each of the functions f and g, either explain why the inverse function does not exist or find the inverse function, stating its domain. [4]

5. 0606_m21_qp_22 Q: 10

The function f is defined by $f(x) = \frac{\sqrt{4x^2 - 1}}{2x}$ for $0.5 \leqslant x \leqslant 1.5$.

The diagram shows a sketch of $y = f(x)$.



(a) **(i)** It is given that f^{-1} exists. Find the domain and range of f^{-1}. [3]

(ii) Find an expression for $f^{-1}(x)$. [3]

(b) The function g is defined by $g(x) = e^{x^2}$ for all real x. Show that $gf(x) = e^{\left(1-\frac{a}{bx^2}\right)}$, where a and b are integers. [2]

6. 0606_s21_qp_22 Q: 13

The functions f and g are defined, for $x > 0$, by

$$f(x) = \frac{2x^2 - 1}{3x},$$

$$g(x) = \frac{1}{x}.$$

(a) Find and simplify an expression for $fg(x)$. [2]

(b) **(i)** Given that f^{-1} exists, write down the range of f^{-1}. [1]

(ii) Show that $f^{-1}(x) = \dfrac{px + \sqrt{qx^2 + r}}{4}$, where p, q and r are integers. [4]

7. 0606_s21_qp_23 Q: 9

(a) The function f is defined, for all real x, by $\mathrm{f}(x) = 13 - 4x - 2x^2$.

(i) Write $\mathrm{f}(x)$ in the form $a + b(x + c)^2$, where a, b and c are constants. [3]

(ii) Hence write down the range of f. [1]

(b) The function g is defined, for $x \geqslant 1$, by $\mathrm{g}(x) = \sqrt{x^2 + 2x - 1}$.

(i) Given that $\mathrm{g}^{-1}(x)$ exists, write down the domain and range of g^{-1}. [2]

(ii) Show that $\mathrm{g}^{-1}(x) = -1 + \sqrt{px^2 + q}$, where p and q are integers. [4]

8. 0606_w21_qp_21 Q: 9

The following functions are defined for $x > 1$.

$$f(x) = \frac{x+3}{x-1} \qquad g(x) = 1 + x^2$$

(a) Find $fg(x)$. [2]

(b) Find $g^{-1}(x)$. [2]

(c) DO NOT USE A CALCULATOR IN THIS PART OF THE QUESTION.

Solve the equation $f(x) = g(x)$. [5]

9. 0606_s19_qp_22 Q: 12

(a) The functions f and g are defined by

$$f(x) = 5x - 2 \quad \text{for } x > 1,$$
$$g(x) = 4x^2 - 9 \quad \text{for } x > 0.$$

(i) State the range of g. [1]

(ii) Find the domain of gf. [1]

(iii) Showing all your working, find the exact solutions of $gf(x) = 4$. [3]

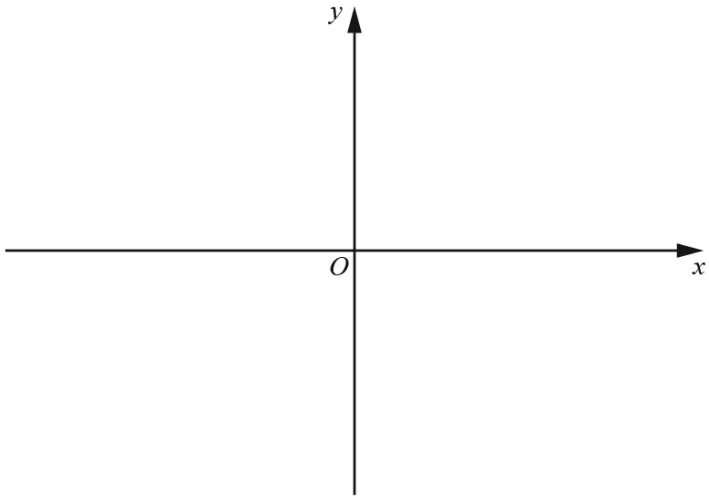
(b) The function h is defined by $\mathrm{h}(x)=\sqrt{x^2-1}$ for $x \leqslant -1$.

(i) State the geometrical relationship between the graphs of $y=\mathrm{h}(x)$ and $y=\mathrm{h}^{-1}(x)$. [1]

(ii) Find an expression for $\mathrm{h}^{-1}(x)$. [3]

10. 0606_s18_qp_22 Q: 10

(a) **(i)** On the axes below, sketch the graph of $y = |(x+3)(x-5)|$ showing the coordinates of the points where the curve meets the x-axis. [2]



(ii) Write down a suitable domain for the function $f(x) = |(x+3)(x-5)|$ such that f has an inverse. [1]

(b) The functions g and h are defined by

$$g(x) = 3x - 1 \qquad \text{for } x > 1,$$
$$h(x) = \frac{4}{x} \qquad \text{for } x \neq 0.$$

(i) Find $hg(x)$. [1]

(ii) Find $(hg)^{-1}(x)$. [2]

(c) Given that $p(a) = b$ and that the function p has an inverse, write down $p^{-1}(b)$. [1]

11. 0606_s18_qp_23 Q: 5

The function f is defined by $\mathrm{f}(x) = \dfrac{1}{2x-5}$ for $x > 2.5$.

