Topical Past Paper Questions Worksheets

AS & A Level Mathematics (9709) Paper 4

[Mechanics]

Exam Series: February/March 2017 - October/November 2024

Format Type B:
Each question is followed by its answer scheme

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Introduction

Each Topical Past Paper Questions Workbook contains a comprehensive collection of hundreds of questions and corresponding answer schemes, presented in worksheet format. The questions are carefully arranged according to their respective chapters and topics, which align with the latest IGCSE or AS/A Level subject content. Here are the key features of these resources:

1. The workbook covers a wide range of topics, which are organized according to the latest syllabus content for Cambridge IGCSE or AS/A Level exams.
2. Each topic includes numerous questions, allowing students to practice and reinforce their understanding of key concepts and skills.
3. The questions are accompanied by detailed answer schemes, which provide clear explanations and guidance for students to improve their performance.
4. The workbook's format is user-friendly, with worksheets that are easy to read and navigate.
5. This workbook is an ideal resource for students who want to familiarize themselves with the types of questions that may appear in their exams and to develop their problem-solving and analytical skills.

Overall, Topical Past Paper Questions Workbooks are a valuable tool for students preparing for IGCSE or AS/A Level exams, providing them with the opportunity to practice and refine their knowledge and skills in a structured and comprehensive manner. To provide a clearer description of this book's specifications, here are some key details:

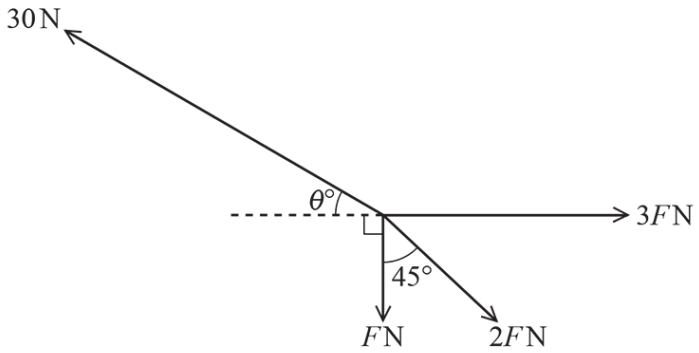
- Title: Cambridge AS & A Level Mathematics (9709) Paper 4 Topical Past Papers
- Subtitle: Exam Practice Worksheets With Answer Scheme
- Examination board: Cambridge Assessment International Education (CAIE)
- Subject code: 9709
- Years covered: February/March 2017 - October/November 2024
- Paper: 4 (Mechanics)
- Number of pages: 1003
- Number of questions: 387

Contents

Chapter 1

Forces and equilibrium

1. 9709_m24_qp_42 Q: 4



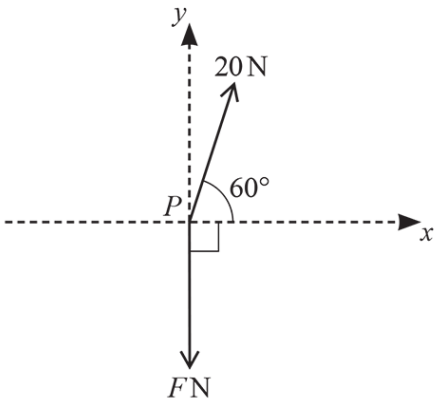
Four coplanar forces act at a point. The magnitudes of the forces are FN, $2F$N, $3F$N and 30 N. The directions of the forces are as shown in the diagram.

Given that the forces are in equilibrium, find the value of F and the value of θ. [6]

Answer:

Question	Answer	Marks	Guidance
	For resolving in either direction	*M1	Correct number of terms allow sign errors; allow sin/cos mix on θ. Forces that need resolving should be resolved.
	$F+2F\cos 45=30\sin\theta$	A1	
	$2F\sin 45+3F=30\cos\theta$	A1	
	$\theta=\tan^{-1}\left(\frac{1+2\cos 45}{2\sin 45+3}\right)$ or $\theta=\sin^{-1}\left(\frac{F+2F\cos 45}{30}\right)$ or $\theta=\cos^{-1}\left(\frac{2F\sin 45+3F}{30}\right)$	DM1	For attempt to find θ. Using their F which can be solved for θ. From equations with correct number of relevant terms, forces that need resolving should be resolved. If $\tan\theta=\frac{\cos\theta}{\sin\theta}$, so have $\theta=\tan^{-1}\left(\frac{2\sin 45+3}{1+2\cos 45}\right)$, then allow M1.
	$\left[F^2(3+2\sin 45)^2+F^2(1+2\cos 45)^2=30^2\Rightarrow\right]$ $F=\sqrt{\frac{30^2}{(3+2\sin 45)^2+(1+2\cos 45)^2}}$ or $F=\frac{30\sin\theta}{1+2\cos 45}$ or $F=\frac{30\cos\theta}{3+2\sin 45}$	DM1	For attempt to find F. From equations with the correct number of relevant terms, forces that need resolving should be resolved. Using *their* θ.
	$F=5.96$ [5.96270…] and $\theta=28.7$ [28.6750…]	A1	Awrt to 5.96 and 28.7. Allow 5.97.
		6	

2. 9709_s24_qp_41 Q: 2



Two forces of magnitudes 20 N and F N act at a point P in the directions shown in the diagram.

(a) Given that the resultant force has no component in the y-direction, calculate the value of F. [2]

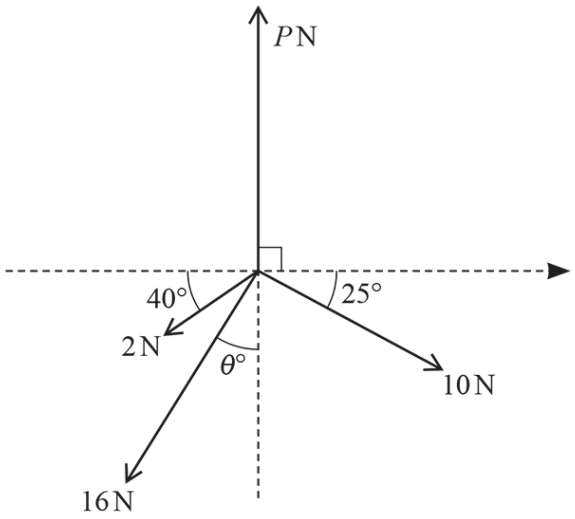
(b) Given instead that $F = 10$, find the magnitude and direction of the resultant force. [5]

Answer:

Question	Answer	Marks	Guidance
(a)	$F = 20\sin 60$	**M1**	Attempt to resolve in y-direction; 2 terms; must be $20\cos 60$ or $20\sin 60$ and must be linked to F (can be implied by the correct answer seen only).
	$= 17.3\,\text{N}$	**A1**	AWRT 17.3 (17.320508…) or $10\sqrt{3}$.
		2	

