Topical Past Paper Questions Worksheets

IGCSE Additional Mathematics (0606)

Paper 1

Exam Series: February/March 2017 – October/November 2024

Format Type B:

Each question is followed by its answer scheme

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Introduction

Each Topical Past Paper Questions Workbook contains a comprehensive collection of hundreds of questions and corresponding answer schemes, presented in worksheet format. The questions are carefully arranged according to their respective chapters and topics, which align with the latest IGCSE or AS/A Level subject content. Here are the key features of these resources:

1. The workbook covers a wide range of topics, which are organized according to the latest syllabus content for Cambridge IGCSE or AS/A Level exams.
2. Each topic includes numerous questions, allowing students to practice and reinforce their understanding of key concepts and skills.
3. The questions are accompanied by detailed answer schemes, which provide clear explanations and guidance for students to improve their performance.
4. The workbook's format is user-friendly, with worksheets that are easy to read and navigate.
5. This workbook is an ideal resource for students who want to familiarize themselves with the types of questions that may appear in their exams and to develop their problem-solving and analytical skills.

Overall, Topical Past Paper Questions Workbooks are a valuable tool for students preparing for IGCSE or AS/A Level exams, providing them with the opportunity to practice and refine their knowledge and skills in a structured and comprehensive manner. To provide a clearer description of this book's specifications, here are some key details:

- Title: Cambridge IGCSE Additional Mathematics (0606) Paper 1 Topical Past Paper Questions
- Subtitle: Exam Practice Worksheets With Answer Scheme
- Examination board: Cambridge Assessment International Education (CAIE)
- Subject code: 0606
- Years covered: February/March 2017 – October/November 2024
- Paper: 1
- Number of pages: 1119
- Number of questions: 581

Contents

Chapter 1

Functions

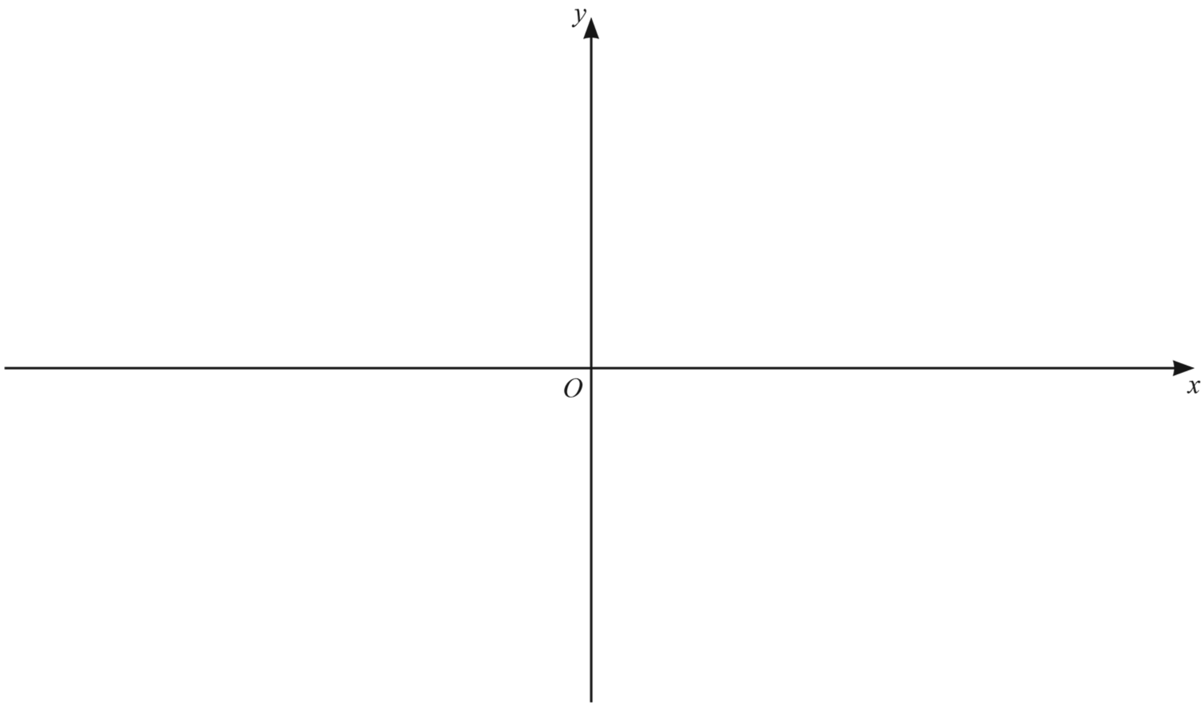
1. 0606_m24_qp_12 Q: 4

A function f is such that $\mathrm{f}(x) = 2 + \mathrm{e}^{-3x}, \quad x \in \mathbb{R}$.

(a) Write down the range of f. [1]

(b) Find an expression for f^{-1}. [2]

(c) On the axes, sketch the graphs of $y = \mathrm{f}(x)$ and $y = \mathrm{f}^{-1}(x)$, stating the coordinates of the points where the curves meet the coordinate axes. State the equations of any asymptotes. Label your curves. [4]

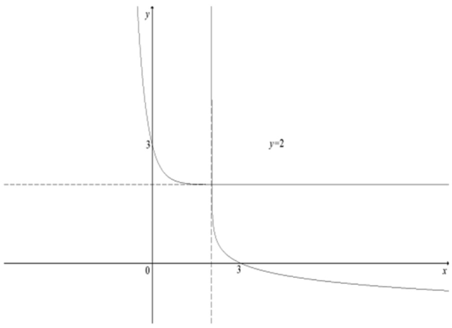


A function g is such that $g(x) = x^{\frac{3}{2}} + 4, \quad x \geqslant 0$.

(d) Find the exact solution of the equation $gf(x) = 12$. [4]

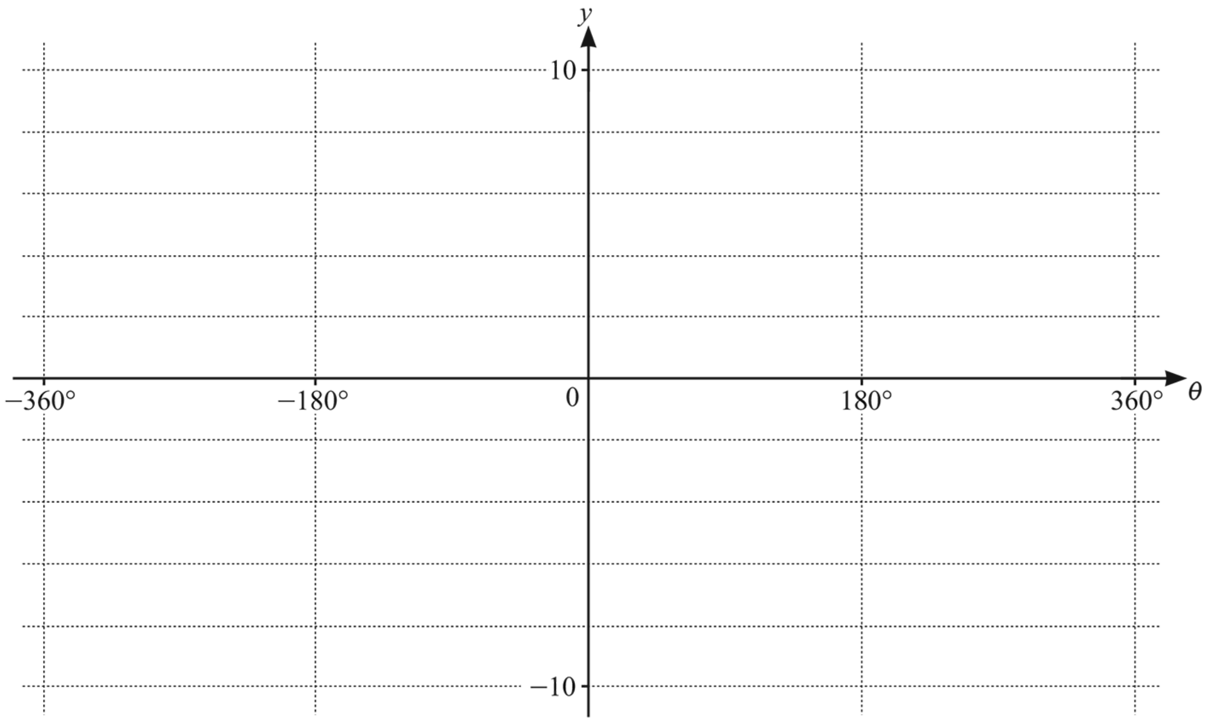
Answer:

