Topical Past Paper Questions Workbook

AS & A Level Mathematics (9709) Paper 6

[Probability & Statistics 2]

Exam Series: May 2015 – May 2022

Format Type B:

Each question is followed by its answer scheme

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Introduction

Each topical past paper questions workbook consists of hundreds of questions and their answer schemes, in the form of worksheets. Questions are assigned to each chapter according to their corresponding topic. Topics, in turn, are based on the items of the latest Cambridge IGCSE or AS/A level syllabus content. This book's specifications are as follows:

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Contents

Chapter 1

The Poisson distribution

1. 9709_m22_qp_62 Q: 7

(a) Two ponds, A and B, each contain a large number of fish. It is known that 2.4% of fish in pond A are carp and 1.8% of fish in pond B are carp. Random samples of 50 fish from pond A and 60 fish from pond B are selected.

Use appropriate Poisson approximations to find the following probabilities.

(i) The samples contain at least 2 carp from pond A and at least 2 carp from pond B. [3]

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(ii) The samples contain at least 4 carp altogether. [3]

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(b) The random variables X and Y have the distributions $\mathrm{Po}(\lambda)$ and $\mathrm{Po}(\mu)$ respectively. It is given that

- $\mathrm{P}(X = 0) = [\mathrm{P}(Y = 0)]^2$,
- $\mathrm{P}(X = 2) = k[\mathrm{P}(Y = 1)]^2$, where k is a non-zero constant.

Find the value of k. [4]

Answer:

Question	Answer	Marks	Guidance
(a)(i)	0.024×50 [= 1.2] and 0.018×60 [= 1.08]	**B1**	
	$(1 - e^{-1.2}(1 + 1.2)) \times (1 - e^{-1.08}(1 + 1.08))$	**M1**	For $(1 - e^{-\lambda}(1+\lambda)) \times (1 - e^{-\mu}(1+\mu))$ any λ, μ ($\lambda \neq \mu$) Allow one end error on either or both terms
	= 0.0991 (3 sf)	**A1**	Unsupported answer scores maximum **SC** B1 B1 **SC** Use of binomial 0.0994 scores B1 only
		3	
(a)(ii)	$\lambda = 0.024 \times 50 + 0.018 \times 60$	**M1**	or *their* 1.2 + 1.08 (NB 0.024+0.018 is M0)
	$1 - e^{-2.28} \times \left(1 + 2.28 + \frac{2.28^2}{2!} + \frac{2.28^3}{3!}\right)$	**M1**	any λ and allow one end error
	= 0.197 (3 sf)	**A1**	Unsupported answer scores maximum **SC** B2
		3	
(b)	$e^{-\lambda} = \left[e^{-\mu}\right]^2 = e^{-2\mu}$	**M1**	
	$e^{-\lambda} \times \frac{\lambda^2}{2} = k\left[e^{-\mu} \times \mu\right]^2$	**M1**	
	$e^{-2\mu} \times 2\mu^2 = k \times e^{-2\mu} \times \mu^2$	**M1**	OE. Use of $\lambda = 2\mu$ to find equation in μ and k only (or λ and k only)
	$k = 2$	**A1**	
		4	

2. 9709_s22_qp_61 Q: 5

Cars arrive at a fuel station at random and at a constant average rate of 13.5 per hour.

(a) Find the probability that more than 4 cars arrive during a 20-minute period. [3]

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(b) Use an approximating distribution to find the probability that the number of cars that arrive during a 12-hour period is between 150 and 160 inclusive. [4]

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Independently of cars, trucks arrive at the fuel station at random and at a constant average rate of 3.6 per 15-minute period.