Question	Answer	Marks	Guidance
(b)	For resolving in any direction	***M1**	Correct number of terms; allow sin/cos mix; allow sign errors.
	(Horizontal component $= X = R\cos\theta) = \pm(20\cos 60)$ [± 10] (Vertical component $= Y = R\sin\theta) = \pm(20\sin 60 - 10)$ [$=\pm 7.3205\ldots$]	**A1**	For both correct.
	Magnitude $= \sqrt{(20\sin 60 - 10)^2 + (20\cos 60)^2}$ [$=12.393136\ldots$]	**DM1**	OE – correct number of terms.
	Angle $= \tan^{-1}\left(\dfrac{20\sin 60 - 10}{20\cos 60}\right)$ [$=36.206023\ldots$]	**DM1**	OE (e.g. reciprocal) - correct number of terms.
	Magnitude = 12.4 N and Direction = 36.2° above (positive) x-axis	**A1**	OE for direction e.g. 36.2° anticlockwise from (positive) x-direction, 36.2° above the horizontal. Possibly seen on a diagram. (Radians: 0.63191… to 3sf or better)
		5	

3. 9709_s24_qp_42 Q: 3



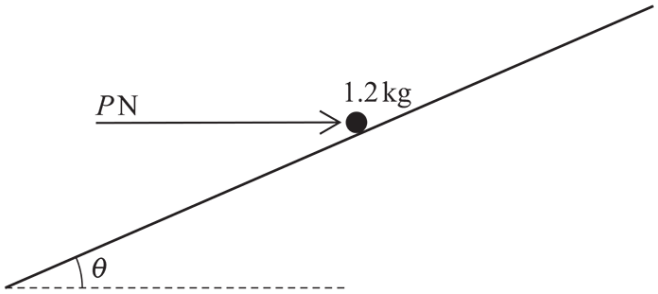
Four coplanar forces of magnitude PN, 10N, 16N and 2N act at a point in the directions shown in the diagram. It is given that the forces are in equilibrium.

Find the values of θ and P. [6]

Answer:

Question	Answer	Marks	Guidance
	Resolving either direction to get an equation	*M1	Correct number of relevant terms; allow sign errors; allow sin/cos mix.
	$10\cos 25 = 2\cos 40 + 16\sin\theta$ $\begin{bmatrix} 9.06307787 = 1.532088886 + 16\sin\theta \\ 7.530988984 = 16\sin\theta \\ \sin\theta = 0.4706868115 \end{bmatrix}$	A1	
	$P = 10\sin 25 + 16\cos\theta + 2\sin 40$ $\begin{bmatrix} P = 4.226182617 + 16\cos\theta + 1.285575219 \\ P = 5.511757837 + 16\cos\theta \end{bmatrix}$	A1	This may be with *their* θ.
	Attempt to solve for $\theta = \sin^{-1}\left(\frac{10\cos 25 - 2\cos 40}{16}\right)$	DM1	From equation(s) with correct number of relevant terms. Must be a numerical expression for θ.
	Attempt to solve for $P = 10\sin 25 + 16\cos(\textit{their}\,\theta) + 2\sin 40$	DM1	From equation(s) with correct number of relevant terms. Using *their* θ. Must be a numerical expression for P.
	$\theta = 28.1$ AND $P = 19.6$	A1	28.07888819 and 19.6285636. AWRT 28.1 and AWRT 19.6 from correct work.
		6	

4. 9709_w24_qp_41 Q: 6



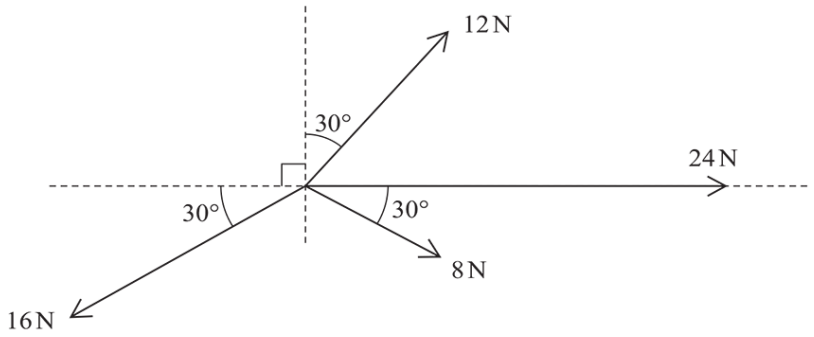
A particle of mass 1.2 kg is placed on a rough plane which is inclined at an angle θ to the horizontal, where $\sin\theta = \frac{7}{25}$. The particle is kept in equilibrium by a horizontal force of magnitude P N acting in a vertical plane containing a line of greatest slope (see diagram). The coefficient of friction between the particle and the plane is 0.15 .

Find the least possible value of P. [6]

Answer:

Question	Answer	Marks	Guidance
	Attempt at resolving perpendicular to the plane to get an equation	*M1	Correct number of relevant terms, allow sign errors, allow sin/cos mix, allow g missing. For reference $R=1.2g\times\cos 16.26...+P\times\sin 16.26...$ - allow with an angle of 16 or better.
	$R=1.2g\times\frac{24}{25}+P\times\frac{7}{25}$	A1	$R=11.52+0.28P$ or $R=\frac{288}{25}+\frac{7}{25}P$.
	Attempt at resolving parallel to the plane to get an equation	*M1	Correct number of relevant terms, allow sign errors, allow sin/cos mix, allow g missing. For reference $F+P\times\cos 16.26...=1.2g\times\sin 16.26...$ allow with an angle of 16 or better.
	$F+P\times\frac{24}{25}=1.2g\times\frac{7}{25}$	A1	$F+0.96P=3.36$ or $F+\frac{24}{25}P=\frac{84}{25}$.
	Use of $F=0.15R$ to get an equation in P only	DM1	Dependent on both previous **M** marks – where R is initially a linear combination of a P component and a weight component (or a mass component). $1.2g\times\frac{7}{25}-P\times\frac{24}{25}=0.15\times\left(1.2g\times\frac{24}{25}+P\times\frac{7}{25}\right)$.
	Solve to get $P=1.63$	A1	Allow $\frac{272}{167}, 1.62874...$
		6	

5. 9709_w24_qp_43 Q: 2



Coplanar forces of magnitudes 16N, 12N, 24N and 8N act at a point in the directions shown in the diagram.