Question	Answer	Marks	Guidance
(a)	$f > 2$	**B1**	
(b)	$f^{-1}(x) = -\frac{1}{3}\ln(x-2)$ or $\frac{1}{3}\ln\left(\frac{1}{x-2}\right)$ isw	2	**M1** for a complete attempt at inverse, allow sign slip but brackets must be used correctly.

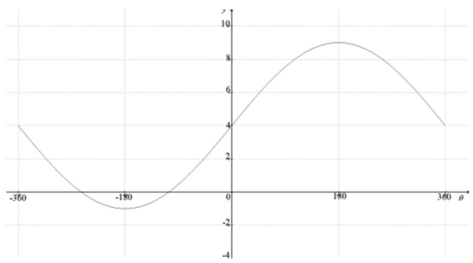
Question	Answer	Marks	Guidance
(c)		4	**B1** for correct $y=\mathrm{f}(x)$ with y-intercept of 3. Must have correct asymptotic behaviour and be in the first and second quadrant. **B1dep** for correct reflection of $y=\mathrm{f}(x)$ to obtain $y=\mathrm{f}^{-1}(x)$ with x-intercept of 3. Must have correct asymptotic behaviour and be in the first and fourth quadrant. **B1** for asymptote of $y=2$ stated or drawn through $y=2$, must have a correctly shaped $y=\mathrm{f}(x)$ **B1** for asymptote of $x=2$ stated or drawn or drawn through $x=2$, must have a correctly shaped $y=\mathrm{f}^{-1}(x)$
(d)	$\left(2+\mathrm{e}^{-3x}\right)^{\frac{3}{2}}+4$ soi	**B1**	For correct order
	$2+\mathrm{e}^{-3x}=4$	**M1**	For forming an equation, must be correct order
	$x=-\frac{1}{3}\ln 2$	2	**M1 dep** for correct attempt to solve for x.

2. 0606_w24_qp_13 Q: 2

On the axes, sketch the graph of $y = 4 + 5\sin\frac{\theta}{2}$, for $-360° \leqslant \theta \leqslant 360°$. State the intercept with the y-axis. [4]



Answer:

Question	Answer	Marks	Guidance
		4	**B1** for correct shape must be a curve with one min in 3rd quadrant and one max in first quadrant and correct endpoints $(-360, 4)$ and $(360, 4)$ Ignore labelling of their maximum point if incorrect coordinates. **depB1** for intercept of 4 on y-axis. Must have the correct shape **depB1** for max in correct position of $(180°,\ 9)$. Must have the correct shape **depB1** for min in correct position of $(-180°,\ -1)$. Must have the correct shape

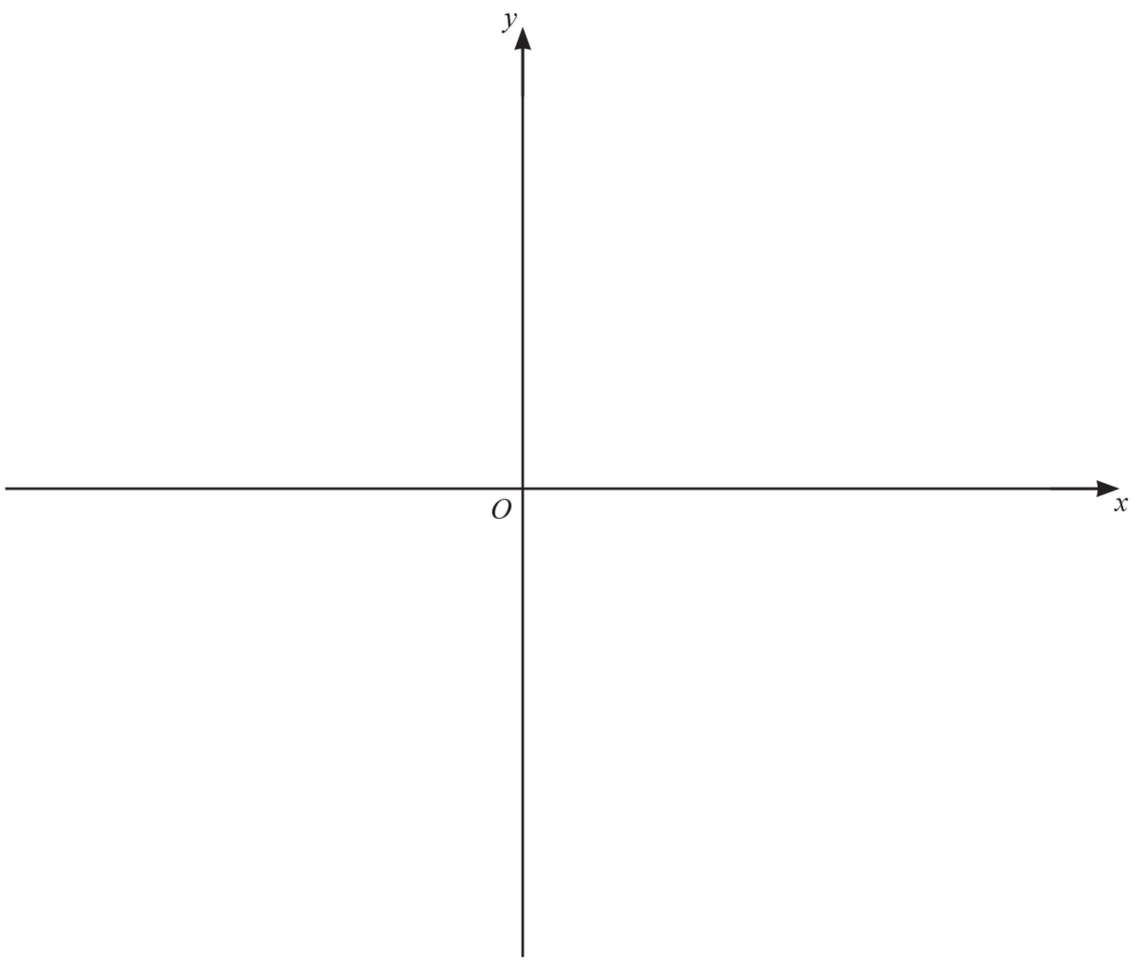
3. 0606_s23_qp_12 Q: 8

It is given that $f(x) = 2\ln(3x-4)$ for $x > a$.