(i) Find an expression for $\mathrm{f}^{-1}(x)$. [2]

(ii) State the domain of $\mathrm{f}^{-1}(x)$. [1]

(iii) Find an expression for $\mathrm{f}^{2}(x)$, giving your answer in the form $\dfrac{ax+b}{cx+d}$, where a, b, c and d are integers to be found. [3]

12. 0606_w18_qp_22 Q: 11

The functions f and g are defined for real values of $x \geqslant 1$ by

$$f(x) = 4x - 3,$$

$$g(x) = \frac{2x+1}{3x-1}.$$

(i) Find $gf(x)$. [2]

(ii) Find $g^{-1}(x)$. [3]

(iii) Solve $fg(x) = x - 1$. [4]

13. 0606_m17_qp_22 Q: 11

The functions f and g are defined by

$$f(x) = \frac{x^2 - 2}{x} \quad \text{for } x \geqslant 2,$$

$$g(x) = \frac{x^2 - 1}{2} \quad \text{for } x \geqslant 0.$$

(i) State the range of g. [1]

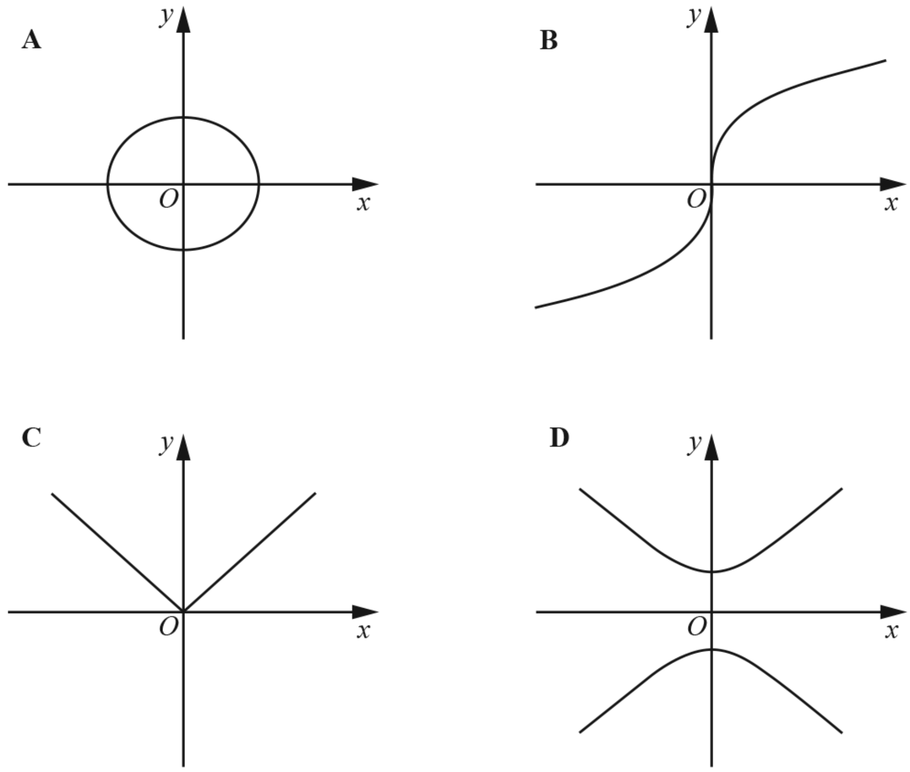
(ii) Explain why fg(1) does not exist. [2]

(iii) Show that $gf(x) = ax^2 + b + \frac{c}{x^2}$, where a, b and c are constants to be found. [3]

(iv) State the domain of gf. [1]

(v) Show that $f^{-1}(x) = \dfrac{x + \sqrt{x^2 + 8}}{2}$. [4]

14. 0606_s17_qp_23 Q: 2



The four graphs above are labelled **A**, **B**, **C** and **D**.

(i) Write down the letter of each graph that represents a function, giving a reason for your choice. [2]

(ii) Write down the letter of each graph that represents a function which has an inverse, giving a reason for your choice. [2]

15. 0606_w17_qp_23 Q: 6

The functions f and g are defined for real values of x by

$$f(x) = (x+2)^2 + 1,$$

$$g(x) = \frac{x-2}{2x-1},\ x \neq \frac{1}{2}.$$

(i) Find $f^2(-3)$. [2]

(ii) Show that $g^{-1}(x) = g(x)$. [3]

(iii) Solve $gf(x) = \frac{8}{19}$. [4]

16. 0606_s14_qp_21 Q: 12

The functions f and g are defined by

$$f(x) = \frac{2x}{x+1} \text{ for } x > 0,$$

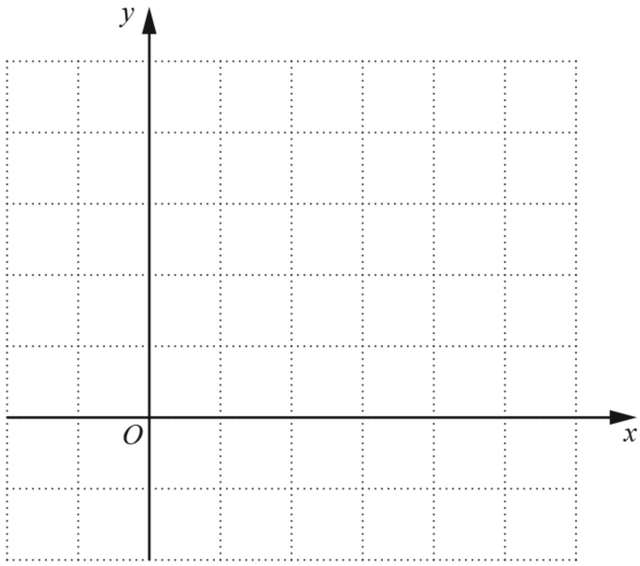
$$g(x) = \sqrt{x+1} \text{ for } x > -1.$$

(i) Find $fg(8)$. [2]

(ii) Find an expression for $f^2(x)$, giving your answer in the form $\frac{ax}{bx+c}$, where a, b and c are integers to be found. [3]

(iii) Find an expression for $g^{-1}(x)$, stating its domain and range. [4]

(iv) On the same axes, sketch the graphs of $y = \mathrm{g}(x)$ and $y = \mathrm{g}^{-1}(x)$, indicating the geometrical relationship between the graphs. [3]



17. 0606_s14_qp_23 Q: 12

The function f is such that $f(x) = 2 + \sqrt{x-3}$ for $4 \leqslant x \leqslant 28$.

