Topical Past Papers

AS & A Level Mathematics (9709) Paper 3

[Pure Mathematics 3]

Exam Series: February/March 2018 – May/June 2025

Format Type B:
Each question is followed by its answer scheme

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Introduction

Each Topical Past Paper Questions Workbook contains a comprehensive collection of hundreds of questions and corresponding answer schemes, presented in worksheet format. The questions are carefully arranged according to their respective chapters and topics, which align with the latest IGCSE or AS/A Level subject content. Here are the key features of these resources:

1. The workbook covers a wide range of topics, which are organized according to the latest syllabus content for Cambridge IGCSE or AS/A Level exams.
2. Each topic includes numerous questions, allowing students to practice and reinforce their understanding of key concepts and skills.
3. The questions are accompanied by detailed answer schemes, which provide clear explanations and guidance for students to improve their performance.
4. The workbook's format is user-friendly, with worksheets that are easy to read and navigate.
5. This workbook is an ideal resource for students who want to familiarize themselves with the types of questions that may appear in their exams and to develop their problem-solving and analytical skills.

Overall, Topical Past Paper Questions Workbooks are a valuable tool for students preparing for IGCSE or AS/A Level exams, providing them with the opportunity to practice and refine their knowledge and skills in a structured and comprehensive manner. To provide a clearer description of this book's specifications, here are some key details:

- Title: Cambridge AS & A Level Mathematics (9709) Paper 3 Topical Past Papers
- Subtitle: Exam Practice Worksheets With Answer Scheme
- Examination board: Cambridge Assessment International Education (CAIE)
- Subject code: 9709
- Years covered: February/March 2018 – May/June 2025
- Paper: 3
- Number of pages: 1392
- Number of questions: 532

Contents

Chapter 1

Algebra

1. 9709_m25_qp_32 Q: 9

The polynomial $6x^3 + ax^2 + bx + 9$ is denoted by $p(x)$, where a and b are constants. It is given that $(x-3)$ is a factor of $p(x)$, and when the first derivative $p'(x)$ is divided by $(x-3)$ the remainder is 72.

(a) Find the values of a and b. [5]

(b) When a and b have the values found in part **(a)**, factorise $p(x)$ completely. [3]

(c) Hence solve the inequality $p(x) < 0$. [2]

Answer:

Question	Answer	Marks	Guidance
(a)	Substitute $x = 3$ or -3 into $p(x)$ and equate to 0 or into $p'(x)$ and equate to 72	**M1***	
	Obtain $162 + 9a + 3b + 9 = 0$	**A1**	OE
	Obtain $162 + 6a + b = 72$	**A1**	OE
	Solve simultaneous equations to obtain either a or b after using $p(\pm 3) = 0$ and $p'(\pm 3) = 72$	**DM1**	
	Obtain $a = -11$ and $b = -24$	**A1**	
		5	

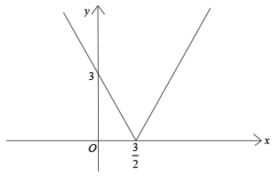
Question	Answer	Marks	Guidance
(b)	Equate $(x-3)(6x^2 + Ax + B)$ to $6x^3 - 11x^2 - 24x + 9$ and obtain equations to solve for A and B or divide $6x^3 - 11x^2 - 24x + 9$ by $x - 3$ and reach $6x^2 \pm 7x$	**M1**	Using *their a* and *their b*. $A - 18 = a = -11$, $B - 3A = -b = -24$, $-3B = 9$. $A = 7$ and $B = -3$. Or reach $6x^2 \pm (\textit{their } a + 18)x$.
	$(x-3)(6x^2 + 7x - 3)$	**A1**	SOI
	Obtain $(x-3)(2x+3)(3x-1)$	**A1**	
			Special Case: If only $(x-3)\left(x+\frac{3}{2}\right)\left(x-\frac{1}{3}\right)$ or $(x-3)(2x+3)(3x-1)$ seen, **SC B1** only (but can gain two marks in **(c)**).
		3	
(c)	Obtain one correct region $x < -\frac{3}{2}$ or $\frac{1}{3} < x < 3$	**B1 FT**	Must be final answer not in working. FT is on the last two brackets (not $(x-3)$).
	Obtain both regions $x < -\frac{3}{2}$, $\frac{1}{3} < x < 3$	**B1 FT**	Allow $x < -\frac{3}{2}$ and $\frac{1}{3} < x < 3$. **SC B1** for $x \leqslant -\frac{3}{2}$, $\frac{1}{3} \leqslant x \leqslant 3$. FT is on the last two brackets (not $(x-3)$). If incorrect factor or factors in **(b)** but correct regions here, allow **SC B1** only.
		2	

2. 9709_s25_qp_31 Q: 1

(a) Sketch the graph of $y = |2x - 3|$. [1]

(b) Solve the inequality $3x - 1 < |2x - 3|$. [2]

Answer:

Question	Answer	Marks	Guidance
(a)		**B1**	Symmetrical. In correct position. Condone if no complete scale shown, but must see 3 and $\frac{3}{2}$ marked. Needs to exist for negative x. Must be intending straight lines. Ignore $y = 3x - 1$ if seen.
		1	
(b)	Obtain critical value $\frac{4}{5}$ from $3x - 1 = 3 - 2x$	**B1**	
	State final answer $x < \frac{4}{5}$	**B1**	
	Alternative Method for Question 1(b)		
	Obtain critical value $\frac{4}{5}$ from $(3x-1)^2 = (3-2x)^2$	**B1**	Ignore $x = -2$ if seen.
	State final answer $x < \frac{4}{5}$	**B1**	
		2	

3. 9709_s25_qp_31 Q: 5

The polynomial $3x^3 + pax^2 + 7a^2x + qa^3$ is denoted by $f(x)$, where p, q and a are constants and $a \neq 0$.

When $f(x)$ is divided by $(x+2a)$ the remainder is $-22a^3$. When $f(x)$ is divided by $(3x-a)$ the remainder is $-a^3$.

Find the values of p and q. [5]

Answer:

Question	Answer	Marks	Guidance
	Use $\mathrm{f}(-2a) = -22a^3$	**M1**	Or use long division and equate a constant remainder to $-22a^3$.
	Obtain $-24a^3 + 4pa^3 - 14a^3 + qa^3 = -22a^3$	**A1**	Must evaluate the terms. OE, e.g. $4p + q = 16$.
	Use $\mathrm{f}\left(\frac{a}{3}\right) = -a^3$	**M1**	Or use long division and equate a constant remainder to $-a^3$.
	Obtain $\frac{1}{9}a^3 + \frac{1}{9}pa^3 + \frac{7}{3}a^3 + qa^3 = -a^3$	**A1**	Must evaluate the terms. OE, e.g. $p + 9q = -31$
	Obtain $p = 5, q = -4$	**A1**	
		5	

4. 9709_s25_qp_32 Q: 2

(a) Expand $(6-x)(1-2x)^{-\frac{3}{2}}$ in ascending powers of x, up to and including the term in x^2, simplifying the coefficients. [4]

(b) State the set of values of x for which the expansion is valid. [1]

Answer:

Question	Answer	Marks	Guidance
(a)	Find the first two terms of the expansion of $(1-2x)^{-\frac{3}{2}}$	**B1**	
	Obtain correct third term $\frac{-\frac{3}{2}\left(-\frac{3}{2}-1\right)}{2!}(-2x)^2$ or $\frac{-\frac{3}{2}\left(-\frac{3}{2}-1\right)}{2!}(2x)^2$	**B1**	$\frac{15}{2}x^2$ Ignore extra terms.
	Multiply *their* 3 term expansion $a+bx+cx^2$ by $(6-x)$ obtaining all necessary terms	**M1**	$6+18x+45x^2-x-3x^2....$ Ignore extra terms.
	$6 + 17x + 42x^2$	**A1**	Ignore extra terms. Allow with the terms in any order.
		4	
(b)	$\lvert x\rvert < \frac{1}{2}$ or $-\frac{1}{2} < x < \frac{1}{2}$ or (-0.5, 0.5) or]-0.5, 0.5[	**B1**	OE B0 for an ambiguous statement. Must be strict inequality.
		1	