(c) Find the probability that the total number of cars and trucks arriving at the fuel station during a 10-minute period is more than 3 and less than 7. [3]

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Answer:

Question	Answer	Marks	Guidance
(a)	$\lambda = 4.5$	B1	
	$1 - e^{-4.5}(1 + 4.5 + \frac{4.5^2}{2!} + \frac{4.5^3}{3!} + \frac{4.5^4}{4!})$	M1	Allow one end error Allow any λ. Poisson expressions must be seen
	= 0.468 (3 sf)	A1	If M0 awarded allow SC B1 for 0.468
		3	
(b)	$\lambda = 162$ ($X \sim$ Po(162) $\Rightarrow X \sim$ N(162, 162))	B1	
	$\frac{149.5 - \text{'162'}}{\sqrt{\text{'162'}}}$ and $\frac{160.5 - \text{'162'}}{\sqrt{\text{'162'}}}$ (= –0.982 and –0.118)	M1	One of these; allow with incorrect or no continuity correction
	Φ('0.982') – ɸ('0.118') oe	M1	Area consistent with *their* values (both standardisations must be seen)
	= 0.290 (3 sf)	A1	Allow 0.29
		4	

Question	Answer	Marks	Guidance
(c)	$\lambda = \frac{13.5}{6} + 3.6 \times \frac{2}{3}$ OE or 4.65	M1	Attempt to find λ
	$e^{-4.65}(\frac{4.65^4}{4!} + \frac{4.65^5}{5!} + \frac{4.65^6}{6!})$	M1	Allow any λ Allow one end error Poisson terms not be seen
	0.494 (3 sf)	A1	If M0 allow SC B1 for 0.494
		3	

3. 9709_s22_qp_62 Q: 3

It is known that 1.8% of children in a certain country have not been vaccinated against measles. A random sample of 200 children in this country is chosen.

(a) Use a suitable approximating distribution to find the probability that there are fewer than 3 children in the sample who have not been vaccinated against measles. [4]

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(b) Justify your approximating distribution. [2]

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Answer:

Question	Answer	Marks	Guidance
(a)	Poisson	**B1**	SOI
	Mean = 3.6	**B1**	Can be awarded for N(3.6, …)
	$e^{-3.6}(1 + 3.6 + \frac{3.6^2}{2})$	**M1**	Allow any λ Allow one end error Expression must be seen
	0.303 (3 s.f.)	**A1**	If M0 awarded allow SC B1 for 0.303 SC Use of binomial: B1 for answer 0.300 (3 sf)
		4	
(b)	[Binomial with] 200 > 50	**B1**	
	[200 × 0.018 =] 3.6 < 5 or [p =] 0.018 < 0.1	**B1**	If B0 B0 then SC n large, p small: B1 or n large $np < 5$: B1 or $n > 50$ and either $np < 5$ or $p < 0.1$: B1
		2	

4. 9709_s22_qp_62 Q: 7

X is a random variable with distribution Po(2.90). A random sample of 100 values of X is taken.

Find the probability that the sample mean is less than 2.88. [5]

Answer:

Question	Answer	Marks	Guidance
	$\bar{X} \sim N(2.9, \frac{2.9}{100})$ OR Totals method N(290, 290)	**B1**	B1 for N(2.9, …) OR N(290, …)
		B1	B1 for Var = $\frac{2.9}{100}$ OR for var = 290 SOI
	$\frac{2.88-2.90}{\sqrt{\frac{2.9}{100}}}$ [= –0.1174] OR $\frac{288-290}{\sqrt{290}}$	**M1**	Standardising with *their* values Allow without –ve sign AND/OR with incorrect continuity correction No mixed methods
	1– Φ('0.1174')	**M1**	For area consistent with *their* values
	0.453 (3 sf)	**A1**	As final answer
	Alternative method for question 7		
	$\bar{X} \sim N(2.9, \frac{2.9}{100})$ OR Totals method N(290, 290)	**B1**	B1 for N(2.9, …) OR N(290, …)
		B1	B1 for Var = $\frac{2.9}{100}$ OR Var = 290 stated or implied
	$\frac{(2.88-\frac{1}{200})-2.90}{\sqrt{\frac{2.9}{100}}}$ [= –0.1468] OR (287.5 – 290)/√290	**M1**	Standardising with *their* values Allow without –ve sign AND/OR with incorrect continuity correction No mixed methods
	1 – ϕ('0.1468')	**M1**	For area consistent with *their* values
	0.442 (3 sf)	**A1**	As final answer
		5	

5. 9709_s22_qp_63 Q: 5

The number of clients who arrive at an information desk has a Poisson distribution with mean 2.2 per 5-minute period.