(a) Write down the least possible value of a. [1]

(b) Write down the range of f. [1]

(c) It is given that the equation $f(x) = f^{-1}(x)$ has two solutions. (You do not need to solve this equation). Using your answer to **part (a)**, sketch the graphs of $y = f(x)$ and $y = f^{-1}(x)$ on the axes below, stating the coordinates of the points where the graphs meet the axes. [4]

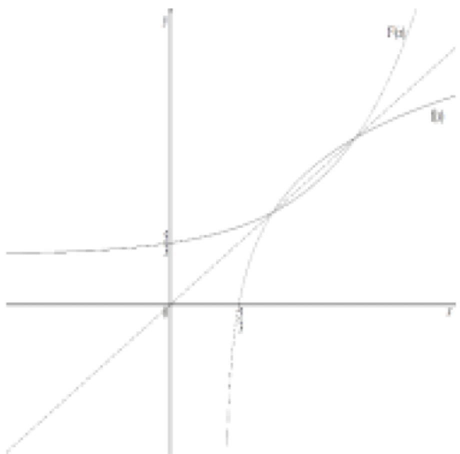


It is given that $g(x) = 2x - 3$ for $x \geqslant 3$.

(d) **(i)** Find an expression for $g(g(x))$. [1]

(ii) Hence solve the equation $fg(g(x)) = 4$ giving your answer in exact form. [3]

Answer:

Question	Answer	Marks	Guidance
(a)	$(a =)\ \frac{4}{3}$ or $1.\dot{3}$	B1	Allow a recurring decimal Must not be an inequality in terms of *a* Allow $x > \frac{4}{3}$
(b)	$f \in \mathbb{R}$ or $-\infty < f < \infty$ or $\mathbb{R}$	B1	Allow y or $f(x)$ but not x.
(c)		4	**B1** for a correct shape for $y = f(x)$ in quadrants 1 and 4 **B1** for $\left(\frac{5}{3}, 0\right)$, must have a correct shape in either quadrant 1 or quadrant 4 **B1** for $y = f^{-1}(x)$, must be a correct shape in quadrants 1 and 2 and intersect twice. **B1** for $\left(0, \frac{5}{3}\right)$, must have a reasonable shape for $y = f^{-1}(x)$ in either the first quadrant or the second quadrant
(d)(i)	$g(g(x)) = 4x - 9$	B1	Must be simplified
(d)(ii)	$fg(g(x)) = 2\ln(12x - 31)$	M1	allow unsimplified, using *their* answer to **(i)**
	$2\ln(12x - 31) = 4$ $x = \frac{e^2 + 31}{12}$	2	**Dep M1** for correct order of operations to solve *their* equation, to get as far as $x = \ldots$ Implied by decimal answer of awrt 3.2 **A1** Must be exact form.

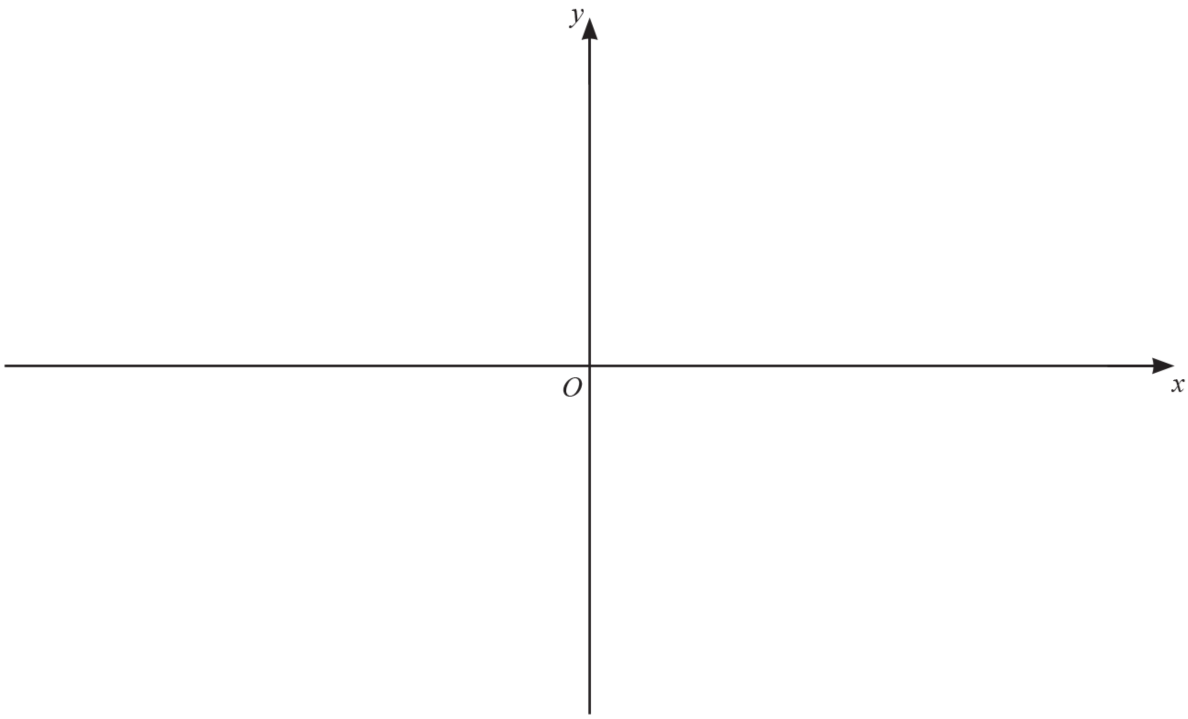
4. 0606_w23_qp_12 Q: 8

(a) It is given that $f : x \to (3x+1)^2 - 4$ for $x \geqslant a$, and that f^{-1} exists.

(i) Find the least possible value of a. [1]

(ii) Using this value of a, write down the range of f. [1]

(iii) Using this value of a, sketch the graphs of $y = f(x)$ and $y = f^{-1}(x)$ on the axes, stating the intercepts with the coordinate axes. [4]

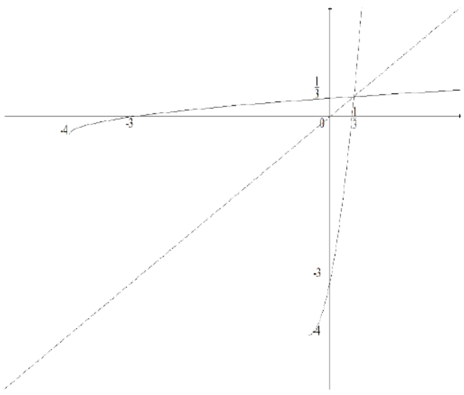


(b) It is given that $\quad g(x) = \ln(2x^2+5) \quad \text{for } x \geqslant 0,$

$$h(x) = 3x-2 \quad \text{for } x \geqslant 0.$$

Solve the equation $\text{hg}(x) = 4$ giving your answer in exact form. [3]

Answer:

Question	Answer	Marks	Guidance
(a)(i)	$a=-\frac{1}{3}$ or $x \geqslant -\frac{1}{3}$	B1	Allow -0.333 or better Allow a correct recurring decimal
(a)(ii)	$f \geqslant -4$	B1	
(a)(iii)		4	**B1** for $y=f(x)$, must have a correct shape (right hand side from the vertex of a quadratic curve), must be a 1:1 function, intersecting each of the *x* and *y* axes once, in quadrants 1, 3 and 4. **B1**, **dependent** on previous **B** for passing through $(0,-3)$ and $\left(\frac{1}{3},\ 0\right)$. **B1 dependent** on first **B1** for $y=f^{-1}(x)$, being a correct reflection of *their* $y=f(x)$, intersecting each of the *x* and *y* axes and $y=f(x)$ once. **B1 dependent** on previous **B** for passing through $(-3,\ 0)$ and $\left(0,\ \frac{1}{3}\right)$.
(b)	$3\left(\ln\left(2x^2+5\right)\right)-2(=4)$	M1	For correct order
	$x=\sqrt{\frac{e^2-5}{2}}$ or exact equivalent	2	**M1 dep** for a correct attempt to deal with logarithms and obtain $x = \ldots$ Allow one arithmetic or sign slip.