5. 9709_s25_qp_33 Q: 1

(a) Sketch the graph of $y = |3x - 2a|$, where a is a positive constant. [1]

(b) Hence or otherwise solve the inequality $|3x - 2a| < x + 5a$. [3]

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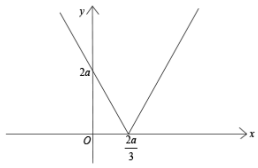
Answer:

Question	Answer	Marks	Guidance
(a)		**B1**	Symmetrical. In correct position. Lines intended to be straight. Must be in both first and second quadrants. Key coordinates must be correct. Ignore $y = x + 5a$ if seen.
		1	

Question	Answer	Marks	Guidance
(b)	Obtain critical value $\frac{7a}{2}$ from $x+5a=3x-2a$	**B1**	Allow if seen in an inequality.
	Obtain critical value $-\frac{3a}{4}$ from $x+5a=2a-3x$	**B1**	Allow if seen in an inequality.
	State final answer $-\frac{3a}{4} < x < \frac{7a}{2}$	**B1**	**SC B1** only for $-\frac{3a}{4} < x < \frac{7a}{2}$ with *their a* from part **(a)**. Allow any equivalent notation. Allow $-\frac{3a}{4} < x$ and $x < \frac{7a}{2}$.
	Alternative Method for Question 1(b)		
	Solve quadratic equation $(3x-2a)^2 = (x+5a)^2$	**M1**	$8x^2 - 22ax - 21a^2 = 0$
	Obtain critical values $-\frac{3a}{4}$ and $\frac{7a}{2}$	**A1**	
	State final answer $-\frac{3a}{4} < x < \frac{7a}{2}$	**A1**	**SC B1** only for $-\frac{3a}{4} < x < \frac{7a}{2}$ with *their a* from part **(a)**. Allow any equivalent notation. Allow $-\frac{3a}{4} < x$ and $x < \frac{7a}{2}$.
		3	

6. 9709_s25_qp_35 Q: 9

(a) Express $\dfrac{12x^2+55x-2}{(3x-2)(x+6)}$ in partial fractions. [5]

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(b) Hence obtain the expansion of $\dfrac{12x^2+55x-2}{(3x-2)(x+6)}$ in ascending powers of x, up to and including the term in x^2. [4]

Answer:

Question	Answer	Marks	Guidance
(a)	State or imply the form $A+\frac{B}{3x-2}+\frac{C}{x+6}$ Values for *A*, *B* and *C* do not need to be substituted into this form to gain full marks	B1	$\frac{Dx+E}{3x-2}+\frac{F}{x+6}$ and $\frac{P}{3x-2}+\frac{Qx+R}{x+6}$ B0 However, can recover all marks from these $\frac{S}{3x-2}+\frac{T}{x+6}$ B0 and can only gain maximum M1A1
	Use a correct method for finding a constant	M1	
	Obtain one of $A = 4$, $B = 6$ and $C = -5$ or $F = -5$ or $P = 6$ or $S = 6$ or $T = -5$	A1	Allow maximum M1A1 for one or more 'correct' values after B0, even if from $\frac{S}{3x-2}+\frac{T}{x+6}$. $D = 12$ $E = -2$ $F = -5$ $P = 6$ $Q = 4$ $R = 19$ $S = 6$ $T = -5$
	Obtain a second value from $\frac{Dx+E}{3x-2} = A+\frac{B}{3x-2}$ or $\frac{Qx+R}{x+6} = A+\frac{C}{x+6}$	A1	
	Obtain a third value	A1	
	Alternative Method for Question 9(a)		
	Divide numerator by denominator and reach quotient of 4 and remainder of $Px+Q$	M1	Or by inspection P or $Q \neq 0$
	Obtain $4+\frac{-9x+46}{(3x-2)(x+6)}$ or $4+\frac{-9x+46}{3x^2+16x-12}$	A1	
	State or imply *their* remainder is of form $\frac{D}{3x-2}+\frac{E}{x+6}$ Values for *D* and *E* do not need to be substituted into this form to gain full marks	B1	

Question	Answer	Marks	Guidance
(a)	Obtain one of $D = 6$, $E = -5$	A1	
	Obtain a second value	A1	
		5	
(b)	Use the correct method to find the first two unsimplified terms of the expansion of $(3x-2)^{-1}$ or $(x+6)^{-1}$ or $(1-\frac{3}{2}x)^{-1}$ or $(1+\frac{1}{6}x)^{-1}$	M1	E.g. $-2^{-1}-2^{-2}(3x)$, or $6^{-1}-6^{-2}(x)$, or $1+\frac{3}{2}x$ or $1-\frac{1}{6}x$.
	Obtain correct unsimplified expansions up to the term in x^2 of each partial fraction $B = 6$ $C = -5$ $A = 4$	A1FT A1FT	The FT is on *B* and *C*, e.g. $\frac{B}{-2}\left(1+\frac{3}{2}x+\left(\frac{3}{2}x\right)^2\right)+\frac{C}{6}\left(1-\frac{1}{6}x+\left(\frac{1}{6}x\right)^2\right)$ OE.
	$\frac{1}{6}-\frac{157}{36}x-\frac{1463}{216}x^2$	A1	OE Do not ISW, e.g. multiplication through by 216. Allow terms in any order.
		4	

7. 9709_m24_qp_32 Q: 1

Find the quotient and remainder when $x^4 - 3x^3 + 9x^2 - 12x + 27$ is divided by $x^2 + 5$. [3]

Answer:

Question	Answer	Marks	Guidance
	Commence division and reach partial quotient of the form $x^2 \pm 3x$ or $x^4 - 3x^3 + 9x^2 - 12x + 27 = (x^2 + 5)(Ax^2 + Bx + C) + Dx + E$ or $Ax^4 + Bx^3 + (5A + C)x^2 + 5Bx + 5C$ and reach $A = 1$ and $B = \pm 3$	**M1**	
	Obtain quotient $x^2 - 3x + 4$	**A1**	$A = 1$, $B = -3$ [$5A + C = 9$ so $C = 4$; $5B + D = -12$ so $D = 3$; $5C + E = 27$ so $E = 7$]. A pair of incorrect statements 'remainder $x^2 - 3x + 4$' and 'quotient $3x + 7$' score **M1 A1 A0**.
	Obtain remainder $3x + 7$	**A1**	
	$$\begin{array}{r r l} & & x^2 - 3x + 4 \\ x^2 + 5 & & x^4 - 3x^3 + 9x^2 - 12x + 27 \\ & & x^4 \qquad\quad + 5x^2 \\ & & - 3x^3 + 4x^2 \\ & & - 3x^3 \qquad\quad - 15x \\ & & + 4x^2 + 3x \\ & & + 4x^2 \qquad\quad + 20 \\ & & + 3x \qquad\quad + 7 \end{array}$$	**3**	

8. 9709_s24_qp_32 Q: 1

(a) Sketch the graph of $y = |x - 2a|$, where a is a positive constant. [1]

(b) Solve the inequality $2x - 3a < |x - 2a|$. [2]

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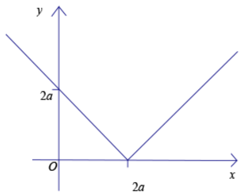
Answer:

Question	Answer	Marks	Guidance
(a)		B1	Correct shape, roughly symmetrical. Both sections should be solid straight lines. If not drawn with a ruler the intention must be clear. Allow construction lines if dashed or clearly fainter. $2a$ marked on each axis (must be $2a$, not just 2). Needs to extend into negative x. If a is given a value, then B0. Ignore $y = 2x - 3a$ if seen.
		1	
(b)	Solve linear equation or inequality to obtain critical value $x = \frac{5}{3}a$ or exact equivalent.	B1	Ignore $x = a$ if seen.
	Obtain $x < \frac{5}{3}a$ or exact equivalent	B1	Accept $x < \frac{10}{6}a$ or $\left(-\infty, \frac{5}{3}a\right)$. Must be strict inequality. Need a clear final solution: $x > a$ or $x < a$ must be rejected if seen as part of the working. Rejection can be implied, e.g. if only the correct inequality is underlined. B0 B0 if a is given a value.
	Alternative Method for Question 1(b)		
	Solve quadratic equation $(2x-3a)^2 = (x-2a)^2$ to obtain critical value $x = \frac{5}{3}a$ or exact equivalent	(B1)	$\left(3x^2 - 8ax + 5a^2 = 0\right)$ Ignore $x = a$ if seen.
	Obtain $x < \frac{5}{3}a$ or exact equivalent	(B1)	Accept $x < \frac{10}{6}a$ or $\left(-\infty, \frac{5}{3}a\right)$. Must be strict inequality. Need a clear final solution: $x > a$ or $x < a$ must be rejected if seen as part of the working. Rejection can be implied, e.g. if only the correct inequality is underlined. B0 B0 if a is given a value.
		2	

9. 9709_s24_qp_32 Q: 2

Express $\frac{6x^2-9x-16}{2x^2-5x-12}$ in partial fractions. [5]

Answer:

Question	Answer	Marks	Guidance
	State or imply the form $A+\frac{B}{2x+3}+\frac{C}{x-4}$	B1	$\frac{Dx+E}{2x+3}+\frac{F}{x-4}$ and $\frac{P}{2x+3}+\frac{Qx+R}{x-4}$ are also valid.
	Use a correct method for finding a constant	M1	**SC**: If score B0, they can score **M1 A1** for one correct constant. B0 M1 A0 available if they substitute two values to form simultaneous equations but get an incorrect answer, or they substitute one value and make an arithmetic error.
	Obtain one of $A=3$, $B=-2$ and $C=4$	A1	**SC**: If the horizontal equation is correct apart from an incorrect value for *A*, the other A marks may be available.
	Obtain a second value	A1	**SC**: If denominator factorised as $\left(x+\frac{3}{2}\right)(x-4)$ can score a maximum of **B0 M1 A1 A1 A0** for a split involving 3 terms.
	Obtain a third value	A1	ISW Statement of the final split is not required.

Question	Answer	Marks	Guidance
	Alternative method for Question 2		
	Divide numerator by denominator	**(M1)**	
	Obtain $3\left(+\frac{Px+Q}{2x^2-5x-12}\right)$	**(A1)**	$\left(3+\frac{6x+20}{(2x+3)(x-4)}\right)$
	State or imply the form $\frac{Px+Q}{2x^2-5x-12}=\frac{D}{2x+3}+\frac{E}{x-4}$	**(B1)**	Must deal with the 3 separately or include it correctly on both sides in their split.
	Obtain one of $D=-2$ and $E=4$	**(A1)**	**SC**: If denominator factorised as $\left(x+\frac{3}{2}\right)(x-4)$, then can score a maximum of **B0 M1 A1 A1 A0** for a split involving three terms.
	Obtain a second value	**(A1)**	ISW Statement of the final split is not required.
		5	

10. 9709_s24_qp_33 Q: 5

Express $\frac{6x^2-2x+2}{(x-1)(2x+1)}$ in partial fractions. [5]

Answer:

Question	Answer	Marks	Guidance
	State or imply the form $A+\frac{B}{x-1}+\frac{C}{2x+1}$	**B1**	
	Use a correct method for finding a constant	**M1**	Correct appropriate method.
	Obtain one of $A = 3$, $B = 2$ and $C = -3$	**A1**	
	Obtain a second value	**A1**	
	Obtain a third value	**A1**	
	Alternative Method for Question 5		
	Divide numerator by denominator to reach $A = 3$	**(M1)**	May be implied by 3 [+] $\frac{ax+b}{(x-1)(2x+1)}$ with a and b not both 0.
	Obtain $3+\frac{x+5}{(x-1)(2x+1)}$	**(A1)**	
	State or imply the form $\frac{D}{x-1}+\frac{E}{2x+1}$	**(B1)**	
	Obtain one of $D = 2$ and $E = -3$	**(A1)**	
	Obtain a second value	**(A1)**	
		5	

11. 9709_w24_qp_31 Q: 1

The polynomial $4x^3 + ax^2 + 5x + b$, where a and b are constants, is denoted by $p(x)$. It is given that $(2x+1)$ is a factor of $p(x)$. When $p(x)$ is divided by $(x-4)$ the remainder is equal to 3 times the remainder when $p(x)$ is divided by $(x-2)$.

Find the values of a and b. [5]

Answer:

Question	Answer	Marks	Guidance
	Substitute $x=-\frac{1}{2}$ and equate the result to zero	**M1**	
	Obtain a correct equation, e.g. $-\frac{4}{8}+\frac{a}{4}-\frac{5}{2}+b=0$	**A1**	$\left(\frac{a}{4}+b=3\right)$ Any equivalent form.
	Substitute $x=2$ and $x=4$ and use $\mathrm{p}(4)=3\mathrm{p}(2)$	**M1**	If using long division, M1 is for correct use of two constant remainders. Condone if 3 is on the wrong side.
	Obtain a correct equation, e.g. $3(32+4a+10+b)=256+16a+20+b$	**A1**	$(-2a+b=75)$ Any equivalent form.
	Obtain $a=-32$ and $b=11$	**A1**	
		5	

12. 9709_w24_qp_31 Q: 7

Let $f(x) = \dfrac{5x^2 + 8x + 5}{(1+2x)(2+x^2)}$.

(a) Express $f(x)$ in partial fractions. [5]

(b) Hence find the coefficient of x^3 in the expansion of $f(x)$. [4]

Answer:

Question	Answer	Marks	Guidance
(a)	State or imply the form $\frac{A}{1+2x}+\frac{Bx+C}{2+x^2}$	B1	
	Use a correct method to find a constant	M1	
	Obtain one of $A=1, B=2$ and $C=3$	A1	
	Obtain a second value	A1	
	Obtain the third value	A1	
		5	
(b)	State $\frac{-1.-2.-3}{3!}(2x)^3$ or -8	B1 FT	Correct term in x^3 or coefficient of x^3 in the expansion of $A(1+2x)^{-1}$. Any equivalent form.
	Use a correct method to obtain the coefficient of x^2 in the expansion of $(2+x^2)^{-1}$ or the coefficient of x^2 in the expansion of $\left(1+\frac{x^2}{2}\right)^{-1}$.	M1	Do not need to deal with 2^{-1} at this stage.
	Obtain $(Bx+C)\times\frac{1}{2}\times-\frac{1}{2}x^2$ or $-\frac{B}{4}x^3$ or $-\frac{B}{4}$	A1 FT	Follow *their* B (and C).
	Obtain final answer $-8\frac{1}{2}$ or $-8\frac{1}{2}x^3$	A1	Or simplified equivalent. Ignore additional terms for other powers of x.
		4	

13. 9709_w24_qp_33 Q: 8

Let $f(x) = \dfrac{7a^2}{(a-2x)(3a+x)}$, where a is a positive constant.