(a) Find the probability that, in a randomly chosen 15-minute period, exactly 6 clients arrive at the desk. [3]

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(b) If more than 4 clients arrive during a 5-minute period, they cannot all be served.

Find the probability that, during a randomly chosen 5-minute period, not all the clients who arrive at the desk can be served. [2]

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(c) Use a suitable approximating distribution to find the probability that, during a randomly chosen 1-hour period, fewer than 20 clients arrive at the desk. [4]

Answer:

Question	Answer	Marks	Guidance
(a)	$\lambda = 6.6$	B1	
	$e^{-6.6} \times \frac{6.6^6}{6!}$	M1	Any λ
	0.156 (3 s.f.)	A1	If M0 awarded SC B1 for 0.156
		3	
(b)	$1 - e^{-2.2}(1 + 2.2 + \frac{2.2^2}{2} + \frac{2.2^3}{3!} + \frac{2.2^4}{4!})$	M1	Allow one end error. Need 1 – … Any λ
	0.0725 (3 s.f.)	A1	If M0 awarded SC B1 for 0.0725
		2	

Question	Answer	Marks	Guidance
(c)	N(26.4, 26.4)	B1	Give at early stage 2.2 × 12
	$\frac{19.5 - '26.4'}{\sqrt{'26.4'}}$ [= –1.343]	M1	Standardising with *their* values. Allow wrong or no continuity correction
	$\phi('-1.343') = 1 - \phi('1.343')$	M1	Area consistent with *their* working
	0.0897 or 0.0896 (3 s.f.)	A1	
		4	

6. 9709_m21_qp_62 Q: 4

On average, 1 in 400 microchips made at a certain factory are faulty. The number of faulty microchips in a random sample of 1000 is denoted by X.

(a) State the distribution of X, giving the values of any parameters. [1]

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(b) State an approximating distribution for X, giving the values of any parameters. [2]

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(c) Use this approximating distribution to find each of the following.

(i) $P(X = 4)$. [2]

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(ii) $P(2 \leqslant X \leqslant 4)$. [2]

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(d) Use a suitable approximating distribution to find the probability that, in a random sample of 700 microchips, there will be at least 1 faulty one. [3]

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Answer:

Question	Answer	Marks	Guidance
(a)	B(1000, $\frac{1}{400}$)	**B1**	Accept Bin and $n = 1000$, $p = \frac{1}{400}$.
		1	
(b)	Po(2.5)	**B2**	B1 for Po. B1 for $\lambda = 2.5$.
		2	
(c)(i)	$e^{-2.5} \times \frac{2.5^4}{4!}$	**M1**	FT *their* **(b)** for Normal must have a continuity correction. Allow any λ
	0.134 (3 sf)	**A1**	CWO
		2	
(c)(ii)	$e^{-2.5}(\frac{2.5^2}{2!} + \frac{2.5^3}{3!} + \frac{2.5^4}{4!})$	**M1**	FT *their* **(b)** for Normal must have a continuity correction. Allow with one term extra or omitted or wrong. Allow any λ.
	0.604 (3 sf)	**A1**	CWO
		2	
(d)	$\lambda = 2.5 \times 0.7$ or $\lambda = 700 \times \frac{1}{400}$ [= 1.75]	**M1**	Must see λ or use of Poisson.
	$1 - e^{-1.75}$	**M1**	Allow any λ. Allow 1 – P(0,1).
	0.826	**A1**	**SC B1** Use of B(700,0.0025) leading to 0.826.
		3	

7. 9709_s21_qp_61 Q: 1

Accidents at two factories occur randomly and independently. On average, the numbers of accidents per month are 3.1 at factory A and 1.7 at factory B.