(i) Find the range of f. [2]

(ii) Find $f^2(12)$. [2]

(iii) Find an expression for $f^{-1}(x)$. [2]

The function g is defined by $g(x) = \frac{120}{x}$ for $x \geqslant 0$.

(iv) Find the value of x for which $gf(x) = 20$. [3]

18. 0606_w14_qp_21 Q: 4

The functions f and g are defined for real values of x by

$$f(x) = \sqrt{x-1} - 3 \qquad \text{for } x > 1,$$

$$g(x) = \frac{x-2}{2x-3} \qquad \text{for } x > 2.$$

(i) Find gf(37). [2]

(ii) Find an expression for $f^{-1}(x)$. [2]

(iii) Find an expression for $g^{-1}(x)$. [2]

19. 0606_s13_qp_21 Q: 11

A one-one function f is defined by $f(x) = (x-1)^2 - 5$ for $x \geqslant k$.

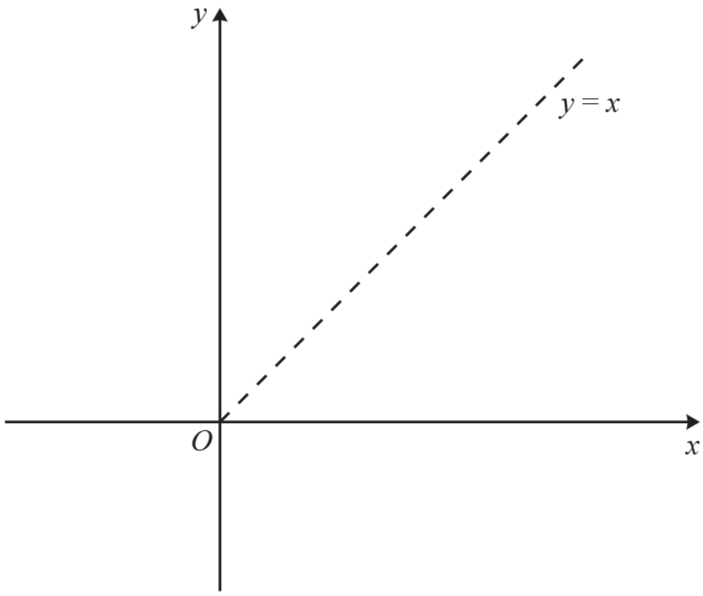
(i) State the least value that k can take. [1]

For this least value of k

(ii) write down the range of f, [1]

(iii) find $f^{-1}(x)$, [2]

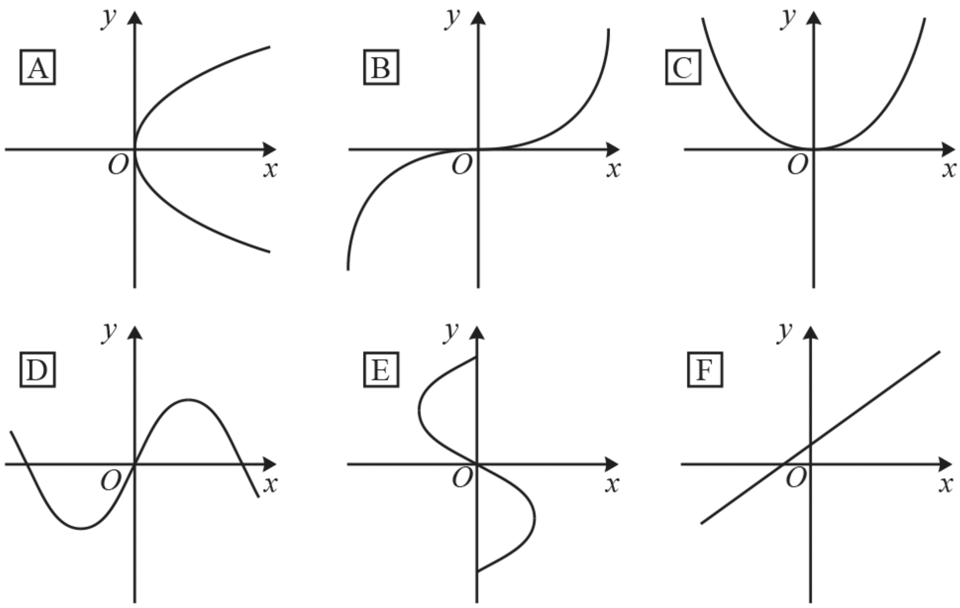
(iv) sketch and label, on the axes below, the graph of $y = f(x)$ and of $y = f^{-1}(x)$, [2]



(v) find the value of x for which $f(x) = f^{-1}(x)$. [2]

20. 0606_s13_qp_22 Q: 3

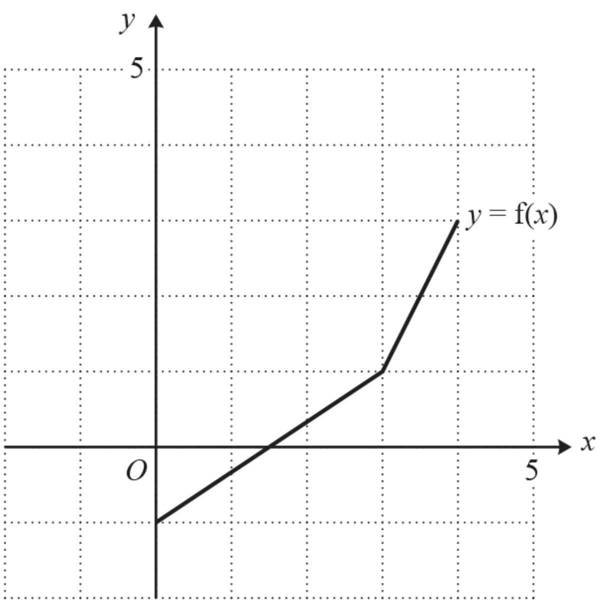
(a)