(a) Express $f(x)$ in partial fractions. [3]

(b) Hence obtain the expansion of $f(x)$ in ascending powers of x, up to and including the term in x^2. [4]

(c) State the set of values of x for which the expansion in part **(b)** is valid. [1]

Answer:

Question	Answer	Marks	Guidance
(a)	State or imply the form $\frac{A}{a-2x}+\frac{B}{3a+x}$ and use a correct method to find a constant	M1	
	Obtain $A=2a$ or $B=a$	A1	
	Obtain $A=2a$ and $B=a$	A1	
		3	
(b)	Use a correct method to obtain the first two terms of the expansion of $(a-2x)^{-1}$ or $\left(1-\frac{2x}{a}\right)^{-1}$ or $(3a+x)^{-1}$ or $\left(1+\frac{x}{3a}\right)^{-1}$	M1	
	Obtain $+2\left(1+\frac{2x}{a}+\frac{4x^2}{a^2}+...\right)$	A1ft	OE. May be unsimplified. Follow *their* A, B for an expansion involving a.
	Obtain $+\frac{1}{3}\left(1-\frac{x}{3a}+\frac{x^2}{9a^2}+...\right)$	A1ft	OE. May be unsimplified. Follow *their* A, B for an expansion involving a.
	Obtain $+\frac{7}{3}+\frac{35x}{9a}+\frac{217x^2}{27a^2}$	A1	Or simplified equivalent. Final answer. Ignore any terms in higher powers of x. Do not ISW, e.g. multiplying by $27a^2$. Condone different order of terms.

Question	Answer	Marks	Guidance
(b)	**Alternative Method for Question 8(b)**		
	Expanding $7a^2(a-2x)^{-1}(3a+x)^{-1}$ from the original question. Use a correct method to obtain the first two terms of the expansion of $(a-2x)^{-1}$ or $\left(1-\frac{2x}{a}\right)^{-1}$ or $(3a+x)^{-1}$ or $\left(1+\frac{x}{3a}\right)^{-1}$	M1	
	Obtain $+7a\left(1+\frac{2x}{a}+\frac{4x^2}{a^2}+...\right)$ or $+\frac{7a}{3}\left(1-\frac{x}{3a}+\frac{x^2}{9a^2}+...\right)$	A1	OE. May be unsimplified. May be implied by the expression shown for the next A1.
	Obtain $+\frac{7}{3}\left(1+\frac{2x}{a}+\frac{4x^2}{a^2}+...\right)\left(1-\frac{x}{3a}+\frac{x^2}{9a^2}+...\right)$	A1	OE. May be unsimplified.
	Obtain $+\frac{7}{3}+\frac{35x}{9a}+\frac{217x^2}{27a^2}$	A1	Or simplified equivalent. Final answer. Ignore any terms in higher powers of x. Do not ISW, e.g. multiplying by $27a^2$. Condone different order of terms.
		4	
(c)	$\lvert x\rvert<\frac{a}{2}$	B1	Or $-\frac{a}{2}<x<\frac{a}{2}$. Mark final answer. Must make a clear statement.
		1	

14. 9709_w24_qp_33 Q: 9

(a) Find the quotient and remainder when $x^4 + 16$ is divided by $x^2 + 4$. [3]

(b) Hence show that $\int_{2}^{2\sqrt{3}} \frac{x^4+16}{x^2+4}\,dx = \frac{4}{3}(\pi+4)$. [5]

Answer:

Question	Answer	Marks	Guidance
(a)	Divide to obtain quotient $x^2 + k$	**M1**	k is a constant.
	Obtain quotient $x^2 - 4$	**A1**	If quotient stated separately, mark at this stage.
	Obtain remainder 32	**A1**	If remainder stated separately, mark at this stage. Need not state which is quotient and remainder, but if stated wrongly, max 2/3. After a correct division, still allow the marks if then written as $x^2 - 4 + \frac{32}{x^2+4}$.
	Alternative Method for Question 9(a)		
	Expands brackets to get $B = 0$	**M1**	$\left(x^2+4\right)\left(x^2+Bx+C\right)+D =$ $x^4 + Bx^3 + \left(C+4\right)x^2 + 4Bx + 4C + D$
	$C = -4$	**A1**	
	$D = 32$	**A1**	Need not state which is quotient and remainder, but if stated wrongly, max 2/3.
		3	

Question	Answer	Marks	Guidance
(b)	$\frac{1}{3}x^3 - 4x$	**B1 FT**	Follow *their* quotient of form $Ax^2 + B$.
	Obtain $p\tan^{-1}qx$ where $q = 2$ or $q = \frac{1}{2}$	**M1**	
	Obtain $16\tan^{-1}\frac{1}{2}x$	**A1 FT**	Follow *their* constant remainder, i.e. $\left(\frac{\textit{their}\text{ constant remainder}}{2}\right)\tan^{-1}\frac{1}{2}x$.
	Use limits correctly in an expression containing $p\tan^{-1}qx$ where $q = 2$ or $q = \frac{1}{2}$ and $rx^3 + sx$	**M1**	Terms need not be evaluated, e.g. $\left[8\sqrt{3} - 8\sqrt{3}\right] + 16\tan^{-1}\sqrt{3} - \left(\frac{8}{3} - 8 + 16\tan^{-1}1\right)$ or $\frac{8}{3} - 8$ can be $-\frac{16}{3}$, $16\tan^{-1}\sqrt{3}$ can be $\frac{16\pi}{3}$, $16\tan^{-1}1$ can be 4π.
	Obtain $\frac{4}{3}\left(\pi + 4\right)$ from full and correct working	**A1**	AG
		5	

15. 9709_m23_qp_32 Q: 3

The polynomial $2x^4 + ax^3 + bx - 1$, where a and b are constants, is denoted by $p(x)$. When $p(x)$ is divided by $x^2 - x + 1$ the remainder is $3x + 2$.

Find the values of a and b. [5]

Answer:

Question	Answer	Marks	Guidance
	Commence division and reach partial quotient $2x^2 + (a \pm 2)x$	M1	$2x^2 + (a + 2)x + a$ need $2x^2 + (a \pm 2)x$ $(x^2 - x + 1)\ 2x^4 + ax^3 + 0x^2 + bx - 1$ $2x^4 - 2x^3 + 2x^2$ $(a + 2)x^3 - 2x^2 + bx$ $(a + 2)x^3 - (a + 2)x^2 + (a + 2)x$ $ax^2 + (b - (a + 2))x - 1$ $ax^2 - ax + a$ $(b - 2)x - (1 + a)$ $3x + 2$ Working backwards from remainder: $2x^2 + (\ldots.)x \pm 3$ **M1** $2x^2 - x - 3$ **A1**
	Obtain correct quotient $2x^2 + (a + 2)x + a$	A1	Allow sign error e.g. in $b - 2$.
	Set *their* linear remainder equal to part of "$3x + 2$" and solve for a or for b	M1	Remainder = $3x + 2 = (b - 2)x - 1 - a$. Allow for just equating x term or constant term.
	Obtain answer $a = -3$	A1	
	Obtain answer $b = 5$	A1	