5. 0606_s21_qp_11 Q: 5

The functions f and g are defined as follows.

$$f(x) = x^2 + 4x \quad \text{for} \quad x \in \mathbb{R}$$

$$g(x) = 1 + e^{2x} \quad \text{for} \quad x \in \mathbb{R}$$

(a) Find the range of f. [2]

(b) Write down the range of g. [1]

(c) Find the exact solution of the equation $fg(x) = 21$, giving your answer as a single logarithm. [4]

Answer:

Question	Answer	Marks	Guidance
(a)	$f \geqslant -4$	**2**	**M1** for a valid method to find the least value of x^2+4x **A1** for $f \geqslant -4$, $y \geqslant -4$ or $f(x) \geqslant -4$
(b)	$g > 1$	**B1**	Allow $y > 1$ or $g(x) > 1$
(c)	$\left(1+e^{2x}\right)^2+4\left(1+e^{2x}\right)[=21]$	**M1**	
	$e^{4x}+6e^{2x}-16=0$ $\left(e^{2x}+8\right)\left(e^{2x}-2\right)=0$	**M1**	**Dep** for quadratic in terms of e^{2x} and attempt to solve to obtain $e^{2x}=k$
	$e^{2x}=2$ $x=\frac{1}{2}\ln k$	**M1**	**Dep** on both previous M marks, for attempt to solve $e^{2x}=k$
	$x=\ln\sqrt{2}$ or $\ln 2^{\frac{1}{2}}$	**A1**	

6. 0606_w21_qp_13 Q: 2

A particle moves in a straight line such that its velocity, v ms^{-1}, at time t seconds after passing through a fixed point O, is given by $v = e^{3t} - 25$. Find the speed of the particle when $t = 1$. [2]

Answer:

Question	Answer	Marks	Guidance
	$v = -4.91$ soi	**B1**	
	Speed = 4.91	**B1**	

7. 0606_s20_qp_12 Q: 5

$$f : x \mapsto (2x+3)^2 \quad \text{for } x > 0$$

(a) Find the range of f. [1]

(b) Explain why f has an inverse. [1]

(c) Find f^{-1}. [3]

(d) State the domain of f^{-1}. [1]

(e) Given that $g : x \mapsto \ln(x+4)$ for $x > 0$, find the exact solution of $fg(x) = 49$. [3]

Answer:

Question	Answer	Marks	Partial Marks
(a)	$f > 9$	**B1**	Allow y but not x
(b)	It is a one-one function because of the restricted domain	**B1**	

Question	Answer	Marks	Partial Marks
(c)	$x = (2y+3)^2$ or equivalent	**M1**	For a correct attempt to find the inverse
	$y = \dfrac{\sqrt{x}-3}{2}$	**M1**	For correct rearrangement
	$f^{-1} = \dfrac{\sqrt{x}-3}{2}$	**A1**	Must have correct notation
(d)	$x > 9$	**B1**	**FT** on *their* (a)
(e)	$f(\ln(x+4)) = 49$	**M1**	For correct order
	$(2\ln(x+4)+3)^2 = 49$ $\ln(x+4) = 2$	**M1**	For correct attempt to solve, dep on previous M mark, as far as $x =$
	$x = e^2 - 4$	**A1**	

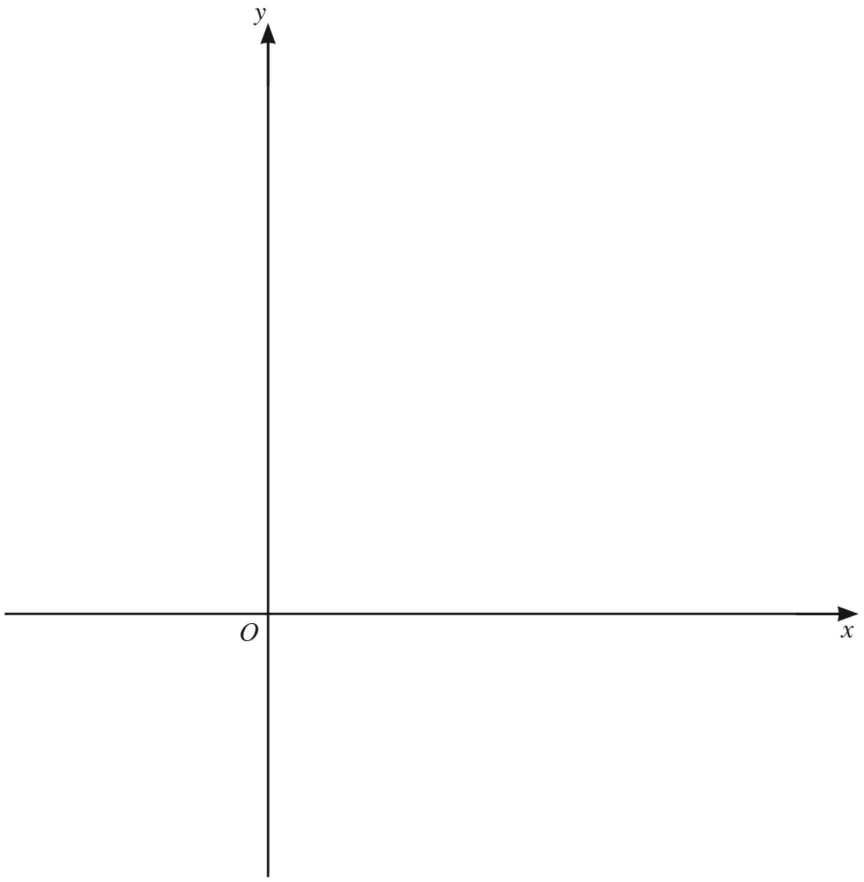
8. 0606_w20_qp_11 Q: 7

It is given that $f(x) = 5\ln(2x+3)$ for $x > -\frac{3}{2}$.

(a) Write down the range of f. [1]

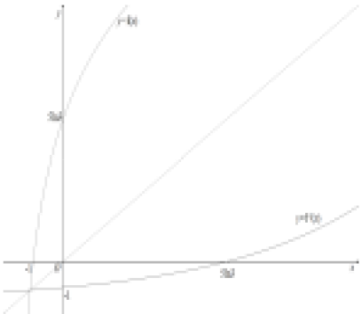
(b) Find f^{-1} and state its domain. [3]

(c) On the axes below, sketch the graph of $y = f(x)$ and the graph of $y = f^{-1}(x)$. Label each curve and state the intercepts on the coordinate axes.



[5]

Answer:

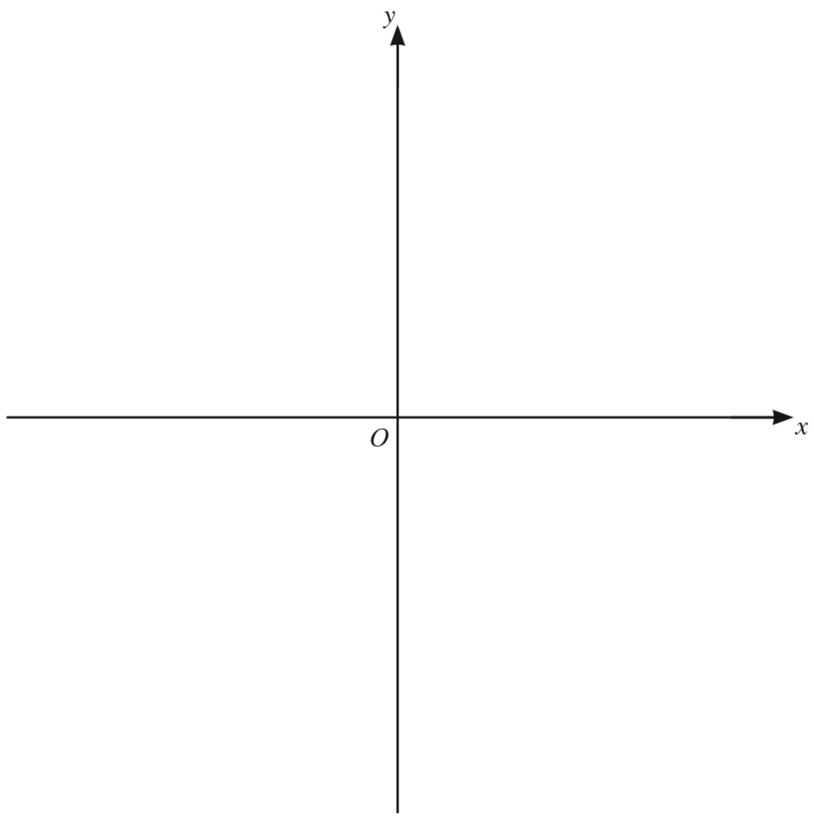
Question	Answer	Marks	Guidance
(a)	$\mathrm{f} \in \mathbb{R}$	**B1**	Allow *y* but not *x*
(b)	$x = 5\ln(2y+3)$ $e^{\frac{x}{5}} = 2y+3$	**M1**	For a complete attempt to obtain inverse
	$\mathrm{f}^{-1}(x) = \dfrac{e^{\frac{x}{5}} - 3}{2}$	**A1**	Must be using correct notation
	Domain $x \in \mathbb{R}$	**B1**	**FT** on *their* **(a)**. Must be using correct notation
(c)		**5**	**B1** for shape of $y = \mathrm{f}(x)$ **B1** for shape of $y = \mathrm{f}^{-1}(x)$ **B1** for $5\ln 3$ or 5.5 and -1 on both axes for $y = \mathrm{f}(x)$ **B1** for $5\ln 3$ or 5.5 and -1 on both axes for $y = \mathrm{f}^{-1}(x)$ **B1** All correct, with apparent symmetry which may be implied be previous 2 B marks or by inclusion of $y = x$, and implied asymptotes, may have one or two points of intersection

9. 0606_w20_qp_12 Q: 6

$$f(x) = x^2 + 2x - 3 \quad \text{for } x \geqslant -1$$

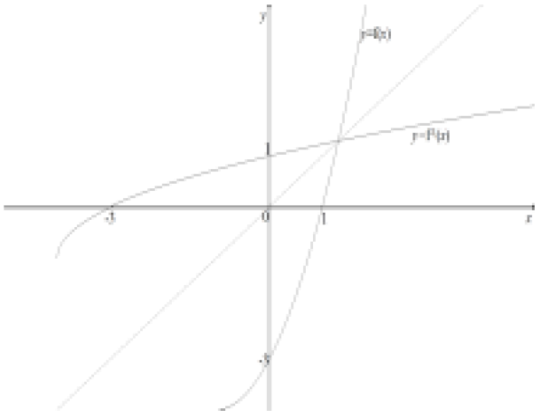
(a) Given that the minimum value of $x^2 + 2x - 3$ occurs when $x = -1$, explain why $f(x)$ has an inverse. [1]

(b) On the axes below, sketch the graph of $y = f(x)$ and the graph of $y = f^{-1}(x)$. Label each graph and state the intercepts on the coordinate axes.



[4]

Answer:

Question	Answer	Marks	Guidance
(a)	It is a one-one function because of the given restricted domain or because $x \geqslant -1$	**B1**	
(b)		4	**B1** for $y = f(x)$ for $x > -1$ only **B1** for 1 on x-axis and –3 on y-axis for $y = f(x)$ **B1** for $y = f^{-1}(x)$ as a reflection of $y = f(x)$ in the line $y = x$, maybe implied by intercepts with axes **B1** for 1 on y-axis and –3 on x-axis for $y = f^{-1}(x)$

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10. 0606_w20_qp_13 Q: 3

(a) $f(x) = 4\ln(2x - 1)$

(i) Write down the largest possible domain for the function f. [1]

(ii) Find $f^{-1}(x)$ and its domain. [3]

(b) $g(x) = x + 5 \quad \text{for } x \in \mathbb{R}$

$h(x) = \sqrt{2x - 3} \quad \text{for } x \geqslant \frac{3}{2}$

Solve $gh(x) = 7$. [3]

Answer:

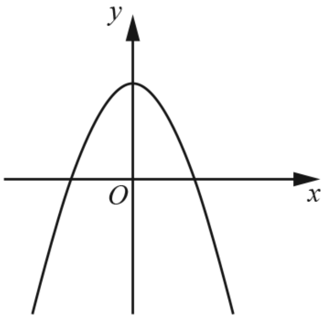
Question	Answer	Marks	Guidance
(a)(i)	$x > \frac{1}{2}$	**B1**	Must be using x

Question	Answer	Marks	Guidance
(a)(ii)	$x = 4\ln(2y-1)$ $e^{\frac{x}{4}} = 2y-1$ $y = \frac{1}{2}\left(1+e^{\frac{x}{4}}\right)$	**M1**	For full method for inverse using correct order of operations
	$f^{-1}(x) = \frac{1}{2}\left(1+e^{\frac{x}{4}}\right)$ or $f^{-1}(x) = \frac{1}{2}\left(1+\sqrt[4]{e^x}\right)$	**A1**	Must be using correct notation
	$x \in \mathbb{R}$	**B1**	
(b)	$\sqrt{2x-3}+5=7$	**M1**	For correct order
	$x = \frac{2^2+3}{2}$	**M1**	Dep on previous M mark, for obtaining x by simplifying and solving using correct order of operations, including squaring
	$x = \frac{7}{2}$ or 3.5	**A1**	

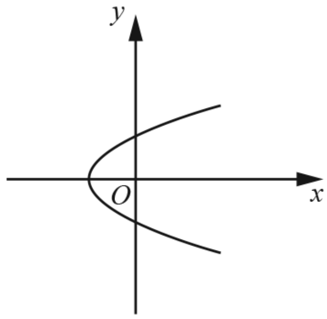
11. 0606_s18_qp_11 Q: 3

Diagrams **A** to **D** show four different graphs. In each case the whole graph is shown and the scales on the two axes are the same.

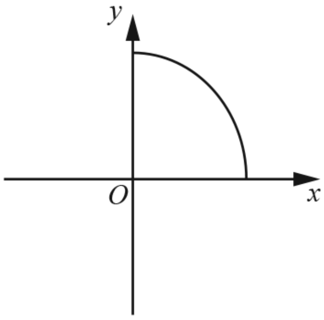
A



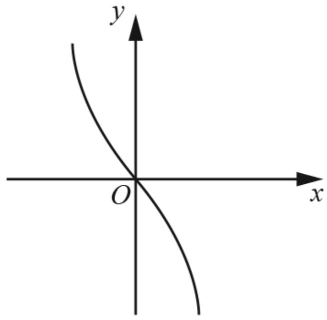
B



C



D



Place ticks in the boxes in the table to indicate which descriptions, if any, apply to each graph. There may be more than one tick in any row or column of the table. [4]

	A	**B**	**C**	**D**
Not a function				
One-one function				
A function that is its own inverse				
A function with no inverse				

Answer:

<table>
<tr>
<td></td>
<td>
<table>
<tr><th>A</th><th>B</th><th>C</th><th>D</th></tr>
<tr><td></td><td>✓</td><td></td><td></td></tr>
<tr><td></td><td></td><td>✓</td><td>✓</td></tr>
<tr><td></td><td></td><td>✓</td><td></td></tr>
<tr><td>✓</td><td></td><td></td><td></td></tr>
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<td>4</td>
<td>B1 for either each row correct or each column correct – mark to candidate’s advantage.</td>
</tr>
</table>

12. 0606_w17_qp_11 Q: 6

(a) Functions f and g are such that, for $x \in \mathbb{R}$,

$$f(x) = x^2 + 3,$$

$$g(x) = 4x - 1.$$

(i) State the range of f. [1]

(ii) Solve $fg(x) = 4$. [3]

(b) A function h is such that $\mathrm{h}(x) = \dfrac{2x+1}{x-4}$ for $x \in \mathbb{R}$, $x \neq 4$.