(i) Write down the letter of each graph which does **not** represent a function. [2]

(ii) Write down the letter of each graph which represents a function that does **not** have an inverse. [2]

(b)



The diagram shows the graph of a function $y = \mathrm{f}(x)$. On the same axes sketch the graph of $y = \mathrm{f}^{-1}(x)$. [2]

Chapter 2

Quadratic functions

21. 0606_s23_qp_22 Q: 1

(a) Solve the inequality $3x^2 - 12x + 16 > 3x + 4$. [3]

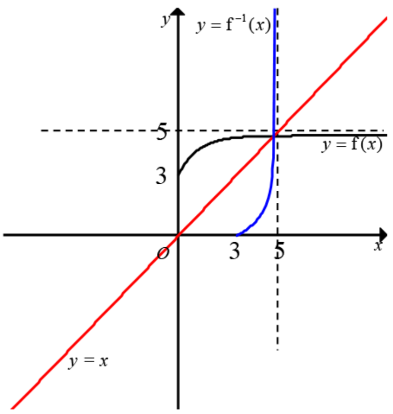
(b) **(i)** Write $3x^2 - 12x + 16$ in the form $a(x+b)^2 + c$ where a, b and c are integers. [3]

(ii) Hence, write down the equation of the tangent to the curve $y = 3x^2 - 12x + 16$ at the minimum point of the curve. [1]

Appendix A

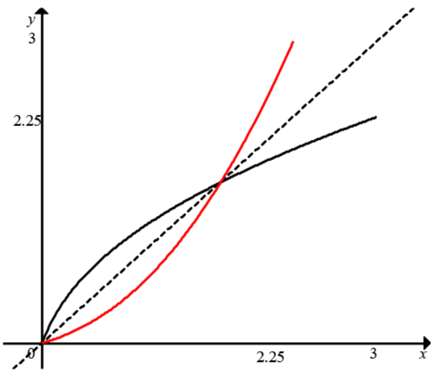
Answers

1. 0606_m23_ms_22 Q: 8

Question	Answer	Marks	Partial Marks
(a)(i)	$3 \leqslant x < 5$	B2	**B1** for $x \geqslant 3$ or for $x < 5$ or for 3 and 5 in an incorrect inequality
(a)(ii)	$x = \sqrt{5x-4}$ and rearrangement to $x^2 - 5x + 4[=0]$	B1	
	Factorises $x^2 - 5x + 4$ or solves *their* $x^2 - 5x + 4 = 0$	M1	
	$x = 4$ only, nfww	A1	
(a)(iii)	Correct pair of graphs. 	4	**B1** for correct shape for f; may not be over correct domain but must have positive y-intercept and appear to tend to an asymptote in the 1st quadrant **B1** for (0, 3) and f in 1st quadrant only; must have attempted correct shape **B1** for asymptote at $y = 5$; must have attempted correct shape **B1** for a correct reflection of *their* f in the line $y = x$ **Maximum of 3 marks if not fully correct**

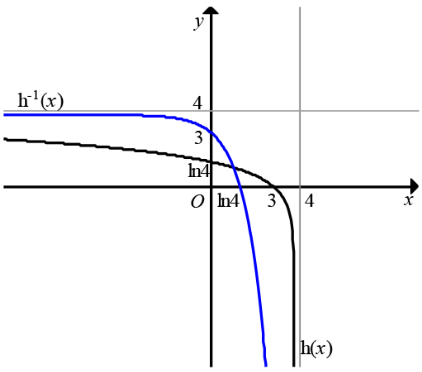
Question	Answer	Marks	Partial Marks
(b)	$f^{-1}(x) = -\ln\frac{5-x}{2}$ or $f^{-1}(x) = \ln\frac{2}{5-x}$ oe	2	**M1** for a complete attempt to find the inverse function with at most one sign or arithmetic error: Putting $y = f(x)$ and changing subject to x and swopping x and y or swopping x and y and changing subject to y
	Correct simplified form e.g. $\left[f^{-1}g(x) =\right] -\ln\frac{2-5x}{2(1-x)}$ or $\left[f^{-1}g(x) =\right]\ln\frac{2-2x}{2-5x}$	2	**M1 FT** for a correct unsimplified form of the function; FT providing of equivalent difficulty

2. 0606_s23_ms_21 Q: 8

Question	Answer	Marks	Partial Marks
(a)(i)	[Domain f^{-1}] $0 \leqslant x \leqslant 2.25$ oe	**B2**	**B1** for either end correct or for 0 and 2.25 in an incorrect inequality
	[Range f^{-1}] $0 \leqslant f^{-1} \leqslant 3$	**B1**	
(a)(ii)	$x = 1.6$ oe or $x = 0$	2	**B1** for each
(a)(iii)		2	**B1** for attempt at correct graph of inverse function drawn over correct domain soi **B1** for correct shape with intersection in approximately correct location
(b)(i)	For a complete method to find the inverse, including changing the subject and swapping the variables	**M1**	
	$\left[g^{-1}(x) =\right]\sqrt[3]{\frac{x^3-3}{8}}$ oe mark final answer	**A1**	
(b)(ii)	$[k =]$ 0	1	
(b)(iii)	$\sqrt[3]{8e^{12x}+3}$ mark final answer	1	