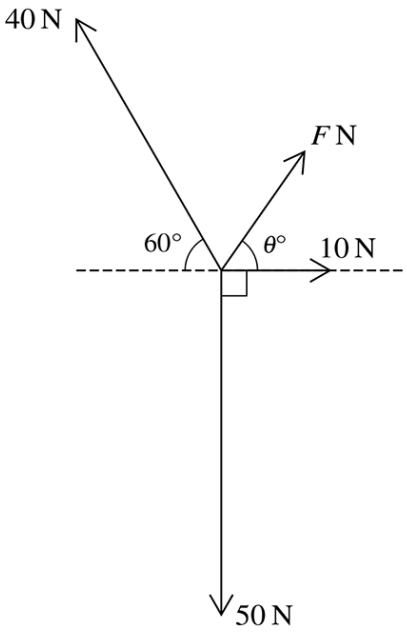
Find the magnitude and direction of the single additional force acting at the same point which will produce equilibrium. [6]

Answer:

Question	Answer	Marks	Guidance
	Resolving either direction	**M1**	With correct number of relevant terms. Allow sin/cos mix. Allow sign errors. Do not allow 'forces to the left = forces to the right' e.g. $12\cos30-8\sin30=16\sin30$ unless subsequently 'corrected'.
	$\pm(12\sin30+24+8\cos30-16\cos30)\left[=F_x \text{ or } F\cos\theta \text{ or } F\sin\alpha\right]$	**A1**	$F_x=\pm\left(30-4\sqrt{3}\right)\quad[=23.07\ldots]$.
	$\pm(12\cos30-8\sin30-16\sin30)\left[=F_y \text{ or } F\sin\theta \text{ or } F\cos\alpha\right]$	**A1**	$F_y=\pm\left(6\sqrt{3}-12\right)\quad[=-1.607\ldots]$.
	$F=\sqrt{\left(6\sqrt{3}-12\right)^2+\left(30-4\sqrt{3}\right)^2}$ $F=\dfrac{30-4\sqrt{3}}{\cos(their\ \theta)}$ $F=\dfrac{6\sqrt{3}-12}{\sin\left(their\ \theta\right)}$	**M1**	Attempt to find F. Must have correct number of relevant terms. (Forces must have or not have components as required). All forces resolved/not resolved as appropriate, but allow consistent sin/cos muddle. Allow use of their θ provided correctly derived from equations with the correct number of relevant terms.
	$\theta=\tan^{-1}\left(\dfrac{6\sqrt{3}-12}{30-4\sqrt{3}}\right)$ $\theta=\cos^{-1}\left(\dfrac{30-4\sqrt{3}}{their\ F}\right)$ Note: this will not give the correct answer unless F given to several significant figures $\theta=\sin^{-1}\left(\dfrac{6\sqrt{3}-12}{their\ F}\right)$	**M1**	Attempt to find θ. Must have correct number of relevant terms. (Forces must have or not have components as required). All forces resolved/not resolved as appropriate, but allow consistent sin/cos muddle. Allow upside down so $\tan^{-1}\left(\dfrac{30-4\sqrt{3}}{6\sqrt{3}-12}\right)$. Allow use of their F provided correctly derived from equations with the correct number of relevant terms. Note: watch for use of $\sin^{-1}\left(\dfrac{6\sqrt{3}-12}{30-4\sqrt{3}}\right)$ or $\sin^{-1}\left(\dfrac{1.607}{23.07}\right)$ which leads to correct answer of angle 4.0° scores M0A0.

Question	Answer	Marks	Guidance
	$F=23.1$ N $\theta=3.99^\circ$ above the negative x-axis oe	**A1**	[23.1277…] Both correct Allow 4.0° but not simply 4° . [3.986...] Allow answers about the direction such as 'Above the west', 'north of west' etc, or clockwise 183.99 from x axis, or resultant sketch with angle indicated. If not specified in working please check original diagram to see if direction specified there instead. Allow a bearing of 274.0°. Allow explanation of direction that could be drawn uniquely. Or e.g. 86.0° to left of the y-axis or 176.0° from the positive x-axis.
		6	

6. 9709_s23_qp_41 Q: 5



Four coplanar forces act at a point. The magnitudes of the forces are F N, 10 N, 50 N and 40 N. The directions of the forces are as shown in the diagram.

(a) Given that the forces are in equilibrium, find the value of F and the value of θ. [6]

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(b) Given instead that $F = 10\sqrt{2}$ and $\theta = 45$, find the direction and the exact magnitude the resultant force. [3]

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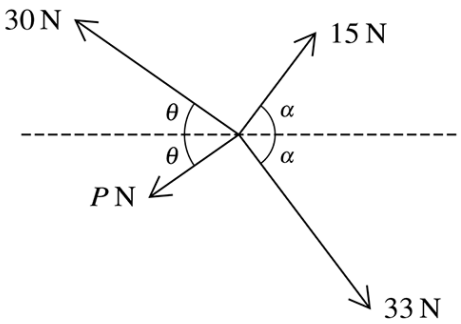
Answer:

Question	Answer	Marks	Guidance
(a)	Resolving either direction.	**M1**	3 terms; allow sign errors and allow sin/cos mix. Must be an equation with either = 0 or with an attempt to balance forces.
	Vertical: $F\sin\theta + 40\sin 60 - 50 = 0$	**A1**	$\left[F\sin\theta = 50 - 20\sqrt{3} = 15.358...\right]$
	Horizontal: $F\cos\theta + 10 - 40\cos 60 = 0$	**A1**	$[F\cos\theta = 10]$
	$\theta = \tan^{-1}(5 - 2\sqrt{3})$	**M1**	Attempt to solve for θ; one missing term in total $\theta = \tan^{-1}1.535898...$.
	$F = \sqrt{15.358...^2 + 10^2}$	**M1**	Attempt to solve for F: one missing term in total.
	$\theta = 56.9,\quad F = 18.3$	**A1**	Both correct (18.327530…, 56.932462…).
		6	
(b)	$(Y =) \pm(10\sqrt{2}\sin 45 + 40\sin 60 - 50)\,[= \pm(20\sqrt{3} - 40)]$	**B1**	Allow non-exact values for $\sqrt{2}$ etc. in correct expression.
	$(X =) \pm(10\sqrt{2}\cos 45 + 10 - 40\cos 60)\,[= 0]$	**B1**	Allow non-exact values for $\sqrt{2}$ etc. in correct expression. Could be implied by correct answer.
	Resultant force is $40 - 20\sqrt{3}$ (N) in the same direction as the 50 (N) force.	**B1**	Allow vertically downwards, south, 180°, negative y-direction. Resultant force must be exact and positive (so $20\sqrt{3} - 40$ is B0).
		3	

7. 9709_s23_qp_42 Q: 3



Coplanar forces of magnitudes 30 N, 15 N, 33 N and P N act at a point in the directions shown in the diagram, where $\tan\alpha = \frac{4}{3}$. The system is in equilibrium.