Question	Answer	Marks	Guidance
	Alternative method for Question 3		
	State $2x^4 + ax^3 + 0x^2 + bx - 1 = \left(x^2 - x + 1\right)\left(2x^2 + Ax + B\right) + 3x + 2$ and form and solve equation(s) to obtain A or B	M1	e.g. $0 = B - A + 2$ and $-1 = B + 2$.
	Obtain $A = -1$, $B = -3$	A1	
	Form and solve equations for a or for b	M1	e.g. $a = A - 2$ or $b = -B + A + 3$.
	Obtain answer $a = -3$	A1	
	Obtain answer $b = 5$	A1	
	Alternative method for Question 3		
	Use remainder theorem with $x = \frac{1 \pm \sqrt{-3}}{2}$ or $x = \frac{1 \pm i\sqrt{3}}{2}$	M1	Allow for correct use of a reasonable attempt at either root in exact or decimal form in the remainder theorem $x^2 = \frac{-1 + \sqrt{-3}}{2}$ $x^3 = -1$ $x^4 = \frac{-1 - \sqrt{-3}}{2}$.
	Obtain $-a + \frac{b}{2} \pm \frac{b\sqrt{-3}}{2} \mp \sqrt{-3} - 2 = \frac{7}{2} \pm \frac{3\sqrt{-3}}{2}$ or $-a + \frac{b}{2} \pm \frac{bi\sqrt{3}}{2} \mp i\sqrt{3} - 2 = \frac{7}{2} \pm \frac{3i\sqrt{3}}{2}$	A1	Expand brackets and obtain exact equation for either root. Accept exact equivalent.
	Solve simultaneous equations, or single equation, for a or for b	M1	
	Obtain answer $a = -3$ from exact working	A1	
	Obtain answer $b = 5$ from exact working	A1	
		5	

16. 9709_s23_qp_31 Q: 2

(a) Sketch the graph of $y = |2x + 3|$. [1]

(b) Solve the inequality $3x + 8 > |2x + 3|$. [3]

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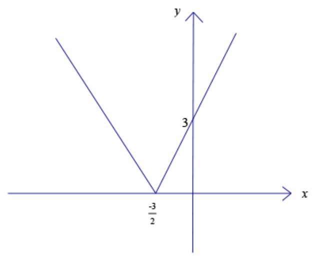
Answer:

Question	Answer	Marks	Guidance
(a)		B1	Show a recognizable sketch graph of $y = \|2x+3\|$. (Ignore any attempt to sketch $y = 3x+8$). Straight lines. Vertex in approximately correct position on x axis. Symmetry.
		1	

Question	Answer	Marks	Guidance
(b)	Find x-coordinate of intersection with $y = 3x+8$	M1	
	Obtain $x = -\frac{11}{5}$	A1	
	State final answer $x > -\frac{11}{5}$ only	A1	$(x > -2.2)$ Do not condone $\geqslant$ for $>$.
	Alternative Method 1		
	Solve the linear inequality $3x+8 > -(2x+3)$, or corresponding linear equation	M1	
	Obtain critical value $x = -\frac{11}{5}$	A1	
	State final answer $x > -\frac{11}{5}$ only	A1	$(x > -2.2)$ Do not condone $\geqslant$ for $>$.
	Alternative Method 2		
	Solve the quadratic inequality $(3x+8)^2 > (2x+3)^2$, or corresponding quadratic equation	(M1)	$5x^2+36x+55$.
	Obtain critical value $x = -\frac{11}{5}$	(A1)	Ignore -5 if seen.
	State final answer $x > -\frac{11}{5}$ only	(A1)	$(x > -2.2)$ Do not condone $\geqslant$ for $>$.
		3	

17. 9709_s23_qp_32 Q: 1

Solve the inequality $|5x - 3| < 2|3x - 7|$. [4]

Answer:

Question	Answer	Marks	Guidance
	State or imply non-modular inequality $(5x-3)^2 < 2^2(3x-7)^2$, or corresponding quadratic equation, or pair of linear equations $(5x - 3) = \pm 2(3x - 7)$	**B1**	$11x^2 - 138x + 187 > 0$.
	Solve a 3-term quadratic, or solve **two** linear equations for x	**M1**	If no working is shown, the M1 is implied by the correct roots for an incorrect quadratic.
	Obtain critical values $x = \frac{17}{11}$ and $x = 11$	**A1**	Accept 1.55 or better.
	State **final** answer $x < \frac{17}{11}$, $x > 11$	**A1**	Strict inequality required. In set notation, allow notation for open sets but not for closed sets e.g. accept $\left(-\infty, \frac{17}{11}\right) \cup (11, \infty)$ or $(-\infty, \frac{17}{11}[\cup]11, \infty)$ but not $(-\infty, \frac{17}{11}] \cup [11, \infty)$. Allow 'or' but not 'and'. Accept $\cup$. Final A0 for $\frac{17}{11} > x > 11$. Exact values expected but ISW if exact inequalities seen followed by decimal approx.
	Alternative Method for Question 1		
	Obtain critical value $x = 11$ from a graphical method, or by inspection, or by solving a linear equation or an inequality	**B1**	
	Obtain critical value $x = \frac{17}{11}$ similarly	**B2**	Accept decimal value.
	State final answer $x < \frac{17}{11}$, $x > 11$	**B1**	Strict inequality required. See notes above.
		4	

18. 9709_s23_qp_33 Q: 10

Let $f(x) = \dfrac{21 - 8x - 2x^2}{(1 + 2x)(3 - x)^2}$.

(a) Express $f(x)$ in partial fractions. [5]

(b) Hence obtain the expansion of $f(x)$ in ascending powers of x, up to and including the term in x^2. [5]

Answer:

Question	Answer	Marks	Guidance
(a)	State or imply the form $\frac{A}{1+2x}+\frac{B}{3-x}+\frac{C}{(3-x)^2}$	B1	Alternative form: $\frac{A}{1+2x}+\frac{Dx+E}{(3-x)^2}$.
	Use a correct method to find a constant	M1	Incorrect format for partial fractions: Allow M1 and a possible A1 if obtain one of these correct values. Max 2/5 Allow M1 even if multiply up by $(1+2x)(3-x)^3$.
	Obtain one of $A=2$, $B=2$ and $C=-3$	A1	Alternative form: obtain one of $A=2$, $D=-2$ and $E=3$.
	Obtain a second value	A1	
	Obtain the third value	A1	Do not need to substitute values back into original form.
		5	If $\frac{A}{1+2x}+\frac{B}{3-x}+\frac{Cx+D}{(3-x)^2}$ B0 but M1 A1 for *A*, A1 for *B* and A1 for *C* and *D*. If *C* = 0 then recovers B1 from above.

Question	Answer	Marks	Guidance
(b)	Use a correct method to obtain the first two terms of one of the unsimplified expansions $(1+2x)^{-1}$, $\left(1-\frac{1}{3}x\right)^{-1}$, $\left(1-\frac{1}{3}x\right)^{-2}$ $(3-x)^{-1}$, $(3-x)^{-2}$	M1	$(1+2x)^{-1}=1+(-1)(2x)+\ldots$ $\left(1-\frac{1}{3}x\right)^{-1}=1+(-1)(-x/3)+\ldots$ $\left(1-\frac{1}{3}x\right)^{-2}=1+(-2)(-x/3)+\ldots$ $(3-x)^{-1}=3^{-1}+(-1)3^{-2}(-x)\ldots$ $(3-x)^{-2}=3^{-2}+(-2)3^{-3}(-x)+\ldots$
	Obtain the correct unsimplified expansions up to the term in x^2 for each partial fraction If correct, should be working with $\frac{2}{1+2x}+\frac{2}{3-x}-\frac{3}{(3-x)^2}$ or $\frac{2}{1+2x}+\frac{-2x+3}{(3-x)^2}$	A1 FT A1 FT A1 FT	Follow through on *their A*, *B*, *C* $A(1+(-1)(2x)+((-1)(-2)/2)(2x)^2+\ldots)$ $\frac{B}{3}(1+(-1)(-x/3)+((-1)(-2)/2)(-x/3)^2+\ldots)$ $\frac{C}{3^2}(1+(-2)(-x/3)+((-2)(-3)/2)(-x/3)^2+\ldots)$. Must be *their* coefficients from **(a)** but may be unsimplified expansions for FT marks. If correct, expect to see $2(1-2x+(2x)^2)$ or $2-4x+8x^2$ $\frac{2}{3}\left(1+\frac{x}{3}+\left(\frac{x}{3}\right)^2\right)$ or $\frac{2}{3}+\frac{2}{9}x+\frac{2}{27}x^2$ $-\frac{1}{3}\left(1+\frac{2x}{3}+(3)\left(\frac{x}{3}\right)^2\right)$ or $-\frac{1}{3}-\frac{2}{9}x-\frac{x^2}{9}$.
	Obtain final answer $\frac{7}{3}-4x+\frac{215}{27}x^2$	A1	Accept $2\frac{1}{3}-4x+7\frac{26}{27}x^2$. No ISW.