3. 0606_s23_ms_23 Q: 8

Question	Answer	Marks	Partial Marks
(a)(i)	$f \leqslant -1$	**1**	

Question	Answer	Marks	Partial Marks
(a)(ii)	$x = -2$ nfww	3	**M1** for $\left[x = f\left(\frac{2\pi}{3}\right) =\right] \sec\left(\frac{2\pi}{3}\right)$ or $\sec^{-1} x = \frac{2\pi}{3}$ **A1** for $\frac{1}{\cos\left(\frac{2\pi}{3}\right)}$ OR **M1** for a complete attempt to find $f^{-1}(x)$; includes swapping the variables **A1** for $f^{-1}(x) = \cos^{-1}\left(\frac{1}{x}\right)$
(a)(iii)	$\frac{\pi}{2} < x < \frac{3\pi}{2}$	1	
(a)(iv)	$gf(x) = 3(\sec^2 x - 1)$	B1	
	$3\tan^2 x = 1$ or $\frac{1}{\cos^2 x} = \frac{4}{3}$ oe	M1	
	$\tan x = [\pm]\sqrt{\frac{1}{3}}$ oe or $\cos x = [\pm]\sqrt{\frac{3}{4}}$ oe and solves for x, soi	M1	
	$x = \frac{5\pi}{6}, \frac{7\pi}{6}$ and no other solutions	A2	**A1** for one correct solution, condoning extras
(b)	Correct diagram with intercepts indicated and asymptotes shown.  	4	**B1** for correct shape for h; may not be over correct domain but must have positive y-intercept and x-intercept and appear to tend to an asymptote in the 4th quadrant **B1** for 3 and ln4 correctly marked; must have attempted correct shape **B1** for the position of the vertical asymptote indicated; must have attempted correct shape **B1** for h^{-1} the reflection of *their* h in the line $y = x$ **Maximum of 3 marks if not fully correct**

4. 0606_w23_ms_21 Q: 9

Question	Answer	Marks	Partial marks
(a)	$2(e^x + 1)^2 - 1$ [= 8]	**M1**	
	$e^x = -1 + \sqrt{\frac{9}{2}}$ oe	**A1**	
	$x = \ln\left(\frac{3}{\sqrt{2}} - 1\right)$ isw or 0.115 or 0.1145[06…] rot to 4 or more dp	**A1**	
(b)	f is not one-one, hence f^{-1} does not exist oe	**B1**	
	$g^{-1}(x) = \ln(x - 1)$	**2**	**M1** for $x = \ln(y - 1)$ and a swop of variables at some point or $y = \ln(x + 1)$ or $e^x = y - 1$ and $y = \ln x - 1$
	$x > 1$	**B1**	

5. 0606_m21_ms_22 Q: 10

(a)(i)	Range f^{-1}: $0.5 \leqslant f^{-1} \leqslant 1.5$	**B1**	
	Domain f^{-1}: $0 \leqslant x \leqslant \frac{2\sqrt{2}}{3}$ oe	**B2**	**B1** for 0 and $\frac{2\sqrt{2}}{3}$ in an incorrect inequality or for $x \geqslant 0$ or $x \leqslant \frac{2\sqrt{2}}{3}$

(a)(ii)	Correctly collects terms ready to factorise e.g. $4x^2 - 4x^2y^2 = 1$ or $4y^2x^2 - 4y^2 = -1$ or simplifies to subject in one term only e.g. $\frac{1}{4y^2} = 1 - x^2$ or $-\frac{1}{4x^2} = y^2 - 1$ oe	**M1**	
	Correctly factorises and/or rearranges at least as far as: $x^2 = \frac{1}{4-4y^2}$ or $y^2 = \frac{-1}{4x^2-4}$ oe	**M1**	**FT** only if of equivalent difficulty
	$\left[f^{-1}(x) =\right]\sqrt{\frac{1}{4-4x^2}}$ or $\left[y =\right]\sqrt{\frac{-1}{4x^2-4}}$ oe, isw	**A1**	
(b)	Correct order of composition: gf(x) = $e^{\left(\frac{\sqrt{4x^2-1}}{2x}\right)^2}$	**M1**	
	$gf(x) = e^{\left(1-\frac{1}{4x^2}\right)}$ isw	**A1**	

6. 0606_s21_ms_22 Q: 13

Question	Answer	Marks	Partial Marks
(a)	$[\text{fg}(x) =] \dfrac{2\left(\frac{1}{x}\right)^2 - 1}{3\left(\frac{1}{x}\right)}$ oe	**M1**	
	$[\text{fg}(x) =] \dfrac{2 - x^2}{3x}$ or $\dfrac{2}{3x} - \dfrac{x}{3}$	**A1**	mark final answer
(b)(i)	$\text{f}^{-1} > 0$	**B1**	
(b)(ii)	$2x^2 - 3xy - 1 = 0$ or $2y^2 - 3xy - 1 = 0$	**B1**	
	Correctly applies quadratic formula: $[x =] \dfrac{-(-3y) \pm \sqrt{(-3y)^2 - 4(2)(-1)}}{2(2)}$ oe or $[y =] \dfrac{-(-3x) \pm \sqrt{(-3x)^2 - 4(2)(-1)}}{2(2)}$ oe	**M1**	**FT** *their* $2x^2 - 3xy - 1 = 0$ or $2y^2 - 3xy - 1 = 0$ with at most one sign error in the equation
	Justifies the positive square root at some point	**B1**	
	$\text{f}^{-1}(x) = \dfrac{3x + \sqrt{9x^2 + 8}}{4}$ cao	**A1**	must be a function of x

7. 0606_s21_ms_23 Q: 9

(a)(i)	$15 - 2(x+1)^2$ isw	**B3**	**B1** for $(x+1)^2$ **B1** for $a = 15$
(a)(ii)	$f \leqslant 15$	**B1**	**STRICT FT** *their a*
(b)(i)	Domain: $x \geqslant \sqrt{2}$	**B1**	
	Range: $g^{-1} \geqslant 1$	**B1**	
(b)(ii)	$x^2 + 2x + (-1 - y^2) = 0$ or $y^2 + 2y + (-1 - x^2) = 0$	**B1**	
	Correctly applies quadratic formula: $[x =]\dfrac{-2 \pm \sqrt{2^2 - 4(1)(-1-y^2)}}{2}$ or $[y =]\dfrac{-2 \pm \sqrt{2^2 - 4(1)(-1-x^2)}}{2}$	**M1**	**FT** *their* $x^2 + 2x + (-1 - y^2) = 0$ or $y^2 + 2y + (-1 - x^2) = 0$ with at most one sign error in the equation
	Justifies the positive square root at some point	**B1**	
	Correct completion to $g^{-1}(x) = -1 + \sqrt{x^2 + 2}$	**A1**	