Find the probability that the total number of accidents in the two factories during a 2-month period is more than 3. [4]

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Answer:

Question	Answer	Marks	Guidance
	$\lambda = (3.1 + 1.7) \times 2$	M1	Attempt combined mean. Allow 3.1 + 1.7 for M1
	$= 9.6$	A1	Correct mean
	$1 - e^{-9.6}\left(1 + 9.6 + \frac{9.6^2}{2} + \frac{9.6^3}{3!}\right)$	M1	Allow incorrect mean. Allow one end error.
	$= 0.986$ (3 sf)	A1	**SC** If 9.6 seen and unsupported 0.986 M1A1B1. **SC** Unsupported correct answer of 0.986 only if 9.6 also not seen scores B2 only.
		4	

8. 9709_s21_qp_61 Q: 5

On average, 1 in 75 000 adults has a certain genetic disorder.

(a) Use a suitable approximating distribution to find the probability that, in a random sample of 10 000 people, at least 1 has the genetic disorder. [3]

(b) In a random sample of n people, where n is large, the probability that no-one has the genetic disorder is more than 0.9.

Find the largest possible value of n. [4]

Answer:

Question	Answer	Marks	Guidance
(a)	$\mathrm{Po}\left(\frac{2}{15}\right)$	M1	SOI. Allow Po(0.133).
	$\mathrm{P}(X \geqslant 1) = 1 - \mathrm{e}^{-\frac{2}{15}}$	M1	Allow incorrect λ allow one end error
	$= 0.125$ (3 sf)	A1	**SC** Partially unsupported final answer: $\mathrm{Po}\left(\frac{2}{15}\right)$ stated B1 then unsupported 0.125 B1 **SC** Use of Binomial (0.1248) B1 only Use of Normal scores M0
		3	

Question	Answer	Marks	Guidance
(b)	$\lambda = \frac{n}{75000}$	B1	
	$\mathrm{e}^{-\frac{n}{75000}} > 0.9$	M1	Allow '=' Allow incorrect λ
	$-\frac{n}{75000} > \ln 0.9 \quad [n < 7902.04]$	M1	Attempt ln both sides
	Largest value of n is 7902	A1	CWO. Must be an integer.
	Alternative method for Question 5(b)		
	$\mathrm{e}^{-\mu} > 0.9$	M1	Allow '='
	$-\mu > \ln 0.9 \quad [\mu < 0.10536]$	M1	Attempt ln both sides
	$n = \mu \times 75000$	B1	
	Largest value of n is 7902	A1	CWO. Must be an integer.
	Alternative method for Question 5(b)		
	$\frac{74999}{75000}$	B1	
	$\left(\frac{74999}{75000}\right)^n > 0.9$	M1	
	$n\ln\frac{74999}{75000} > \ln 0.9$	M1	Attempt ln or log both sides
	Largest value of n is 7901	A1	CWO Must be an integer
		4	

9. 9709_s21_qp_62 Q: 7

Customers arrive at a particular shop at random times. It has been found that the mean number of customers who arrive during a 5-minute interval is 2.1.

(a) Find the probability that exactly 4 customers arrive during a 10-minute interval. [2]

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(b) Find the probability that at least 4 customers arrive during a 20-minute interval. [2]

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(c) Use a suitable approximating distribution to find the probability that fewer than 40 customers arrive during a 2-hour interval. [4]

Answer:

Question	Answer	Marks	Guidance
(a)	$e^{-4.2} \times \frac{4.2^4}{4!}$	M1	P(4), allow any λ
	0.194 (3 sf)	A1	As final answer. **SC** Unsupported correct answer scores B1 only.
		2	
(b)	$1-e^{-8.4}\left(1+8.4+\frac{8.4^2}{2}+\frac{8.4^3}{3!}\right)$	M1	Allow M1 with incorrect λ. Accept one end error.
	0.968 (3 sf)	A1	As final answer. **SC** Unsupported correct answer scores B1 only.
		2	
(c)	N(50.4, 50.4)	M1	SOI
	$\frac{39.5-50.4}{\sqrt{50.4}}$ $[=-1.535]$	M1	Allow wrong or no continuity correction. Must have √
	Φ('–1.535') = 1 – Φ('1.535')	M1	For correct probability area consistent with *their* working.
	0.0624 (3 sf) or 0.0623	A1	
		4	