(i) Find $\mathrm{h}^{-1}(x)$ and state its range. [4]

(ii) Find $\mathrm{h}^{2}(x)$, giving your answer in its simplest form. [3]

Answer:

(a)(i)	$f \geqslant 3$	**B1**	must be using a correct notation
(a)(ii)	$(4x-1)^2+3=4$	**M1**	correct order
	solution of resulting quadratic equation	**DM1**	
	$x=0,\ x=\frac{1}{2}$	**A1**	both required
(b)(i)	$xy-4y=2x+1$	**M1**	'multiplying out'
	$x(y-2)=4y+1$ $x=\frac{4y+1}{y-2}$	**M1**	collecting together like terms
	$h^{-1}(x)=\frac{4x+1}{x-2}$	**A1**	correct answer with correct notation
	Range $h^{-1}\neq 4$	**B1**	must be using a correct notation
(b)(ii)	$h^2(x)=h\left(\frac{2x+1}{x-4}\right)$ $=\frac{2\left(\frac{2x+1}{x-4}\right)+1}{\left(\frac{2x+1}{x-4}\right)-4}$	**M1**	dealing with h^2 correctly
	dealing with fractions within fractions	**M1**	
	$=\frac{5x-2}{17-2x}$ oe	**A1**	

Chapter 2

Quadratic functions

13. 0606_w24_qp_13 Q: 3

Find the values of k for which the equation $4x^2 - k = 4kx - 2$ has no real roots. [4]

Answer:

Question	Answer	Marks	Guidance
	$4x^2 - 4kx - k + 2[=0]$	**B1**	soi
	$k^2 + k - 2$ Critical values -2, 1	**2**	**M1** for use of discriminant on *their* three-term quadratic equation to obtain two critical values
	$-2 < k < 1$	**A1**	Strict inequality

14. 0606_s23_qp_13 Q: 2

(a) Write $2x^2+5x+3$ in the form $2(x+a)^2+b$, where a and b are rational numbers. [2]

(b) Hence write down the coordinates of the stationary point on the curve $y=2x^2+5x+3$. [2]

(c) Solve the inequality $2x^2+5x+3<\frac{15}{8}$. [3]

Answer:

Question	Answer	Marks	Guidance
(a)	$2\left(x+\frac{5}{4}\right)^2-\frac{1}{8}$ oe	2	**B1** for $2\left(x+\frac{5}{4}\right)^2$ **B1** for $-\frac{1}{8}$
(b)	$\left(-\frac{5}{4},\ -\frac{1}{8}\right)$ oe	2	**B1 FT** for each on *their* (a).

Question	Answer	Marks	Guidance
(c)	Use of *their* (a) or expansion to 3 term quadratic (= 0), to obtain two critical values.	**M1**	
	$-\frac{9}{4}$, $-\frac{1}{4}$	**A1**	For both critical values
	$-\frac{9}{4} < x < -\frac{1}{4}$	**A1**	

15. 0606_w23_qp_12 Q: 9

Solve the equation $12x^{\frac{2}{3}} - 5x^{-\frac{2}{3}} - 11 = 0$ for $x > 0$. Give your answer correct to one decimal place. [4]

Answer:

Question	Answer	Marks	Guidance
	$12\left(x^{\frac{2}{3}}\right)^2 - 11\left(x^{\frac{2}{3}}\right) - 5 = 0$	**B1**	For recognition of a 3-term quadratic equation in terms of $x^{\frac{2}{3}}$ or a suitable substitution
	$\left(3x^{\frac{2}{3}}+1\right)\left(4x^{\frac{2}{3}}-5\right)=0$ $x^{\frac{2}{3}} = -\frac{1}{3}$, $x^{\frac{2}{3}} = \frac{5}{4}$	**2**	**M1** for attempt to solve a 3-term quadratic equation in the form $12u^2 \pm 11u \pm 5 = 0$ to obtain at least one solution in the form $x^{\frac{2}{3}} = ...$ or ‘u’ = … **A1** for at least one correct solution.
	$x = 1.4$ only	**A1**	

16. 0606_m22_qp_12 Q: 1

Find the values of k such that the line $y = 9kx + 1$ does not meet the curve $y = kx^2 + 3x(2k + 1) + 4$.

[5]

Answer:

Question	Answer	Marks	Guidance
	$9kx + 1 = kx^2 + 3(2k+1)x + 4$, leading to $kx^2 + x(3-3k) + 3 \ [=0]$	**M1**	For equating the two equations and attempt to obtain a 3 term quadratic equation equated to zero.
	$(3-3k)^2 - (4 \times 3k)$ oe	**M1**	**Dep** on previous M mark for attempt to use the discriminant in any form
	$3k^2 - 10k + 3$ oe	**M1**	**Dep** on previous M mark for simplification to a 3 term quadratic expression in terms of k
	Critical values 3 and $\frac{1}{3}$	**A1**	For both
	$\frac{1}{3} < k < 3$	**A1**	Mark the final answer

17. 0606_m22_qp_12 Q: 2

DO NOT USE A CALCULATOR IN THIS QUESTION.

Solve the equation $(3-5\sqrt{3})x^2+(2\sqrt{3}+5)x-1=0$, giving your solutions in the form $a+b\sqrt{3}$, where a and b are rational numbers. [6]

Answer:

Question	Answer	Marks	Guidance
	$x=$ $\dfrac{-(2\sqrt{3}+5)\pm\sqrt{(2\sqrt{3}+5)^2-4(3-5\sqrt{3})(-1)}}{2(3-5\sqrt{3})}$	**M1**	For the use of the quadratic formula
	$x=$ $\dfrac{-(2\sqrt{3}+5)\pm\sqrt{12+20\sqrt{3}+25+12-20\sqrt{3}}}{2(3-5\sqrt{3})}$	**M1**	For expansion of the square root, must see at least 4 terms
	$x=\dfrac{-12-2\sqrt{3}}{2(3-5\sqrt{3})}$ oe, $x=\dfrac{2-2\sqrt{3}}{2(3-5\sqrt{3})}$ oe	**A1**	For both
	$x=\dfrac{-12-2\sqrt{3}}{2(3-5\sqrt{3})}\times\dfrac{3+5\sqrt{3}}{3+5\sqrt{3}}$ oe or $x=\dfrac{2-2\sqrt{3}}{2(3-5\sqrt{3})}\times\dfrac{3+5\sqrt{3}}{3+5\sqrt{3}}$ oe with an attempt to simplify	**M1**	For attempt to rationalise at least one of *their* solutions (must be similar structure) Sufficient detail must be seen, at least 3 terms in the numerator
	$\dfrac{1}{2}+\dfrac{\sqrt{3}}{2}$	**A1**	Must have sufficient detail shown
	$\dfrac{2}{11}-\dfrac{\sqrt{3}}{33}$	**A1**	Must have sufficient detail shown

18. 0606_w22_qp_11 Q: 4

DO NOT USE A CALCULATOR IN THIS QUESTION.