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8. 0606_w21_ms_21 Q: 9

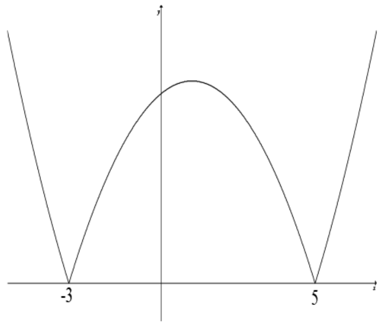
(a)	$[\text{fg}(x) =]\dfrac{x^2 + 4}{x^2}$ oe, final answer	**2**	**B1** for an attempt at the correct order of composition with at most one error
(b)	Complete, correct method to find the inverse	**M1**	
	$\left[g^{-1}(x) =\right]\sqrt{x-1}$ final answer	**A1**	

(c)	$x^3 - x^2 - 4 = 0$	**M1**	condone one sign or arithmetic error
	Shows $x - 2$ is a factor or shows that $x = 2$ is a solution	**M1**	
	Uses $x - 2$ is a factor to find $x^2 + x + 2$	**B2**	**B1** for a quadratic factor with 2 terms correct
	Indicates that $x^2 + x + 2$ has no real roots and states $x = 2$ as the only solution	**A1**	dep on all previous marks awarded

9. 0606_s19_ms_22 Q: 12

(a)(i)	$g > -9$	**B1**	
(a)(ii)	$x > 1$	**B1**	
(a)(iii)	$[\text{gf}(x) =]\ \ 4(5x-2)^2 - 9$	**B1**	
	$100x^2 - 80x - 38 = 0$ or $(5x-2)^2 = \frac{45+9}{4}$	**M1**	
	$[x =]\ \frac{-(-80) \pm \sqrt{(-80)^2 - 4(100)(-38)}}{2(100)}$ leading to $\frac{4+3\sqrt{6}}{10}$ oe only or $\frac{1}{5}\left(2+\sqrt{\frac{54}{4}}\right)$ or better only	**A1**	
(b)(i)	(They are) **reflection**s (of each other) in (the line) $\boldsymbol{y = x}$ oe	**B1**	
(b)(ii)	$x^2 = y^2 + 1$ or $y^2 = x^2 + 1$	**M1**	
	$x = [\pm]\sqrt{y^2+1}$ or $y = [\pm]\sqrt{x^2+1}$	**A1**	
	$-\sqrt{x^2+1}$ nfww	**A1**	

10. 0606_s18_ms_22 Q: 10

(a)(i)		B2	**B1** for correct shape **B1** for roots marked on the graph or seen nearby provided graph drawn and one root is negative and one is positive
(a)(ii)	Any correct domain	B1	
(b)(i)	$\frac{4}{3x-1}$	B1	mark final answer
(b)(ii)	Correct method for finding inverse function e.g. swopping variables **and** changing subject or vice versa; or indicates $(\text{hg})^{-1}(x) = \text{g}^{-1}\text{h}^{-1}(x)$ and finds $\text{g}^{-1}(x) = \frac{x+1}{3}$ and $\text{h}^{-1}(x) = \frac{4}{x}$	M1	**FT** only if *their*hg(x) of the form $\frac{a}{bx+c}$ where a, b and c are integers
	$\left[(\text{hg})^{-1}(x) =\right] \frac{1}{3}\left(\frac{4}{x}+1\right)$ oe isw or $\left[(\text{hg})^{-1}(x) =\right] \frac{4+x}{3x}$ oe isw	A1	**FT** *their* $(\text{hg})^{-1}(x) = \frac{a-cx}{bx}$ oe If **M0** then **SC1** for *their* hg(x) of the form $y = \frac{a}{x} + b$ oe leading to *their* $(\text{hg})^{-1}(x)$ of the form $y = \frac{a}{x-b}$ isw
(c)	a cao	B1	

11. 0606_s18_ms_23 Q: 5

(i)	Putting $y = f(x)$, changing subject to x and swopping x and y or vice versa	**M1**	
	$f^{-1}(x) = \frac{1}{2}\left(\frac{1}{x}+5\right)$ or $\frac{5x+1}{2x}$ oe isw	**A1**	
(ii)	$x > 0$ oe	**B1**	
(iii)	$\dfrac{1}{2\left(\frac{1}{2x-5}\right)-5}$	**B1**	
	$\dfrac{1}{\frac{2-5(2x-5)}{2x-5}}$ oe	**M1**	**FT** if expression of equivalent difficulty e.g. $\dfrac{1}{\left(\frac{1}{2x-5}\right)-5}$
	Completes to $\dfrac{2x-5}{-10x+27}$ oe final answer	**A1**	

12. 0606_w18_ms_22 Q: 11

(i)	$gf(x) = \dfrac{2(4x-3)+1}{3(4x-3)-1}$	**M1**	
	$= \dfrac{8x-5}{12x-10}$	**A1**	
(ii)	$y(3x-1) = 2x+1$ or $x(3y-1) = 2y+1$	**B1**	
	$(3y-2)x = y+1$ or $(3x-2)y = x+1$	**M1**	
	$g^{-1}(x) = \dfrac{x+1}{3x-2}$	**A1**	
(iii)	$4\left(\dfrac{2x+1}{3x-1}\right)-3[=x-1]$	**B1**	
	$3x^2-3x-6$ oe	**B1**	
	$3(x+1)(x-2)$	**M1**	
	$x = 2$ only	**A1**	