10. 9709_s21_qp_63 Q: 1

The number of goals scored by a team in a match is independent of other matches, and is denoted by the random variable X, which has a Poisson distribution with mean 1.36. A supporter offers to make a donation of \$5 to the team for each goal that they score in the next 10 matches.

Find the expectation and standard deviation of the amount that the supporter will pay. [5]

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Answer:

Question	Answer	Marks	Guidance
	$\lambda = 10 \times 1.36$ [= 13.6]	**M1**	
	E(amount) = 5×13.6 = [$]68	**A1**	
	Var(amount) = $5^2 \times 13.6$ [= 340]	**M1**	$5^2 \times \ldots$
		M1	$\ldots \times$ *their* λ
	Standard Deviation = [$]18.4(4) (3 s.f.)	**A1**	CAO condone $2\sqrt{85}$
		5	

11. 9709_s21_qp_63 Q: 5

Most plants of a certain type have three leaves. However, it is known that, on average, 1 in 10 000 of these plants have four leaves, and plants with four leaves are called 'lucky'. The number of lucky plants in a random sample of 25 000 plants is denoted by X.

(a) State, with a justification, an approximating distribution for X, giving the values of any parameters. [2]

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Use your approximating distribution to answer parts **(b)** and **(c)**.

(b) Find $P(X \leqslant 3)$. [2]

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(c) Given that $P(X = k) = 2P(X = k + 1)$, find k. [2]

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The number of lucky plants in a random sample of n plants, where n is large, is denoted by Y.

(d) Given that $P(Y \geqslant 1) = 0.963$, correct to 3 significant figures, use a suitable approximating distribution to find the value of n. [3]

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Answer:

Question	Answer	Marks	Guidance
(a)	Po(2.5)	**B1**	Accept Poisson with mean = 2.5 not just np = 2.5
	n = 25 000 > 50 and np (or λ) = 2.5 which is < 5 or n = 25 000 > 50 and p = 0.0001 < 0.1	**B1**	Must see 2.5 (or 0.0001) and 25000 OE, not just np < 5 (or p < 0.1) and n > 50
		2	
(b)	$e^{-2.5}(1 + 2.5 + \frac{2.5^2}{2} + \frac{2.5^3}{3!})$	**M1**	Any λ, accept one end error. FT binomial from part **(a)** scores M1 only for equivalent binomial expressions FT normal from part **(a)** must use correct continuity correction and tables scores M1 only for complete method
	0.758 (3 sf)	**A1**	Unsupported answer of 0.758 scores B1 instead of M1A1
		2	
(c)	$e^{-2.5} \times \frac{2.5^k}{(k)!} = 2e^{-2.5} \times \frac{2.5^{k+1}}{(k+1)!}$	**M1**	Any λ FT binomial from **(a)** scores M1 only for equivalent binomial expression FT from **(a)** normal for equivalent expressions continuity correction must be included
	$k = 4$	**A1**	No errors seen **SC** $k = 4$ unsupported scores B1 only, but see full Poisson expressions for P(4) and P(5) and 0.134 scores M1A1
		2	

Question	Answer	Marks	Guidance
(d)	$1 - e^{-\lambda} = 0.963$	**M1**	Accept *their* attempt at λ
	$\lambda = -\ln 0.037$ (= 3.2968 or 3.30 or 3.3)	**M1**	Correct use of lns
	n = 33 000 (3 sf)	**A1**	Allow n = 32 950 to 33 050 (must be an integer) **SC** use of binomial leading to 32 967 scores B1 for $(0.9999)^n = 0.037$ B1 for 33 000 to 3sf (32 967)
		3	

12. 9709_w21_qp_61 Q: 2

The number of enquiries received per day at a customer service desk has a Poisson distribution with mean 45.2. If more than 60 enquiries are received in a day, the customer service desk cannot deal with them all.