(a) Show that $\left(\frac{14.4}{30-P}\right)^2 + \left(\frac{28.8}{P+30}\right)^2 = 1$. [4]

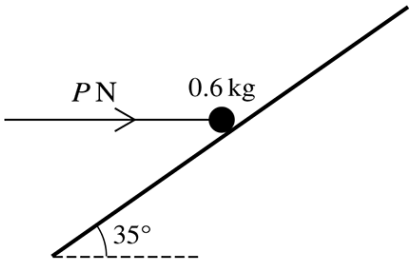
(b) Verify that $P = 6$ satisfies this equation and find the value of θ. [2]

Answer:

Question	Answer	Marks	Guidance
(a)	Resolving either direction.	**M1**	Correct number of terms, allow sign errors, allow sin/cos mix. Do not allow with just $\sin\alpha$ and $\cos\alpha$.
	$(33+15)\times\frac{3}{5}=P\cos\theta+30\cos\theta$ OR $(33+15)\cos\left(\tan^{-1}\frac{4}{3}\right)=P\cos\theta+30\cos\theta$ OR $19.8+9=P\cos\theta+30\cos\theta$	**A1**	OE, but see note for final A1. Allow: $28.8=(P+30)\cos\theta$ $(33+15)\cos 53(.1)=P\cos\theta+30\cos\theta$ $19.81+9.01=P\cos\theta+30\cos\theta$ $19.86+9.03=P\cos\theta+30\cos\theta$.
	$15\times\frac{4}{5}+30\sin\theta=33\times\frac{4}{5}+P\sin\theta$ OR $15\sin\left(\tan^{-1}\frac{4}{3}\right)+30\sin\theta=33\sin\left(\tan^{-1}\frac{4}{3}\right)+P\sin\theta$ OR $12+30\sin\theta=26.4+P\sin\theta$	**A1**	OE, but see note for final A1. Allow: $14.4=(30-P)\sin\theta$ $15\sin 53(.1)+30\sin\theta=33\sin 53(.1)+P\sin\theta$ $12.00+30\sin\theta=26.39+P\sin\theta$ $11.98+30\sin\theta=26.35+P\sin\theta$.
	[Use $\cos^2\theta+\sin^2\theta=1$ with] $\cos\theta=\frac{28.8}{P+30}$ and $\sin\theta=\frac{14.4}{30-P}$ to get $\left(\frac{14.4}{30-P}\right)^2+\left(\frac{28.8}{P+30}\right)^2=1$	**A1**	AG. Must have evidence of where 28.8 and 14.4 come from. A0 for any error seen. A0 if use of inexact angles seen. Any inexact decimals seen for force components, i.e. if 14.4 and/or 28.8 have come from rounding to 3sf, scores M1A1A1A0 max 3/4. If exact values of $\sin\alpha$ and $\cos\alpha$ not shown (e.g. $28.8=(P+30)\cos\theta$ or $14.4=(30-P)\sin\theta$ from no working), this scores M1A1A1A0 max 3/4 marks.
		4	

Question	Answer	Marks	Guidance
(b)	Sub $P=6$ into $\left(\frac{14.4}{30-P}\right)^2+\left(\frac{28.8}{P+30}\right)^2$ to get $\left[\left(\frac{14.4}{24}\right)^2+\left(\frac{28.8}{36}\right)^2\right]=\left(\frac{3}{5}\right)^2+\left(\frac{4}{5}\right)^2=0.36+0.64=1$	**B1**	Must see either $\left(\frac{3}{5}\right)^2+\left(\frac{4}{5}\right)^2=1$ or $0.36+0.64=1$ as minimum working.
	$\theta=36.9$	**B1**	AWRT 36.9 .
		2	

8. 9709_s23_qp_42 Q: 5



A particle of mass 0.6 kg is placed on a rough plane which is inclined at an angle of 35° to the horizontal. The particle is kept in equilibrium by a horizontal force of magnitude P N acting in a vertical plane containing a line of greatest slope (see diagram). The coefficient of friction between the particle and plane is 0.4.

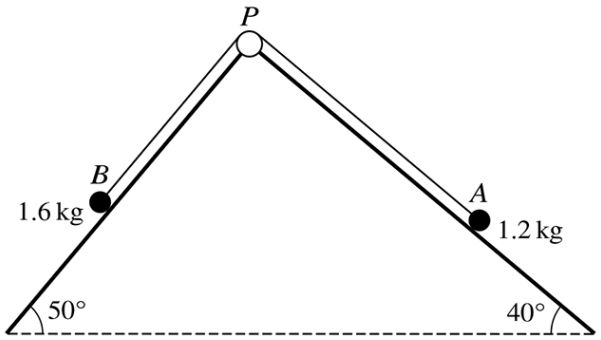
Find the least possible value of P. [6]

Answer:

Question	Answer	Marks	Guidance
	Attempt at resolving parallel or perpendicular to the plane.	*M1	3 terms, allow sign errors, allow sin/cos mix, allow g missing. Forces that need resolving should be resolved, forces that do not need resolving should not be resolved.
	$R = P\sin 35 + 0.6g\cos 35$ $\left[R = (0.573\ldots)P + 4.914\ldots\right]$	A1	
	$F + P\cos 35 = 0.6g\sin 35$ $\left[F + (0.819\ldots)P = 3.441\ldots\right]$	A1	Their F.
	Use of $F = 0.4R$	*M1	Where R is initially a linear combination of a P component and a weight component (or a mass component).
	Solve for P.	DM1	From equations with the correct number of relevant resolved terms. $R = \dfrac{0.6g}{\cos 35 + 0.4\sin 35} = 5.7222$. Must get to P = …, e.g. $P = \dfrac{0.6g\sin 35 - 0.4\times 0.6g\cos 35}{\cos 35 + 0.4\sin 35}$ If no working seen, allow this mark if correct solution for their equations. If $F \leqslant 0.4R$ used, it should be used correctly. e.g. $0.6g\sin 35 - P\cos 35 \leqslant 0.4(P\sin 35 + 0.6g\cos 35)$.
	$P = 1.41$	A1	AWRT 1.41 . If $P \geqslant 1.41$ seen, must then state the least value explicitly for A1.

Question	Answer	Marks	Guidance
	Alternative for Question 5: Resolving vertically and horizontally		
	Attempt at resolving vertically or horizontally.	*M1	3 terms, allow sign errors, allow sin/cos mix, allow g missing. Forces that need resolving should be resolved, forces that do not need resolving should not be resolved.
	$R\cos 35 + F\sin 35 = 0.6g$	A1	*Their* F or R.
	$P + F\cos 35 = R\sin 35$	A1	*Their* F or R.
	Use of $F = 0.4R$	*M1	To get 2 equations, one in R (or F) and the other in P and R (or P and F) from resolved equations with correct number of relevant terms. Allow g missing.
	Solve for P	DM1	From equations with the correct number of relevant resolved terms. May see $R = \dfrac{0.6g}{\cos 35 + 0.4\sin 35} = 5.7222$; Must get to P = …, e.g. $P = \dfrac{0.6g\sin 35 - 0.4\times 0.6g\cos 35}{\cos 35 + 0.4\sin 35}$. If no working seen, allow this mark if correct solution for *their* equations.
	$P = 1.41$	A1	AWRT 1.41 .
		6	

9. 9709_w23_qp_41 Q: 5



The diagram shows a particle A, of mass 1.2 kg, which lies on a plane inclined at an angle of 40° to the horizontal and a particle B, of mass 1.6 kg, which lies on a plane inclined at an angle of 50° to the horizontal. The particles are connected by a light inextensible string which passes over a small smooth pulley P fixed at the top of the planes. The parts AP and BP of the string are taut and parallel to lines of greatest slope of the respective planes. The two planes are rough, with the same coefficient of friction, μ, between the particles and the planes.