13. 0606_m17_ms_22 Q: 11

(i)	$g \geqslant -\frac{1}{2}$	**B1**	
(ii)	$g(1) = 0$ valid comment e.g. domain of f is $x \geqslant 2$	**B1** **B1**	 **B1** for either
(iii)	$\dfrac{\left(\frac{x^2-2}{x}\right)^2 - 1}{2}$	**M1**	or $\dfrac{\left(x-\frac{2}{x}\right)^2 - 1}{2}$
	$\left(\frac{x^2-2}{x}\right)^2 = \frac{x^4-4x^2+4}{x^2}$ soi	**B1**	or $\left(x-\frac{2}{x}\right)^2 = x^2 - 4 + \frac{4}{x^2}$
	$\frac{1}{2}x^2 - \frac{5}{2} + \frac{2}{x^2}$	**A1**	or correct 3 term equivalent or $a = 0.5$, $b = -2.5$, $c = 2$
(iv)	$x \geqslant 2$	**B1**	
(v)	$x^2 - yx - 2 = 0$	**B1**	or $y^2 - xy - 2 = 0$
	$[x =]\frac{-(-y) \pm \sqrt{(-y)^2 - 4(1)(-2)}}{2}$	**M1**	or $[y =]\frac{-(-x) \pm \sqrt{(-x)^2 - 4(1)(-2)}}{2}$
	Explains why negative square root should be discarded	**B1**	at some point
	$f^{-1}(x) = \frac{x + \sqrt{x^2+8}}{2}$	**A1**	allow $y = \frac{x + \sqrt{x^2+8}}{2}$ If zero scored, allow **SC2** for showing correctly that the inverse of the given f^{-1} is f.

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14. 0606_s17_ms_23 Q: 2

(i)	*B* and *C* with valid reason	**B2**	**B1** for one graph and valid reason or both graphs and no reason
(ii)	*B* only with valid reason	**B2**	**B1** for graph *B* or valid reason

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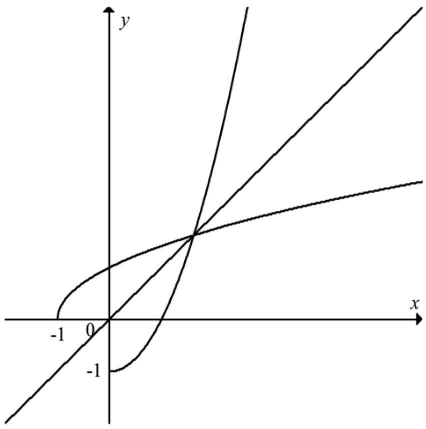
15. 0606_w17_ms_23 Q: 6

(i)	$f^2 = f(f)$ used algebraic $([(x+2)^2+1]+2)^2+1$	**M1**	numerical or algebraic
	17	**A1**	
(ii)	$x = \dfrac{y-2}{2y-1}$	**M1**	change x and y
	$2xy - x = y - 2 \rightarrow y(2x-1) = x-2$	**M1**	**M1dep** multiply, collect y terms, factorise
	$y = \dfrac{x-2}{2x-1}$ $[= g(x)]$	**A1**	correct completion
(iii)	$gf(x) = \dfrac{[(x+2)^2+1]-2}{2[(x+2)^2+1]-1}$ oe	**B1**	
	$\dfrac{(x+2)^2-1}{2(x+2)^2+1} = \dfrac{8}{19}$ $3(x+2)^2 = 27$ oe $3x^2+12x-15=0$	**M1**	*their* gf $= \dfrac{8}{19}$ and simplify to quadratic equation
	solve quadratic	**M1**	**M1dep** Must be of equivalent form
	$x=1$ $x=-5$	**A1**	

16. 0606_s14_ms_21 Q: 12

(i)	f(3) $\frac{6}{4}$ oe	M1 A1	or $\text{fg}(x)=\frac{2\sqrt{(x+1)}}{\sqrt{(x+1)}+1}$
(ii)	$\frac{2\left(\frac{2x}{x+1}\right)}{\frac{2x}{x+1}+1}$	M1	allow omission of 2(…..) in numerator or (…..) + 1 in denominator, but not both.
	A correct and valid step in simplification	dM1	e.g. multiplying numerator and denominator by $x+1$, , or simplifying $\frac{2x}{x+1}+1$ to $\frac{2x+x+1}{x+1}$
	Correctly simplified to $\frac{4x}{3x+1}$	A1	
(iii)	Putting $y = \text{g}(x)$, changing subject to x and swopping x and y or vice versa	M1	condone $x = y^2 - 1$; reasonable attempt at correct method
	$\text{g}^{-1}(x) = x^2 - 1$	A1	condone y =….. , f^{-1} =……
	(Domain) $x > 0$ (Range) $\text{g}^{-1}(x) > -1$	B1 B1	condone $y > -1$ $\text{f}^{-1} > -1$
(iv)		B1 + B1 B1	correct graphs; –1 need not be labelled but could be implied by 'one square' idea of reflection or symmetry in line $y = x$ must be stated.

17. 0606_s14_ms_23 Q: 12

(i)	$3 < f < 7$	**B1,B1**	If **B0** then **SC1** for $3 \le f \le 7$
(ii)	$f(12) = 5$	**B1**	$f^2(x)$ $\sqrt{\left(\sqrt{(x-3)}+2-3\right)}+2$ earns **B1**
	$(f(5) =)\ 2+\sqrt{2}$	**B1**	
(iii)	Clear indication of method $f^{-1}(x) = (x-2)^2 + 3$	**M1** **A1**	condone $y = (x-2)^2 + 3$
(iv)	$gf(x) = \dfrac{120}{\sqrt{(x-3)}+2}$	**B1**	
	Attempt to solve *their* $gf(x) = 20$	**M1**	
	$x = 19$	**A1**	