Use a suitable approximating distribution to find the probability that, on a randomly chosen day, the customer service desk cannot deal with all the enquiries that are received. [4]

Answer:

Question	Answer	Marks	Guidance
	N(45.2, 45.2)	**B1**	SOI
	$\frac{60.5-45.2}{\sqrt{45.2}}$ [= 2.276]	**M1**	Allow with wrong or no continuity correction.
	1 –ϕ(‘2.276’)	**M1**	
	0.0114	**A1**	
		4	

13. 9709_w21_qp_62 Q: 5

In a certain large document, typing errors occur at random and at a constant mean rate of 0.2 per page.

(a) Find the probability that there are fewer than 3 typing errors in 10 randomly chosen pages. [2]

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(b) Use an approximating distribution to find the probability that there are more than 50 typing errors in 200 randomly chosen pages. [4]

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In the same document, formatting errors occur at random and at a constant mean rate of 0.3 per page.

(c) Find the probability that the total number of typing and formatting errors in 20 randomly chosen pages is between 8 and 11 inclusive. [3]

Answer:

Question	Answer	Marks	Guidance
(a)	$e^{-2}(1 + 2 + \frac{2^2}{2!})$	**M1**	$P(X < 3)$ any λ. Allow one end error.
	0.677 (3sf)	**A1**	Unsupported correct answer scores SC **B1** only.
		2	
(b)	N(40, 40)	**M1**	SOI
	$\frac{50.5 - 40}{\sqrt{40}}$ [= 1.660]	**M1**	For standardising with *their* values. Allow with wrong or no cc must have square root.
	$P(z > \text{'}1.660\text{'}) = 1 - \Phi(\text{'}1.660\text{'})$	**M1**	Correct area consistent with *their* working.
	0.0485 or 0.0484 (3sf)	**A1**	
		4	
(c)	$\lambda = 10$	**B1**	Condone mean = 10.
	$e^{-10}\left(\frac{10^8}{8!} + \frac{10^9}{9!} + \frac{10^{10}}{10!} + \frac{10^{11}}{11!}\right)$	**M1**	Allow any λ (allow one end error).
	0.477 (3sf)	**A1**	Unsupported correct answer scores SC **B2** only.
		3	

14. 9709_w21_qp_63 Q: 2

The number of enquiries received per day at a customer service desk has a Poisson distribution with mean 45.2. If more than 60 enquiries are received in a day, the customer service desk cannot deal with them all.

Use a suitable approximating distribution to find the probability that, on a randomly chosen day, the customer service desk cannot deal with all the enquiries that are received. [4]

Answer:

Question	Answer	Marks	Guidance
	N(45.2, 45.2)	**B1**	SOI
	$\frac{60.5-45.2}{\sqrt{45.2}}$ [= 2.276]	**M1**	Allow with wrong or no continuity correction.
	1 – ϕ('2.276')	**M1**	
	0.0114	**A1**	
		4	

15. 9709_m20_qp_62 Q: 1

The booklets produced by a certain publisher contain, on average, 1 incorrect letter per 30 000 letters, and these errors occur randomly. A randomly chosen booklet from this publisher contains 12 500 letters.

Use a suitable approximating distribution to find the probability that this booklet contains at least 2 errors. [3]

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Answer:

Question	Answer	Marks	Guidance
	$(\lambda =)\, \frac{5}{12} = 0.417$ or better	**B1**	
	$1 - e^{-\frac{5}{12}}(1 + \frac{5}{12})$	**M1**	$1 - P(X = 0$ or $1)$, by Poisson, using any λ, allow $1 - P(X = 0$ or 1 or $2)$ for M1
	$= 0.0661$ or 0.0662 (3 sf)	**A1**	Final answer **SC** use of Binomial (from 0.06607…) B1 only
		3	