Find the value of μ for which the system is in limiting equilibrium. [7]

Answer:

Question	Answer	Marks	Guidance
	Resolving parallel to the slope at A or B to form an equation.	***M1**	Correct number of terms; allow sign errors; allow sin/cos mix.
	$1.6g\sin 50 - T - F_B = 0$	**A1**	If using the same Fs, then M1A1A0B1 max.
	$T - F_A - 1.2g\sin 40 = 0$	**A1**	System equation (must be four different terms): $1.6g\sin 50 - F_B - F_A - 1.2g\sin 40 = 0$ only scores M1A1A1. Any sign errors scores M1 only.
	$R_A = 1.2g\cos 40$ or $R_B = 1.6g\cos 50$	***B1**	Either correct. Must be explicitly linked to the correct contact (so could be seen on a diagram), or as part of a resolving parallel to the slope equation(s) (so must be combined with μ).
	$F_A = 1.2g\mu\cos 40$ or $F_B = 1.6g\mu\cos 50$	***M1**	Use of $F = \mu R$ at either A or B. Must be explicitly linked to the correct contact (could be seen on a diagram) or as part of a resolving parallel to the slope equation(s). Allow sin/cos mix error only.
	$1.6g\sin 50 - 1.6g\mu\cos 50 = 1.2g\sin 40 + 1.2g\mu\cos 40$	**DM1**	Eliminating T, F_A and F_B to form an equation in μ only.
	$\left[\mu = \dfrac{1.6g\sin 50 - 1.2g\sin 40}{1.2g\cos 40 + 1.6g\cos 50} \Rightarrow\right] \mu = 0.233$	**A1**	0.23326119…
		7	

10. 9709_w23_qp_42 Q: 4

A particle P of mass 0.2 kg lies at rest on a rough horizontal plane. A horizontal force of 1.2 N is applied to P.

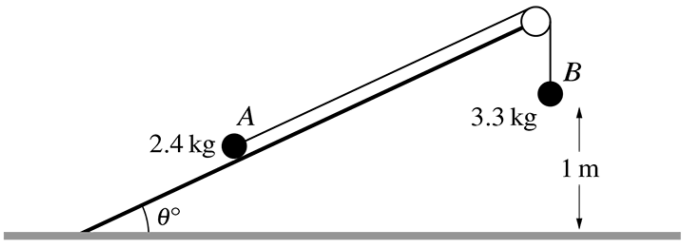
(a) Given that P is in limiting equilibrium, find the coefficient of friction between P and the plane. [3]

(b) Given instead that the coefficient of friction between P and the plane is 0.3, find the distance travelled by P in the third second of its motion. [4]

Answer:

Question	Answer	Marks	Guidance
(a)	$R = 0.2g$	**B1**	
	$1.2 = \mu \times 0.2g$	**M1**	Resolve horizontally and using $F = \mu R$ to get an equation in μ; 2 relevant terms.
	$\mu = 0.6$	**A1**	oe
		3	
(b)	$1.2 - 0.3 \times 0.2g = 0.2a$	***M1**	Resolve horizontally using Newton's Second Law; 3 relevant terms; allow sign errors; $R = 0.2g$ only.
	$a = 3$	**A1**	$0.6 = 0.2a$ only seen, allow with BOD, but if 0.6 as friction being used as resultant force, this is M0A0.
	$s_3 = 0 + \frac{1}{2} \times 3 \times 3^2 \left[= 13.5 \right] s_2 = 0 + \frac{1}{2} \times 3 \times 2^2 \left[= 6 \right]$	**DM1**	For use of $s = ut + \frac{1}{2}at^2$ (or a complete method) to find a distance at least once with $u = 0$ and their positive a and $t = 2$ or $t = 3$.
	Distance = $13.5 - 6 = 7.5$ m	**A1**	www
		4	

11. 9709_w23_qp_43 Q: 7



Particles A and B, of masses 2.4 kg and 3.3 kg respectively, are connected by a light inextensible string that passes over a smooth pulley which is fixed to the top of a rough plane. The plane makes an angle of $\theta°$ with horizontal ground. Particle A is on the plane and the section of the string between A and the pulley is parallel to a line of greatest slope of the plane. Particle B hangs vertically below the pulley and is 1 m above the ground (see diagram). The coefficient of friction between the plane and A is μ.

(a) It is given that $\theta = 30$ and the system is in equilibrium with A on the point of moving directly up the plane.

Show that $\mu = 1.01$ correct to 3 significant figures. [5]

(b) It is given instead that $\theta = 20$ and $\mu = 1.01$. The system is released from rest with the string taut.

Find the total distance travelled by A before coming to instantaneous rest. You may assume that A does not reach the pulley and that B remains at rest after it hits the ground. [8]

Answer:

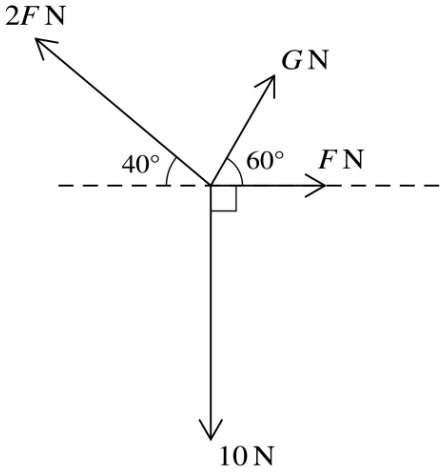
Question	Answer	Marks	Guidance
(a)	Resolving for both particles or for the system to form equation(s)	**M1***	Must have correct number of terms. Allow sign errors. Allow sin/cos mix. Allow g missing. M0 if acceleration included unless subsequently equated to zero. Masses must be appropriate for their equation(s). Forces must have components (or not) as required.
	Either $T - F - 2.4g\sin 30 = 0$ AND $3.3g - T = 0$ Or $3.3g - F - 2.4g\sin 30 = 0$	**A1**	Both correct or system correct. May get $F = 21$. Can be with a wrong non-zero F.
	$R = 2.4g\cos 30$ $\left[= 12\sqrt{3} = 20.7846\ldots\right]$	**B1**	
	Use of $F = \mu R$ to get an equation in μ only $\left[3.3g - 2.4g\mu\cos 30 - 2.4g\sin 30 = 0\right]$	**DM1**	Must be from F dimensionally correct and single term R which is equal to a component the 2.4 kg weight. Allow consistent sin/cos mix but must be different components of weight. F and R must be numerical expressions.
	$\mu = 1.01$ [sight of 1.01036… or 1.0104]	**A1**	AG perhaps from one of $\mu = \frac{3.3g - 24\sin 30}{2.4g\cos 30} = \frac{33 - 12}{12\sqrt{3}} = \frac{21}{12\sqrt{3}} = \frac{7\sqrt{3}}{12} = \frac{21}{20.7846\ldots}$ $= \frac{21}{20.8}$ Do not allow unless evidence of 30 substituted for θ. E.g.: sight of 1.01036… or 1.0104.
		5	