18. 0606_w14_ms_21 Q: 4

(i)	$f(37) = 3$ or $gf(x) = \dfrac{\sqrt{x-1}-3-2}{2\left(\sqrt{x-1}-3\right)-3}$	**B1**	
	$gf(37) = \dfrac{3-2}{6-3} = \dfrac{1}{3}$	**B1**	
(ii)	$y = \sqrt{x-1}-3 \rightarrow (y+3)^2 = x-1$	**M1**	Rearrange and square in any order
	$(x+3)^2 + 1 = f^{-1}(x)$ oe isw	**A1**	Interchange x and y and complete
(iii)	$y = \dfrac{x-2}{2x-3}$ $2xy - 3y = x - 2 \rightarrow 2xy - x = 3y - 2$	**M1**	Multiply and collect like terms
	$\dfrac{3x-2}{2x-1} = g^{-1}(x)$ oe	**A1**	Interchange and complete Mark final answer

19. 0606_s13_ms_21 Q: 11

(i)	1	**B1**	Not a range for *k*, but condone $x = 1$ and $x \geqslant 1$
(ii)	$f \geqslant -5$	**B1**	Not *x*, but condone *y*
(iii)	Method of inverse	**M1**	Do not reward poor algebra but allow slips
	$1+\sqrt{x+5}$	**A1**	Must be $f^{-1} = ...$or $y = ...$
(iv)	f: Positive quadratic curve correct range and domain	**B1**	Must cross *x*-axis
	f^{-1}: Reflection of f in $y = x$	**B1√**	√*their* f*(x)* sketch Condone slight inaccuracies unless clear contradiction.
(v)	Arrange $f(x) = x$ or $f^{-1}(x) = x$ to 3 term quadratic = 0	**M1**	
	4 only www	**A1**	Allow $x = 4$ with no working. Condone (4, 4). Do not allow final **A** mark if –1 also given in answer

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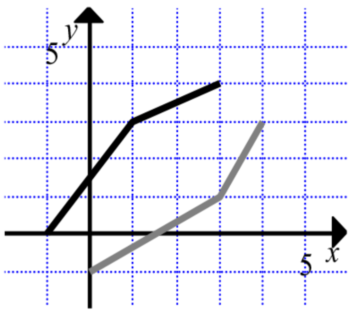
20. 0606_s13_ms_22 Q: 3

(a) (i)	A and E	**B2**	1 mark for each **B1** for 1 extra, **B0** if 2 or more extras
(ii)	C and D	**B2**	1 mark for each **B1** if 1 extra, **B0** if 2 or more extras
(b)		**B2**	(–1, 0), (1, 3), (3, 4) or **B1** for two points correct and joined or for three points correct but clearly not joined

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21. 0606_s23_ms_22 Q: 1

Question	Answer	Marks	Partial Marks
(a)	$3x^2 - 15x + 12$ [* 0] oe where * is any inequality sign or =	B1	
	Factorises or solves *their* 3-term quadratic	M1	**FT** *their* 3-term quadratic
	$x < 1$ or $x > 4$ mark final answer	A1	
(b)(i)	$3(x-2)^2 + 4$	3	**B2** for $3(x-2)^2$ or **B1** for $(x-2)^2$ or $a = 3$, $b = -2$ and **B1** for $a(x+b)^2 + 4$ with numerical values of a and b or $c = 4$
(b)(ii)	$y =$ *their* 4	B1	**STRICT FT** *their* 4 from part **(i)**

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22. 0606_w23_ms_21 Q: 1

Question	Answer	Marks	Partial marks
(a)	$-3(x+2)^2 + 31$	B4	**B2** for $-3(x+2)^2$ or **B1** for $(x+2)^2$ or $a = -3$ and $b = 2$ **B2** for $c = 31$ or **B1** for $-4 \times -3 + 19$ soi
(b)	Maximum value 31 when $x = -2$	B2	**Strict FT** *their c* from part **(a)** and −*their b* from part **(a)** **B1** for either without contradiction
(c)	$-3\left(\sqrt{u}+2\right)^2 = -31$ oe	M1	**FT** an expression of correct form from part **(a)**
	Rearranges as far as $\sqrt{u} = -2 \pm \sqrt{\frac{31}{3}}$	A1	
	1.48 cao or 1.475[13…] rot to 3 or more dp or $\frac{43 - 4\sqrt{93}}{3}$ isw	A1	

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23. 0606_w23_ms_23 Q: 2

Question	Answer	Marks	Guidance
	Uses $b^2 - 4ac$ [*0] $k^2 - 4(4k - 15)$ [*0]	M1	where * is any inequality sign or =;
	$k^2 - 16k + 60$ [*0]	A1	
	$(k-6)(k-10)[*0]$	DM1	**FT** *their* 3-term quadratic; **dep** on previous **M1**
	$6 < k < 10$	A1	Mark final answer
	Alternative method		
	$2x + k$ [= 0]	(M1)	
	$-\frac{k^2}{4} + 4k - 15[*0]$ oe or $-x^2 - 8x - 15[*0]$ oe	(A1)	
	Solves or factorises $(k-6)(k-10)[*0]$ or $(x + 5)(x + 3)$ [*0] **and** $x = -5$, $x = -3$	(DM1)	**FT** *their* 3-term quadratic; **dep** on previous **M1**
	$6 < k < 10$	(A1)	Mark final answer

24. 0606_m22_ms_22 Q: 4

Question	Answer	Marks	Partial Marks
	Eliminates one unknown e.g. $\frac{x^2}{4} + \frac{1}{9}\left(\frac{3}{2x}\right)^2 = 1$	M1	
	Rearranges to solvable form e.g. $x^4 - 4x^2 + 1 = 0$	A1	
	Solves : $\frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(1)}}{2}$	M1	**dep** on attempt to eliminate one unknown and having a 3-term quadratic in x^2
	$x^2 = 2 \pm \sqrt{3}$ oe isw or 3.7320[5...] and 0.2679[4...]	A1	
	$x = \pm 1.932$ or $x = \pm 0.518$	A1	