Question	Answer	Marks	Guidance
(b)	**Alternative Method for Question 10(b)**		
	For the form $\frac{A}{1+2x}+\frac{Dx+E}{(3-x)^2}$	M1*	For the first two terms of an expanded partial fraction, following their *A*, *D*, *E*.
	Obtain the correct unsimplified expansions up to the term in x^2 for each partial fraction	A1FT A1FT	$A(1+(-1)(2x)+((-1)(-2)/2)(2x)^2+\ldots)+$ $(Dx+E)\frac{1}{3^2}(1+(-2)(-x/3)+((-2)(-3)/2)(-x/3)^2+)$ $2(1-2x+(2x)^2+\ldots)$ $+\frac{-2x+3}{3^2}\left(1+\frac{2x}{3}+(3)\left(\frac{x}{3}\right)^2+\ldots\right)$.
	Multiply out fully	DM1	Provided $DE\neq 0$. Ignore cubic terms and above. Allow error in one term but all terms must be present. If correct, expect to see $2-4x+8x^2-\frac{2}{9}x-\frac{4}{27}x^2+\frac{1}{3}+\frac{2}{9}x+\frac{1}{9}x^2$
	Obtain final answer $\frac{7}{3}-4x+\frac{215}{27}x^2$	A1	Accept $2\frac{1}{3}-4x+7\frac{26}{27}x^2$. No ISW

Question	Answer	Marks	Guidance
(b)	**Alternative Method for Question 10(b): Maclaurin's Series**		
	Correct derivatives for $A(1+2x)^{-1}$, $B(3-x)^{-1}$ and $C(3-x)^{-2}$ $(-1)(2)A(1+2x)^{-2}$, $(-1)(-1)B(3-x)^{-2}$ and $(-2)(-1)C(3-x)^{-3}$	**B1 FT**	
	One of following $(-2)(2)(-1)(2)A(1+2x)^{-3}$, $(-2)(-1)(-1)(-1)B(3-x)^{-3}$ and $(-3)(-1)(-2)(-1)C(3-x)^{-4}$	**B1 FT**	
	All correct	**B1 FT**	
	Substitute in $f(0)+xf'(0)+\frac{x^2}{2}f''(0)$	**M1**	
	Obtain final answer $\frac{7}{3}-4x+\frac{215}{27}x^2$	**A1**	Accept $2\frac{1}{3}-4x+7\frac{26}{27}x^2$. No ISW
		5	

19. 9709_w23_qp_31 Q: 10

Let $f(x) = \dfrac{24x + 13}{(1 - 2x)(2 + x)^2}$.

(a) Express $f(x)$ in partial fractions. [5]

(b) Hence obtain the expansion of $f(x)$ in ascending powers of x, up to and including the term in x^2. [5]

(c) State the set of values of x for which the expansion in **(b)** is valid. [1]

Answer:

Question	Answer	Marks	Guidance
(a)	State or imply the form $\frac{A}{1-2x}+\frac{B}{2+x}+\frac{C}{(2+x)^2}$	B1	
	Use a correct method for finding a coefficient	M1	$A(2+x)^2+B(1-2x)(2+x)+C(1-2x)$ $=24x+13$.
	Obtain one of $A=4$, $B=2$ and $C=-7$	A1	If errors in equating still allow A marks for A and C.
	Obtain a second value	A1	
	Obtain the third value	A1	Mark the form $\frac{A}{1-2x}+\frac{Dx+E}{(2+x)^2}$, where $A=4$, $D=2$ and $E=-3$, B1 M1 A1 A1 A1 as above. If there are extra term in partial fractions, that is 4 unknowns A, B, D and E then B0 unless recover at end, e.g. by setting $B=0$. If B set to any value other than 0 and all coefficients correctly found to their new values then allow all A marks, but still B0 for partial fraction expression. Hence A1 for each coefficient, but nothing for coefficient set to specific value. Another case of extra term in partial fraction expression, namely $+F$, mark as above but need $F=0$ to recover B1.
		5	

Question	Answer	Marks	Guidance
(b)	Use a correct method to find the first two terms of the expansion of $(1-2x)^{-1}$, $(2+x)^{-1}$, $(2+x)^{-2}$, $\left(1+\frac{x}{2}\right)^{-1}$ or $\left(1+\frac{x}{2}\right)^{-2}$	M1	Symbolic coefficients are not sufficient for the M1.
	Obtain correct un-simplified expansions up to the term in x^2 of each partial fraction	A1 FT	$A\left(1+(-1)(-2x)+\frac{(-1)(-2)}{2}(-2x)^2+..\right)$ $A=4$.
		A1 FT	$\frac{B}{2}\left(1+(-1)\left(\frac{x}{2}\right)+\frac{(-1)(-2)}{2}\left(\frac{x}{2}\right)^2+...\right)$ $B=2$.
		A1 FT	$\frac{C}{4}\left(1+(-2)\left(\frac{x}{2}\right)+\frac{(-2)(-3)}{2}\left(\frac{x}{2}\right)^2+...\right)$ $C=-7$ $=4(1+2x+4x^2)+2/2(1-x/2+x^2/4)-7/4(1-x+3x^2/4)$ $=(4+1-7/4)+(8-1/2+7/4)\,x+(16+1/4-21/16)\,x^2$ The FT is on A, B, C.

Question	Answer	Marks	Guidance
	Obtain final answer $\frac{13}{4}+\frac{37}{4}x+\frac{239}{16}x^2$	A1	OE $(Dx+E)/4\ [1+(-2)(x/2)+(-2)(-3)(x/2)^2/2\ \ldots]$ $D=2 \quad E=-3$ The FT is on A, D, E. Maclaurin's Series f(0) = 13/4 B1 f'(0) = 37/4 B1 f''(0) = 239/8 B1. $\frac{13}{4}+\frac{37}{4}x+\frac{239}{8}x^2/2$ or equivalent M1 A1. If $1+\frac{37}{4}x+\frac{239}{8}x^2/2$ then M0 A0 unless *their* f(0) actually is 1. For the A, D, E form of fractions, give M1 A1FT A1FT for the expanded partial fractions, then, if $D \neq 0$, M1 for multiplying out fully, and A1 for the final answer. If final answer has been multiplied throughout (e.g. by 16) then A0 at the end
		5	
10(c)	$\lvert x\rvert<\frac{1}{2}$	B1	OE
		1	

20. 9709_w23_qp_32 Q: 1

(a) Sketch the graph of $y = |4x - 2|$. [1]

(b) Solve the inequality $1 + 3x < |4x - 2|$. [4]

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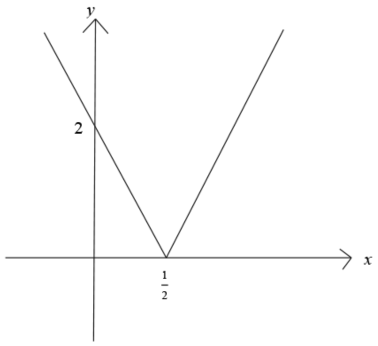
Answer:

Question	Answer	Marks	Guidance
(a)		**B1**	Show a recognizable sketch graph of $y=\|4x-2\|$. Roughly symmetrical. Should extend into the second quadrant. Ignore $y=4x-2$ below the axis if intention is clear e.g. dashed or the required lines are clearly bolder. Some indication of scale on **both** axes – accept dashes. Must go beyond (0, 2) and (1, 2). Ignore any attempt to sketch $y=1+3x$.
		1	

Question	Answer	Marks	Guidance
(b)	Obtain critical value $x=3$	**B1**	Allow incorrect inequality. Allow if later rejected. Allow $\frac{21}{7}$.
	Solve the linear equation $1+3x=2-4x$	**M1**	Or corresponding linear inequality.
	Obtain critical value $\frac{1}{7}$	**A1**	Allow 0.143 or better. Allow incorrect inequality. Allow if later rejected.
	Obtain final answer $x<\frac{1}{7}$ [or] $x>3$	**A1**	Or equivalent. Allow with a comma, or nothing between. Strict inequalities only. Exact values. A0 for $\frac{1}{7}>x>3$ A0 for $x<\frac{1}{7}$ **and** $x>3$.
	Alternative method for question 1(b)		
	Solve the quadratic inequality $(4x-2)^2>(1+3x)^2$, or corresponding quadratic equation	**M1**	e.g. $7x^2-22x+3=0$. Available if they start with the correct equation / inequality, have a correct method for squaring (i.e. not $(a+b)^2=a^2+b^2$) and a correct method for solving. Need to obtain at least one critical value.
	Obtain critical value $x=3$	**A1**	Allow incorrect inequality. Allow if later rejected. Allow $\frac{21}{7}$.
	Obtain critical value $\frac{1}{7}$	**A1**	Allow 0.143 or better. Allow incorrect inequality. Allow if later rejected.
	Obtain final answer $x<\frac{1}{7}$ [or] $x>3$	**A1**	Or equivalent. Strict inequalities only. Allow with a comma, or nothing between. Exact values. A0 for $\frac{1}{7}>x>3$ A0 for $x<\frac{1}{7}$ and $x>3$.
		4	

21. 9709_w23_qp_33 Q: 3

The polynomial $2x^3 + ax^2 + bx + 6$, where a and b are constants, is denoted by $p(x)$. When $p(x)$ is divided by $(x + 2)$ the remainder is -38 and when $p(x)$ is divided by $(2x - 1)$ the remainder is $\frac{19}{2}$.

Find the values of a and b. [5]

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Answer:

Question	Answer	Marks	Guidance
	$2(-2)^3 + a(-2)^2 + b(-2) + 6 = -38$ Allow errors $x+2\,\overline{)\,2x^3 + ax^2 \quad +bx \quad +6}$ quotient $2x^2+(a-4)x+b-2a+8$ $\underline{2x^3 + 4x^2}$ $(a-4)x^2 + bx$ $\underline{(a-4)x^2 + (2a-8)x}$ $(b-2a+8)x + 6$ $\underline{(b-2a+8)x + 2b - 4a + 16}$ $4a - 2b - 10$	**M1**	Substitute $x = -2$ and equate the result to -38 or divide by $x + 2$ to obtain quadratic quotient, and equate constant remainder to -38.
	Obtain a correct evaluated equation, e.g. $-16 + 4a - 2b + 6 = -38$ or $4a - 2b = -28$	**A1**	
	$2\left(\frac{1}{2}\right)^3 + a\left(\frac{1}{2}\right)^2 + b\left(\frac{1}{2}\right) + 6 = \frac{19}{2}$ Allow errors $2x-1\,\overline{)\,2x^3 + ax^2 \quad +bx \quad +6}$ quotient $x^2 + \frac{a+1}{2}x + \frac{b}{2} + \frac{a}{4} + \frac{1}{4}$ $\underline{2x^3 - x^2}$ $(a+1)x^2 + bx$ $\underline{(a+1)x2 - \left(\frac{a}{2} + \frac{1}{2}\right)x}$ $\left(b + \frac{a}{2} + \frac{1}{2}\right)x + 6$ $\underline{\left(b + \frac{a}{2} + \frac{1}{2}\right)x - \left(\frac{b}{2} + \frac{a}{4} + \frac{1}{4}\right)}$ $6 + \frac{b}{2} + \frac{a}{4} + \frac{1}{4}$	**M1**	Substitute $x = \frac{1}{2}$ and equate the result to $\frac{19}{2}$ or divide by $2x - 1$ to obtain quadratic quotient, and equate constant remainder to $\frac{19}{2}$.
	Obtain a correct evaluated equation, e.g. $\frac{1}{4} + \frac{a}{4} + \frac{b}{2} + 6 = \frac{19}{2}$ or $\frac{a}{4} + \frac{b}{2} = \frac{13}{4}$	**A1**	
	Obtain $a = -3$ and $b = 8$	**A1**	ISW
		5	

22. 9709_w23_qp_33 Q: 9

Let $f(x) = \dfrac{17x^2 - 7x + 16}{(2 + 3x^2)(2 - x)}$.

(a) Express $f(x)$ in partial fractions. [5]

(b) Hence obtain the expansion of $f(x)$ in ascending powers of x, up to and including the term in x^3. [5]

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(c) State the set of values of x for which the expansion in **(b)** is valid. Give your answer in an exact form. [1]

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Answer:

Question	Answer	Marks	Guidance
(a)	State or imply the form $\frac{Ax+B}{2+3x^2}+\frac{C}{2-x}$	B1	If incorrect partial fractions e.g. $A = 0$ or $Ax^2 + B$ then M1, A1 A0 for correct C. Only allow single A1 even if other coefficients correct. B1 recoverable by a correct form end statement.
	Use a correct method for finding a coefficient	M1	e.g. $(Ax + B)(2 - x) + C(2 + 3x^2)$ $= (3C - A)x^2 + (2A - B)x + (2B + 2C)$ $= 17x^2 - 7x + 16$.
	Obtain one of $A = -2$, $B = 3$ and $C = 5$	A1	If error present in above still allow A1 for C.
	Obtain a second value	A1	
	Obtain the third value	A1	Extra term in partial fractions, $D/(2 + 3x^2)$, that is 4 unknowns A, B, C and D then B0 unless recover at end, e.g. by setting B or $D = 0$. If B or D set to any value other than 0 and all coefficients correctly found to their new values then allow all A marks, but still B0 for partial fraction expression unless $B + D$ combined. Hence A1 for each coefficient, but nothing for coefficient set to specific value. Another case of extra term in partial fraction expression, namely $+ F$, mark as above but need $F = 0$ to recover B1.
		5	