Question	Answer	Marks	Guidance
(b)	Using Newton's second law for both particles or the system	**M1***	Must have correct number of terms. Allow sign errors. Allow sin/cos mix. Allow g missing. Masses must be appropriate for their equation(s). Forces must have components (or not) as required.
	Either $3.3g - T = 3.3a$ and $T - F - 2.4g\sin 20 = 2.4a$ $\left[T - 22.778 - 8.208 = 2.4a\right]$ or $\left[T - 30.986 = 2.4a\right]$ or $3.3g - F - 2.4g\sin 20 = (2.4 + 3.3)a$ $\left[2.013367\ldots = 5.7a\right]$	**A1**	Both correct or system equation correct. Can be with a wrong non-zero F.
	$F = 1.01 \times 2.4g\cos 20$ $\left[= 22.778\right]$	**B1**	For correct expression for F.
	Attempt to solve for a $a = 0.353$ [0.353222…]	**DM1**	Using their F Must get to 'a ='. If sin/cos mix must be consistent.
	$v^2 = 2 \times 0.353 \times 1$ [= 0.706444…] or $\left[v = 0.841\right]$ Or $1 = 0 + \frac{1}{2} \times 0.353t^2 \Rightarrow t = 2.3795 \Rightarrow v = 0.353 \times 2.38$	**A1FT**	FT their value of $a \neq \pm g$ to get an expression for v^2 or v. Can be implied by awrt 0.84 for v or awrt 0.71 for v^2. This mark does not depend on previous A or B mark, but both Ms must have been awarded.
	Using Newton's second law on A after B reaches the ground $-F - 2.4g\sin 20 = 2.4a$ $\left[-1.01 \times 2.4g\cos 20 - 2.4g\sin 20 = 2.4a\right]$ $\left[-22.78814\ldots - 8.20848\ldots = 2.4a\right]$	**M1***	Must have correct number of terms. Allow sign errors. Allow sin/cos mix. Allow g missing. $\left[\Rightarrow a = -12.911\ldots\right]$
	Use of suvat to find s $\left[0 = their\,0.841^2 + 2 \times their - 12.911\ldots \times s \Rightarrow s = 0.027358\ldots\right]$	**DM1**	Using their $a \neq \pm g$. Must get to 's ='. May find and use $t = 0.0651$.

Question	Answer	Marks	Guidance
(b)	Total distance = 1.03 m	**A1**	
		8	
	Alternative method using energy for first 5 marks		
	$[\text{KE gained} =]\frac{1}{2}\times(2.4+3.3)v^2\,[=2.85v^2]$	**B1**	
	$[\text{PE lost} =]3.3g\times1-2.4g\times1\sin20\,[=24.791\ldots]$ =	**B1**	Allow omission of 1 in either or both terms.
	[Friction =] $1.01\times2.4g\cos20\,[=22.778\ldots]$	**B1**	For correct expression for F.
	$\frac{1}{2}\times(2.4+3.3)v^2=3.3g\times1-2.4g\times1\sin20-1.01\times2.4g\cos20\times1$ Or $2.85v^2=24.791\ldots-22.778\ldots$	**M1**	For attempt at energy equation. Allow sign errors, allow sin/cos mix but must have sin/cos where needed. Correct number of terms, dimensionally correct. Allow omission of 1 in any or all the three relevant terms. Must have cos 20 and sin 20.
	To get a correct expression for v^2 $[v^2=0.706444\ldots$ or $v=0.841\ldots]$	**A1**	Can be implied by awrt 0.84 for v or awrt 0.71 for v^2 if expression not seen.

Question	Answer	Marks	Guidance
(b)	**Alternative method using energy for final 3 marks**		
	KE = $\frac{1}{2}\times2.4\times0.841^2$	**M1**	Using their v^2.
	$1.01\times2.4g\cos20\times s+2.4g\sin20\times s=\frac{1}{2}\times2.4\times0.841^2$ $[\Rightarrow s=0.027358..]$	**M1**	For attempt at 3 term energy equation and solved to get to 's ='. Allow sign errors, allow consistent sin/cos mix but must have sin/cos where needed. Correct number of terms, dimensionally correct.
	Total distance = 1.03 m	**A1**	

12. 9709_m22_qp_42 Q: 5



Four coplanar forces act at a point. The magnitudes of the forces are 10 N, F N, G N and $2F$ N. The directions of the forces are as shown in the diagram.

(a) Given that the forces are in equilibrium, find the values of F and G. [5]

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(b) Given instead that $F = 3$, find the value of G for which the resultant of the forces is perpendicular to the 10 N force. [2]

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Answer:

Question	Answer	Marks	Guidance
(a)	Attempt to resolve vertically or horizontally	**M1**	Correct number of terms.
	$G \sin 60° + 2F \sin 40° - 10 = 0$	**A1**	Correct resolution vertically.
	$F + G \cos 60° - 2F \cos 40° = 0$	**A1**	Correct resolution horizontally.
	Attempt to solve simultaneously for F or G	**M1**	From equations with 3 relevant terms in each
	$F = 4.53$, $G = 4.82$	**A1**	For both correct.
		5	
(b)	$G \sin 60° + 2 \times 3 \sin 40° - 10 = 0$	**M1**	Resolve forces parallel to the 10 N force and equate this expression to zero, 3 terms.
	$G = 7.09$ to 3 sf	**A1**	
		2	

13. 9709_s22_qp_41 Q: 3

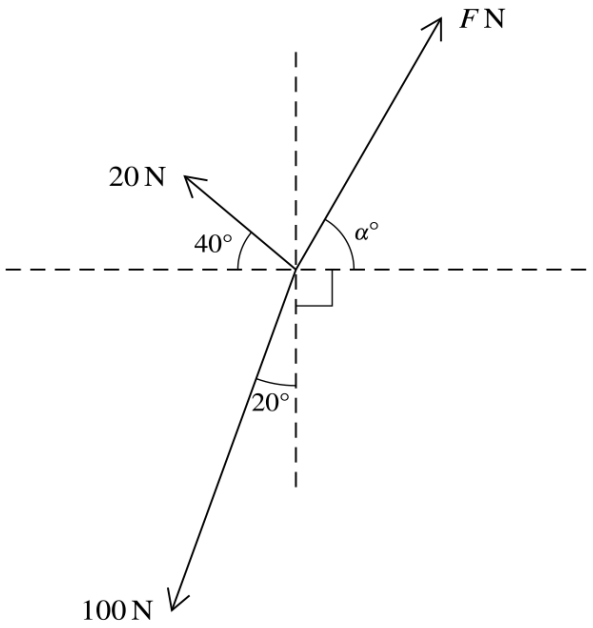
A crate of mass 300 kg is at rest on rough horizontal ground. The coefficient of friction between the crate and the ground is 0.5. A force of magnitude X N, acting at an angle α above the horizontal, is applied to the crate, where $\sin\alpha = 0.28$.