Solve the equation $\left(\sqrt{5}-1\right)x^2 - 2x - \left(\sqrt{5}+1\right) = 0$, giving your answers in the form $a + b\sqrt{5}$, where a and b are constants. [6]

Answer:

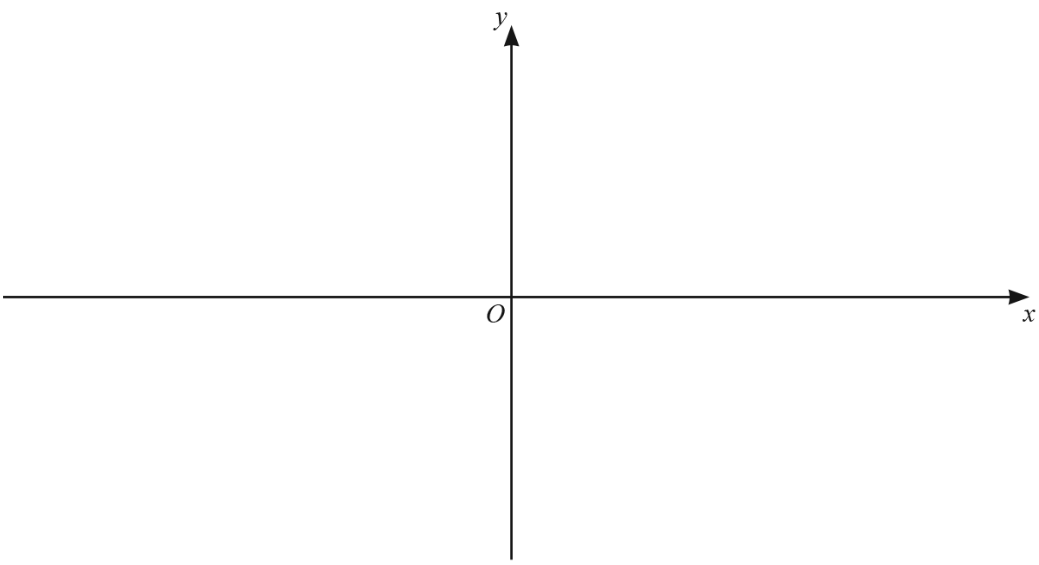
Question	Answer	Marks	Guidance
	$x=\dfrac{2\pm\sqrt{4+4\left(\sqrt{5}-1\right)\left(\sqrt{5}+1\right)}}{2\left(\sqrt{5}-1\right)}$	**M1**	For a correct use of the quadratic formula with sufficient detail
	$x=\dfrac{2\pm2\sqrt{5}}{2\left(\sqrt{5}-1\right)}$ or $x=\dfrac{1\pm\sqrt{5}}{\left(\sqrt{5}-1\right)}$	**2**	**Dep M1** for attempt to simplify to obtain 2 real roots **A1** for either
	$x=\dfrac{\left(\sqrt{5}+1\right)}{\left(\sqrt{5}-1\right)}\times\dfrac{\left(\sqrt{5}+1\right)}{\left(\sqrt{5}+1\right)}$	**M1**	For attempt at rationalisation
	$x=\dfrac{3}{2}+\dfrac{\sqrt{5}}{2}$	**A1**	
	$x=-1$	**B1**	

19. 0606_m21_qp_12 Q: 4

(a) Show that $2x^2+5x-3$ can be written in the form $a(x+b)^2+c$, where a, b and c are constants. [3]

(b) Hence write down the coordinates of the stationary point on the curve with equation $y=2x^2+5x-3$. [2]

(c) On the axes below, sketch the graph of $y=|2x^2+5x-3|$, stating the coordinates of the intercepts with the axes. [3]



(d) Write down the value of k for which the equation $|2x^2+5x-3|=k$ has exactly 3 distinct solutions. [1]

Answer:

Question	Answer	Marks	Guidance
(a)	$2\left(x+\frac{5}{4}\right)^2-\frac{49}{8}$	3	**B1** for $b=\left(x+\frac{5}{4}\right)^2$ or $(x+1.25)^2$ **B1** for $c=-\frac{49}{8}$ or -6.125
(b)	$\left(-\frac{5}{4}, -\frac{49}{8}\right)$ oe	2	**B1** for $-\frac{5}{4}$ as part of a set of coordinates or $x=-\frac{5}{4}$, **FT** on $-$ *their b* **B1** for $-\frac{49}{8}$ as part of a set of coordinates or $y=-\frac{49}{8}$ **FT** on *their c* Need to be using *their* answer to **(a)** and not using differentiation as 'Hence'. **B1** for $-\frac{5}{4},-\frac{49}{8}$
(c)		3	**B1** for correct shape, with maximum in the second quadrant and cusps on the x-axes and reasonable curvature for $x<-3$ and $x>0.5$. **B1** for $(-3,\ 0)$ and $(0.5,\ 0)$ either seen on the graph or stated, must have attempted a correct shape **B1** for $(0,\ 3)$ either seen on the graph or stated, must have attempted a correct shape
(d)	$\frac{49}{8}$ oe	B1	**FT** on *their* $\lvert c\rvert$ from **(a)** Allow $\frac{49}{8}$ from other methods

20. 0606_s21_qp_13 Q: 1

Find the possible values of the constant k such that the equation $kx^2+4kx+3k+1=0$ has two different real roots. [4]

Answer:

Question	Answer	Marks	Guidance
	$(4k)^2 - 4k(3k+1)$	**M1**	For use of the discriminant to obtain a two term quadratic expression.
	$4k^2 - 4k = 0$	**M1**	Dep to find critical values, allow if only one is found
	$k = 0,\ k = 1$	**A1**	For both critical values
	$k < 0 \quad k > 1$	**A1**	

21. 0606_m20_qp_12 Q: 2

Find the values of k for which the line $y = kx + 3$ is a tangent to the curve $y = 2x^2 + 4x + k - 1$. [5]

Answer:

	$2x^2+4x+k-1=kx+3$ $2x^2+(4-k)x+(k-4)=0$	**2**	**M1** for attempt to equate the line and curve and simplify to a 3 term quadratic equation = 0 **A1** for a correct equation, allow equivalent form
	$(4-k)^2=4\times2\times(k-4)$	**M1**	Use of discriminant in any form
	$k^2-16k+48=0$ $k=12,\ k=4$ Do not isw	**2**	Dep **M1** on previous M mark, for attempt to solve a quadratic equation in k **A1** for both
	Alternative 1		
	$2x^2+4x+k-1=kx+3$ $2x^2+(4-k)x+(k-4)=0$	**(2**	**M1** for attempt to equate the line and curve and simplify **A1** for a correct equation, allow equivalent form
	$k=4x+4$ $2\left(\frac{k-4}{4}\right)^2+(4-k)\left(\frac{k-4}{4}\right)+(k-4)=0$	**M1**	Equating gradients and substitution to obtain a quadratic equation in terms of k
	$k^2-16k+48=0$ $k=12$ and $k=4$ Do not isw	**2)**	Dep **M1** on previous M mark, for attempt to solve a quadratic equation in k **A1** for both
	Alternative 2		
	$2x^2+4x+k-1=kx+3$ $2x^2+(4-k)x+(k-4)=0$	**(2**	**M1** for attempt to equate the line and curve and simplify **A1** for a correct equation, allow equivalent form
	$k=4x+4$ $2x^2-4x=0$ $x=0,\ 2$	**M1**	Equating gradients and substitution to obtain a quadratic equation in terms of x and solution of this equation to obtain 2 x values
	$k=4x+4$ $k=12$ and $k=4$ Do not isw	**2)**	Dep **M1** on previous M mark, for substitution of their x values to obtain k values **A1** for both

22. 0606_s20_qp_11 Q: 4

DO NOT USE A CALCULATOR IN THIS QUESTION.