Question	Answer	Marks	Guidance
(b)	Use a correct method to find the first two terms of the expansion $(2-x)^{-1} = 2^{-1} + (-1)\,2^{-2}(-x) + [(-1)(-2)2^{-3}(-x)^2/2!]$, $\left(1+\frac{3x^2}{2}\right)^{-1} = 1 - \frac{3x^2}{2}$ or $\left(1-\frac{x}{2}\right)^{-1} = 1 - \left(\frac{-x}{2}\right)$	M1	Symbolic coefficients are not sufficient for the M1.
	$\frac{Ax+B}{2}\left[1+(-1)\frac{3x^2}{2}\ldots\right]$ $\quad A=-2 \quad B=3$ $\frac{C}{2}\left[1+(-1)\left(\frac{-x}{2}\right)+\frac{(-1)(-2)}{2}\left(\frac{-x}{2}\right)^2 + \frac{(-1)(-2)(-3)}{6}\left(\frac{-x}{2}\right)^3\ldots\right]$ $\quad C=5$	A1 FT	Obtain correct un-simplified expansions up to the term in x^3 of each partial fraction.
	$=\frac{3-2x}{2}\left(1-\frac{3x^2}{2}\right)+\frac{5}{2}\left(1+\frac{x}{2}+\frac{x^2}{4}+\frac{x^3}{8}\right)$ $=\left(\frac{3}{2}+\frac{5}{2}\right)+\left(-1+\frac{5}{4}\right)x+\left(-\frac{9}{4}+\frac{5}{8}\right)x^2+\left(\frac{3}{2}+\frac{5}{16}\right)x^3$	A1 FT	Unsimplified $(2 - x)^{-1}$ expanded correctly, error in simplifying before their C is involved in the expression, allow A1FT when their C is introduced. The FT is on A, B, C.
	Multiply expansion of $\left(1+\frac{3x^2}{2}\right)^{-1}$ $\left(\text{must reach } 1\pm\frac{3x^2}{2}\right)$ by $Ax + B$, where $AB \neq 0$, up to the term in x^3. Allow if used $Cx + D$ ($Ax + B$ miscopied).	M1	Allow either ± 2 or $\pm 2^{-1}$ outside bracket or missing. Allow one error in actual multiplication to acquire the 4 terms [all terms needed]. Ignore errors in higher powers.

Question	Answer	Marks	Guidance
(b)	Obtain final answer $4+\frac{1}{4}x-\frac{13}{8}x^2+\frac{29}{16}x^3$, or equivalent [If final answer has been multiplied throughout, e.g. by 16 then A0 at the end]	**A1**	Maclaurin's Series: f(0) = 4 B1 f'(0) = 1/4 B1. f''(0) = –13/4 and f'''(0) = 87/8 B1. $4+\frac{1}{4}x-\frac{\frac{13}{4}x^2}{2}+\frac{\frac{87}{8}x^3}{6}$ or equivalent M1 A1. If $1+\frac{1}{4}x-\frac{\frac{13}{4}x^2}{2}+\frac{\frac{87}{8}x^3}{6}$ then M0 A0 unless their f(0) actually is 1.
		5	
(c)	State answer $\lvert x\rvert<\sqrt{\frac{2}{3}}$ or $-\sqrt{\frac{2}{3}}<x<\sqrt{\frac{2}{3}}$ clear conclusion required	**B1**	Or exact equivalent. Strict inequality.
		1	

23. 9709_m22_qp_32 Q: 1

Solve the inequality $|2x + 3| > 3|x + 2|$. [4]

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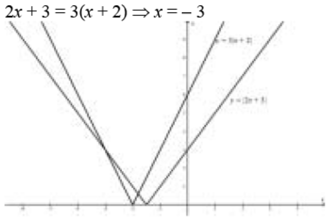
Answer:

Question	Answer	Marks	Guidance
	State or imply non-modular inequality $(2x+3)^2 > 3^2(x+2)^2$, or corresponding quadratic equation, or pair of linear equations	**B1**	
	Make a reasonable attempt at solving a 3-term quadratic, or solve two linear equations for x	**M1**	Quadratic formula or $(5x + 9)(x + 3)$
	Obtain critical values $x = -3$ and $x = -\frac{9}{5}$	**A1**	OE
	State final answer $-3 < x < -\frac{9}{5}$ or $x > -3$ and $x < -\frac{9}{5}$	**A1**	[Do not condone $\leqslant$ for $<$ in the final answer.] No ISW
	Alternative method for question 1		
	Obtain critical value $x = -3$ from a graphical method, or by solving a linear equation or linear inequality	**B1**	$2x + 3 = 3(x + 2) \Rightarrow x = -3$ 
	Obtain critical value $x = -\frac{9}{5}$ similarly	**B2**	
	State final answer $-3 < x < -\frac{9}{5}$ or $x > -3$ and $x < -\frac{9}{5}$	**B1**	[Do not condone $\leqslant$ for $<$ in the final answer.] No ISW
		4	

24. 9709_S22_qp_31 Q: 2

(a) Expand $(2 - x^2)^{-2}$ in ascending powers of x, up to and including the term in x^4, simplifying the coefficients. [4]

(b) State the set of values of x for which the expansion is valid. [1]

Answer:

Question	Answer	Marks	Guidance
(a)	State a correct unsimplified version of the x^2 or the x^4 term of the expansion of $(2-x^2)^{-2}$ or $\left(1-\frac{1}{2}x^2\right)^{-2}$	**M1**	$\frac{1}{4}\left(1+2\frac{x^2}{2}+\frac{-2.-3}{2}\left(\frac{x^2}{2}\right)^2 ...\right)$ Symbolic binomial coefficients are not sufficient for the M1.
	State correct first term $\frac{1}{4}$	**B1**	Accept 2^{-2}.
	Obtain the next two terms $\frac{1}{4}x^2+\frac{3}{16}x^4$	**A1 A1**	A1 for each one correct ISW. Full marks for $\frac{1}{4}\left(1+x^2+\frac{3}{4}x^4\right)$ ISW.
			SC allow **M1 A1 A1** for $\frac{1}{4}$ and $1+x^2+\frac{3}{4}x^4$ SOI. **SC** allow **M1 A1** for $1+x^2+\frac{3}{4}x^4$
		4	
(b)	State answer $\lvert x\rvert<\sqrt{2}$	**B1**	Or $-\sqrt{2}<x<\sqrt{2}$.
		1	

25. 9709_S22_qp_31 Q: 5

The polynomial $ax^3 - 10x^2 + bx + 8$, where a and b are constants, is denoted by $p(x)$. It is given that $(x - 2)$ is a factor of both $p(x)$ and $p'(x)$.

(a) Find the values of a and b. [5]

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(b) When a and b have these values, factorise $\mathrm{p}(x)$ completely. [3]

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Answer:

Question	Answer	Marks	Guidance
(a)	Substitute $x = 2$, equate to zero	**M1**	Or divide by $x - 2$ and equate constant remainder to zero.
	Obtain a correct equation, e.g. $8a - 40 + 2b + 8 = 0$	**A1**	Seen or implied in subsequent work.
	Differentiate p(x), substitute $x = 2$ and equate result to zero	**M1**	Or divide by $x - 2$ and equate constant remainder to zero.
	Obtain $12a - 40 + b = 0$, or equivalent	**A1**	SOI in subsequent work.
	Obtain $a = 3$ and $b = 4$	**A1**	
	Alternative method for question 5(a)		
	State or imply $(x-2)^2$ is a factor	**M1**	
	$\text{p}(x)=(x-2)^2(ax+2)$	**A1**	
	Obtain an equation in b	**M1**	
	e.g. by comparing coefficients of x: $b=4a-8$	**A1**	
	Obtain $a = 3$ and $b = 4$	**A1**	
			SC If uses $x = -2$ in both equations allow **M1** and allow **A1** for $a = -3$, $b = -4$.
		5	

Question	Answer	Marks	Guidance
(b)	Attempt division by $(x - 2)$	**M1**	The M1 is earned if division reaches a partial quotient of $ax^2 + kx$, or if inspection has an unknown factor ax^2+ex+f and an equation in e and/or f. Where a has the value found in part **5(a)**.
	Obtain quadratic factor $3x^2 - 4x - 4$	**A1**	
	Obtain factorisation $(3x+2)(x-2)(x-2)$	**A1**	
	Alternative method for question 5(b)		
	State or imply $(x-2)^2$ is a factor	**B1**	
	Attempt division by $(x-2)^2$, reaching a quotient $ax + k$ or use inspection with unknown factor $cx + d$ reaching a value for c or for d	**M1**	
	Obtain factorisation $(3x+2)(x-2)^2$	**A1**	Accept $3\left(x+\frac{2}{3}\right)(x-2)^2$.
		3	