Find the greatest value of X for which the crate remains at rest. [5]

Answer:

Question	Answer	Marks	Guidance
	For attempt at resolving horizontally or vertically	**M1**	Allow sin/cos mix. Allow sign error. Allow g missing. Correct number of terms.
	$R = 300g - 0.28X$ or $R = 300g - X\sin 16.3$	**A1**	$\alpha = 16.26\ldots$
	$0.96X - F = 0$ or $0.96X - 0.5(300g - X\sin\alpha) = 0$ Or $X\cos 16.3 - F = 0$ or $X\cos 16.3 - 0.5(300g - X\sin\alpha) = 0$	**A1**	Or using *their* F
	Use of $F = 0.5R$	**M1**	Use to get an equation in X only. Allow sin/cos mix. Allow sign error. Allow g missing. Must be from 2 term R, which is a linear combination of $300(g)$ and a component of X
	$X = 1360$ [1363.63…]	**A1**	
		5	

14. 9709_s22_qp_41 Q: 4



Three coplanar forces of magnitudes 20 N, 100 N and F N act at a point. The directions of these forces are shown in the diagram.

Given that the three forces are in equilibrium, find F and α. [6]

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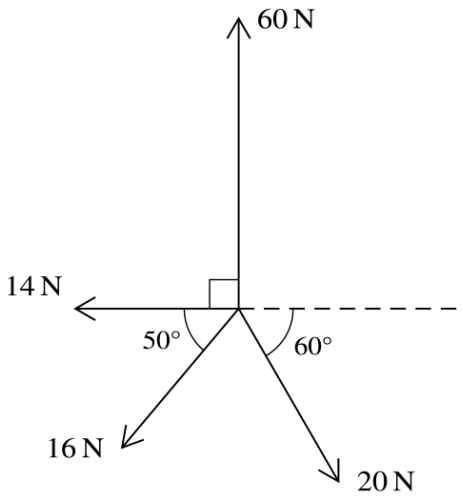
Answer:

Question	Answer	Marks	Guidance
	Attempt to resolve in any direction	**M1**	For resolving. Allow sin/cos mix. Allow sign error. Correct number of terms.
	$F\cos\alpha - 20\cos 40 - 100\sin 20 = 0$ [$F\cos\alpha = 15.320\ldots + 34.202\ldots = 49.5229\ldots$]	**A1**	
	$F\sin\alpha + 20\sin 40 - 100\cos 20 = 0$ [$F\sin\alpha = 93.969\ldots - 12.855\ldots = 81.1135\ldots$]	**A1**	
	$F = \sqrt{(49.5229\ldots)^2 + (81.1135\ldots)^2}$	**M1**	OE; Attempt to solve for F; one term missing in total
	$\alpha = \tan^{-1}\left(\frac{81.1135\ldots}{49.5229\ldots}\right)$	**M1**	OE; Attempt to solve for α; one term missing in total
	$F = 95(.0)$, $\alpha = 58.6$	**A1**	$F = 95.0364\ldots$ and $\alpha = 58.5943\ldots$
	Alternative mark scheme for question 4: For candidates who use cosine and/or sine rule		
	Attempt at cosine rule from triangle of forces	**M1**	Must use lengths 100 and 20 with a suitable angle
	$F^2 = 100^2 + 20^2 - 2\times 100\times 20\cos 70$	**A1**	Correct
	$F = 95[.0]$	**A1**	
	$\frac{95.0364}{\sin 70} = \frac{20}{\sin\beta}$ OR $\frac{95.0364}{\sin 70} = \frac{100}{\sin\gamma}$	**M1**	Attempt at sin rule
		A1	where $\beta = (70-\alpha)$ where $\gamma = (40+\alpha)$
	$\alpha = 58.6$	**A1**	$\alpha = 58.5943\ldots$

Question	Answer	Marks	Guidance
	Alternative mark scheme for question 4: For candidates who resolve in other directions		
	Attempt to resolve (e.g. parallel or perpendicular to 100 N)	**M1**	For resolving. Allow sin/cos mix. Allow sign error. Correct number of terms.
	$F\sin(\alpha+20) + 20\sin 20 - 100 = 0$ $\left[F\sin(\alpha+20) = 93.159\ldots\right]$	**A1**	
	$F\cos(\alpha+20) - 20\cos 20 = 0$ $\left[F\cos(\alpha+20) = 18.793\ldots\right]$	**A1**	
	$F = \sqrt{93.159\ldots^2 + 18.793^2}$	**M1**	OE; Attempt to solve for F; one term missing in total
	$\alpha = \tan^{-1}\left(\frac{93.159\ldots}{18.793\ldots}\right) - 20$	**M1**	OE; Attempt to solve for α; one term missing in total
	$F = 95[.0]$, $\alpha = 58.6$	**A1**	$F = 95.0364\ldots$ and $\alpha = 58.5943\ldots$
		6	

15. 9709_s22_qp_42 Q: 2



Coplanar forces of magnitudes 60 N, 20 N, 16 N and 14 N act at a point in the directions shown in the diagram.