Find the positive solution of the equation $(5+4\sqrt{7})x^2+(4-2\sqrt{7})x-1=0$, giving your answer in the form $a+b\sqrt{7}$, where a and b are fractions in their simplest form. [5]

Answer:

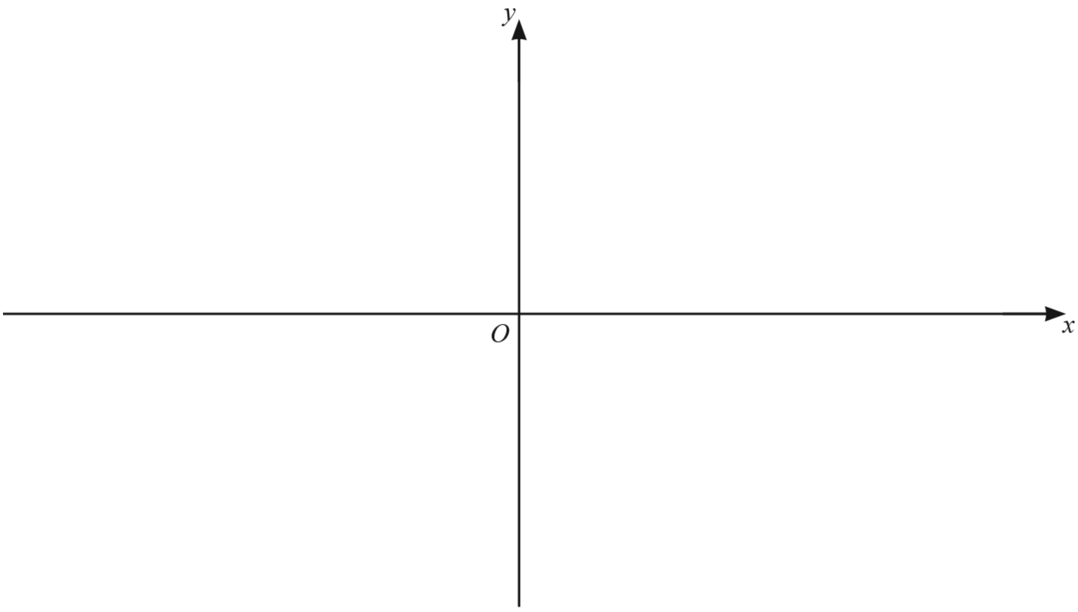
Question	Answer	Marks	Partial Marks
	$x=\frac{-(4-2\sqrt{7})+\sqrt{(4-2\sqrt{7})^2-4(5+4\sqrt{7})(-1)}}{2(5+4\sqrt{7})}$	**M1**	For correct use of quadratic formula, allow inclusion of $\pm$ until final answer
	$x=\frac{-(4-2\sqrt{7})+\sqrt{16+28-16\sqrt{7}+20+16\sqrt{7}}}{2(5+4\sqrt{7})}$ $x=\frac{-(4-2\sqrt{7})+8}{2(5+4\sqrt{7})}$	**M1**	For attempt to simplify discriminant, must see attempt at expansion and subsequent simplification
	$x=\frac{4+2\sqrt{7}}{2(5+4\sqrt{7})}$ or $x=\frac{2+\sqrt{7}}{(5+4\sqrt{7})}$	**A1**	For either
	$x=\frac{2+\sqrt{7}}{(5+4\sqrt{7})}\times\frac{5-4\sqrt{7}}{5-4\sqrt{7}}$ $x=\frac{10+5\sqrt{7}-8\sqrt{7}-28}{25-112}$	**M1**	For attempt to rationalise, must see attempt at expansion and subsequent simplification
	$x=\frac{6}{29}+\frac{\sqrt{7}}{29}$	**A1**	

23. 0606_s20_qp_13 Q: 4

(a) Write $2x^2+3x-4$ in the form $a(x+b)^2+c$, where a, b and c are constants. [3]

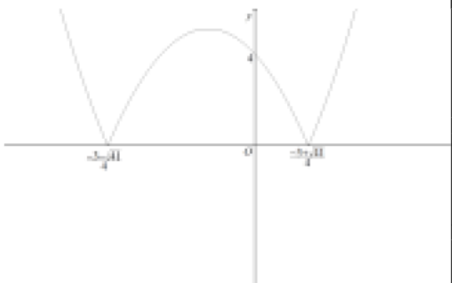
(b) Hence write down the coordinates of the stationary point on the curve $y=2x^2+3x-4$. [2]

(c) On the axes below, sketch the graph of $y=|2x^2+3x-4|$, showing the exact values of the intercepts of the curve with the coordinate axes. [3]



(d) Find the value of k for which $|2x^2+3x-4|=k$ has exactly 3 values of x. [1]

Answer:

Question	Answer	Marks	Partial Marks
(a)	$2\left(x+\frac{3}{4}\right)^2-\frac{41}{8}$	**B3**	**B1** for 2 **B1** for $\frac{3}{4}$ **B1** for $-\frac{41}{8}$
(b)	$\left(-\frac{3}{4}, -\frac{41}{8}\right)$	**B2**	**B1** for $-\frac{3}{4}$ or **FT** on *their* $-b$ **B1** for $-\frac{41}{8}$ or **FT** on *their* c
(c)		**B1**	For shape with max in 2nd quadrant
		B1	For x-intercepts $\frac{-3\pm\sqrt{41}}{4}$
		B1	For y-intercept of 4 and cusps
(d)	$\frac{41}{8}$	**B1**	**FT** on *their* c

24. 0606_m19_qp_12 Q: 2

On the axes below, sketch the graph of the curve $y = |2x^2 - 5x - 3|$, stating the coordinates of any points where the curve meets the coordinate axes.

[4]

Answer:

		**4**	**B1** for general shape with maximum point in 1st quadrant **B1** for $\left(-\frac{1}{2},0\right)$ and $(3,0)$ soi **B1** for $(0,3)$ soi **B1 dep** on first B1, with cusps and correct shape for $x<-\frac{1}{2}$ and $x>3$

25. 0606_s19_qp_12 Q: 2

Do not use a calculator in this question.

Find the coordinates of the points of intersection of the curve $y=(2x+3)^2(x-1)$ and the line $y=3(2x+3)$. [5]

Answer:

	Either: $(2x+3)^2(x-1)=3(2x+3)$ $(2x+3)(2x^2+x-6)(=0)$	**M1**	For attempt to equate line and curve and attempt to simplify to $2x+3\ \times$ a quadratic factor or cancelling $2x+3$ and obtaining a quadratic factor
	$(2x+3)(2x^2+x-6)=0$ $(2x+3)(2x-3)(x+2)=0$	**M1**	**Dep** for attempt at 3 linear factors from a linear term and a quadratic term
	$\left(-\frac{3}{2},0\right)$	**B1**	
	$\left(\frac{3}{2},18\right)$	**A1**	**Dep** on first M mark only
	$(-2,-3)$	**A1**	**Dep** on first M mark only
	Or: $(2x+3)^2(x-1)=3(2x+3)$ $4x^3+8x^2-9x-18(=0)$	**M1**	For attempt to equate line and curve and attempt to simplify to a cubic equation, by collecting like terms
	$(x+2)(4x^2-9)$ $(2x-3)(2x^2+7x+6)$ $(2x+3)(2x^2+x-6)$ $(2x+3)(2x-3)(x+2)(=0)$	**M1**	**Dep** For attempt to find a factor from a 4 term cubic equation (usually $x+2$), do long division oe to obtain a quadratic factor and factorise this quadratic factor
	$\left(-\frac{3}{2},0\right)$	**A1**	
	$\left(\frac{3}{2},18\right)$	**A1**	
	$(-2,-3)$	**A1**	