Find the magnitude and direction of the resultant force. [6]

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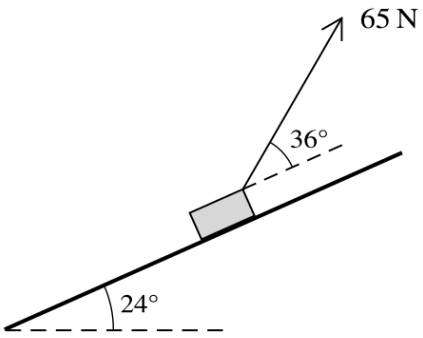
Answer:

Question	Answer	Marks	Guidance
	Resolving either direction	**M1**	3 terms; allow sign errors and allow sin/cos mix
	$(X=)\pm(20\cos 60-14-16\cos 50)\quad[=\mp 14.2846\ldots\ldots]$	**A1**	
	$(Y=)\pm(60-20\sin 60-16\sin 50)\quad[=\pm 30.42278\ldots\ldots]$	**A1**	
	$R=\sqrt{(14.2846\ldots)^2+(30.42278\ldots)^2}$	**M1**	Attempt to solve for R; one missing term in total
	$\theta=\tan^{-1}\left(\frac{30.42278\ldots}{14.2846\ldots}\right)\left[=\tan^{-1}(2.1297\ldots\ldots)\right]$ OR $\alpha=\tan^{-1}\left(\frac{14.2846\ldots}{30.42278\ldots}\right)\left[=\tan^{-1}(0.4596\ldots\ldots)\right]$	**M1**	Attempt to solve for θ or α; one missing term in total
	$R=33.6$ N Direction is 64.8° above the 14 N force or 25.2° above the negative x-axis or 25.2° left of the 60 N force or bearing 335° or 115° anticlockwise from the positive x-axis	**A1**	Both correct. OE; allow 64.9, 25.1 Giving an angle only is insufficient. Direction may be seen on a diagram, with minimum of arrow on resultant. Arrows on both components only is A0 as it doesn't show the direction of the resultant. However the direction is stated, it must be able to be drawn uniquely.
		6	

16. 9709_s22_qp_42 Q: 5



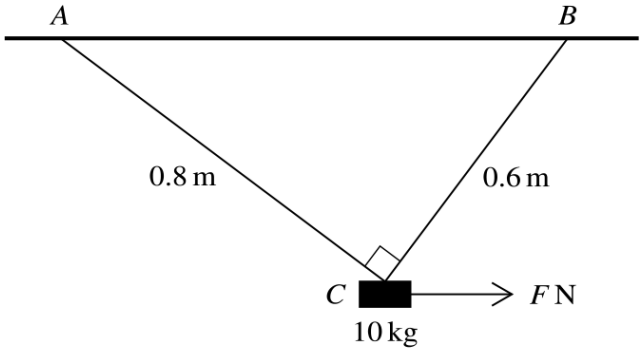
A block of mass 12 kg is placed on a plane which is inclined at an angle of 24° to the horizontal. A light string, making an angle of 36° above a line of greatest slope, is attached to the block. The tension in the string is 65 N (see diagram). The coefficient of friction between the block and plane is μ. The block is in limiting equilibrium and is on the point of sliding up the plane.

Find μ. [6]

Answer:

Question	Answer	Marks	Guidance
	Attempt at resolving parallel to the plane	*M1	3 terms. Allow sign errors, sin/cos mix. Allow g missing, otherwise dimensionally correct.
	$65\cos 36 = 12g \times \sin 24 + F$	A1	$F = 3.777707\ldots\ldots$
	Attempt at resolving perpendicular to the plane	*M1	3 terms. Allow sign errors, sin/cos mix. Allow g missing, otherwise dimensionally correct.
	$12g \times \cos 24 = R + 65\sin 36$	A1	$R = 71.419\ldots\ldots$
	Use $F = \mu R$ $\left[\mu = \frac{65\cos 36 - 12g \times \sin 24}{12g \times \cos 24 - 65\sin 36} = \frac{52.586 - 48.808}{109.625 - 38.206} = \frac{3.777\ldots}{71.419\ldots}\right]$	DM1	To get an equation in μ only. Dependent on two previous M marks. Allow g missing
	$\mu = 0.0529$	A1	Allow AWRT 0.053 Do not accept fractional equivalent.
		6	

17. 9709_s22_qp_43 Q: 4



The diagram shows a block of mass 10 kg suspended below a horizontal ceiling by two strings AC and BC, of lengths 0.8 m and 0.6 m respectively, attached to fixed points on the ceiling. Angle $ACB = 90°$. There is a horizontal force of magnitude F N acting on the block. The block is in equilibrium.

(a) In the case where $F = 20$, find the tensions in each of the strings. [5]

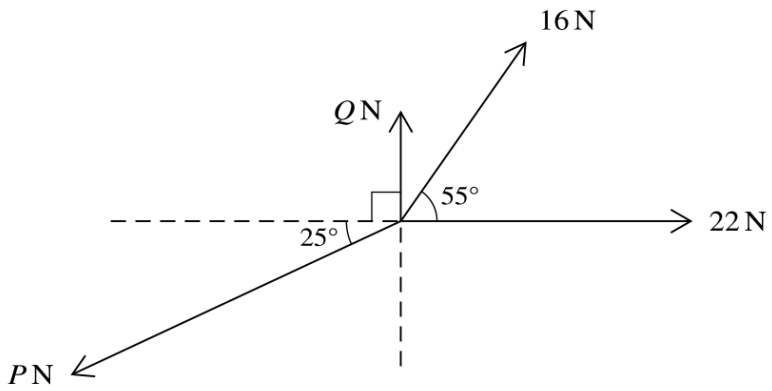
(b) Find the greatest value of F for which the block remains in equilibrium in the position shown. [3]

Answer:

Question	Answer	Marks	Guidance
(a)	$T_A \times 0.8 - T_B \times 0.6 - 20 = 0$ or $T_A \times 0.6 + T_B \times 0.8 - 10g = 0$	**M1**	Resolving horizontally or vertically
	$T_A \times 0.8 - T_B \times 0.6 - 20 = 0$	**A1**	
	$T_A \times 0.6 + T_B \times 0.8 - 10g = 0$	**A1**	
	$0.8T_A - \frac{0.6(10g - 0.6T_A)}{0.8} = 20 \quad \rightarrow \quad T_A = \ldots$	**M1**	Attempt to solve simultaneously
	$T_A = 76$ N, $T_B = 68$ N	**A1**	
		5	

Question	Answer	Marks	Guidance
(b)	$T_A \times 0.6 - 10g = 0 \Rightarrow T_A = \frac{500}{3}$	**B1**	From using $T_B = 0$
	$T_A \times 0.8 - F = 0$	**M1**	
	$F = \frac{400}{3}$	**A1**	Allow $F = 133$ to 3 s.f.
		3	

18. 9709_w22_qp_41 Q: 1



Coplanar forces of magnitudes PN, QN, 16N and 22N act at a point in the directions shown in the diagram. The forces are in equilibrium.

Find the values of P and Q. [5]

Answer:

Question	Answer	Marks	Guidance
	Attempt at resolving horizontally or vertically	**M1**	Allow sign errors, allow sin/cos mix. 3 terms.
	$P\cos 25 = 22 + 16\cos 55$	**A1**	
	$Q + 16\sin 55 = P\sin 25$	**A1**	Allow *their P*.
	Attempt to solve for P or Q	**M1**	No missing/extra terms.
	$P = 34.4 \quad Q = 1.43$	**A1**	$P = 34.40025941,\ Q = 1.431745128$.
		5	