Topical Past Paper Questions Workbook

AS & A Level Mathematics (9709) Paper 1 [Pure Mathematics 1]

May/June 2015 – February/March 2022

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Introduction

Each topical past paper questions book consists of hundreds of questions and their answer schemes in the form of worksheets. Questions are assigned to each chapter according to their related topic. Topics, in turn, are based on the items of the latest Cambridge IGCSE or AS/A level syllabus content. This book's specifications are as follows:

Title: AS & A Level Mathematics (9709) Paper 1 Topical Past Paper Questions Workbook
Subtitle: Exam Practice Worksheets With Answer Scheme
Examination board: Cambridge Assessment International Education (CAIE)
Subject code: 9709
Years covered: May/June 2015 – February/March 2022
Paper: 1
Number of pages: 1005
Number of questions: 527

Contents

Chapter 1

Quadratics

1. 9709_m21_qp_12 Q: 2

By using a suitable substitution, solve the equation

$$(2x-3)^2 - \frac{4}{(2x-3)^2} - 3 = 0. \qquad [4]$$

2. 9709_s21_qp_11 Q: 6

The equation of a curve is $y = (2k - 3)x^2 - kx - (k - 2)$, where k is a constant. The line $y = 3x - 4$ is a tangent to the curve.

Find the value of k. [5]

3. 9709_s21_qp_12 Q: 1

(a) Express $16x^2 - 24x + 10$ in the form $(4x + a)^2 + b$. [2]

(b) It is given that the equation $16x^2 - 24x + 10 = k$, where k is a constant, has exactly one root.

Find the value of this root. [2]

4. 9709_s20_qp_11 Q: 5

The equation of a line is $y = mx + c$, where m and c are constants, and the equation of a curve is $xy = 16$.

(a) Given that the line is a tangent to the curve, express m in terms of c. [3]

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(b) Given instead that $m = -4$, find the set of values of c for which the line intersects the curve at two distinct points. [3]

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5. 9709_s19_qp_13 Q: 1

The function f is defined by $f(x) = x^2 - 4x + 8$ for $x \in \mathbb{R}$.

(i) Express $x^2 - 4x + 8$ in the form $(x - a)^2 + b$. [2]

(ii) Hence find the set of values of x for which $f(x) < 9$, giving your answer in exact form. [3]

6. 9709_s18_qp_13 Q: 1

Express $3x^2 - 12x + 7$ in the form $a(x + b)^2 + c$, where a, b and c are constants. [3]

7. 9709_w18_qp_11 Q: 1

Showing all necessary working, solve the equation $4x - 11x^{\frac{1}{2}} + 6 = 0$. [3]

8. 9709_m17_qp_12 Q: 1

Find the set of values of k for which the equation $2x^2 + 3kx + k = 0$ has distinct real roots. [4]

9. 9709_s16_qp_11 Q: 6

(a) Find the values of the constant m for which the line $y = mx$ is a tangent to the curve $y = 2x^2 - 4x + 8$. [3]

(b) The function f is defined for $x \in \mathbb{R}$ by $f(x) = x^2 + ax + b$, where a and b are constants. The solutions of the equation $f(x) = 0$ are $x = 1$ and $x = 9$. Find

(i) the values of a and b, [2]

(ii) the coordinates of the vertex of the curve $y = f(x)$. [2]

10. 9709_w16_qp_11 Q: 1

(i) Express $x^2 + 6x + 2$ in the form $(x + a)^2 + b$, where a and b are constants. [2]

(ii) Hence, or otherwise, find the set of values of x for which $x^2 + 6x + 2 > 9$. [2]

11. 9709_s15_qp_13 Q: 1

Express $2x^2 - 12x + 7$ in the form $a(x + b)^2 + c$, where a, b and c are constants. [3]

12. 9709_w15_qp_13 Q: 3

(i) Express $3x^2 - 6x + 2$ in the form $a(x + b)^2 + c$, where a, b and c are constants. [3]

(ii) The function f, where $f(x) = x^3 - 3x^2 + 7x - 8$, is defined for $x \in \mathbb{R}$. Find $f'(x)$ and state, with a reason, whether f is an increasing function, a decreasing function or neither. [3]

Chapter 2

Functions

13. 9709_m22_qp_12 Q: 5

(a) Express $2x^2 - 8x + 14$ in the form $2[(x - a)^2 + b]$. [2]

The functions f and g are defined by

$$\mathrm{f}(x) = x^2 \quad \text{for } x \in \mathbb{R},$$
$$\mathrm{g}(x) = 2x^2 - 8x + 14 \quad \text{for } x \in \mathbb{R}.$$

(b) Describe fully a sequence of transformations that maps the graph of $y = \mathrm{f}(x)$ onto the graph of $y = \mathrm{g}(x)$, making clear the order in which the transformations are applied. [4]

14. 9709_m22_qp_12 Q: 9

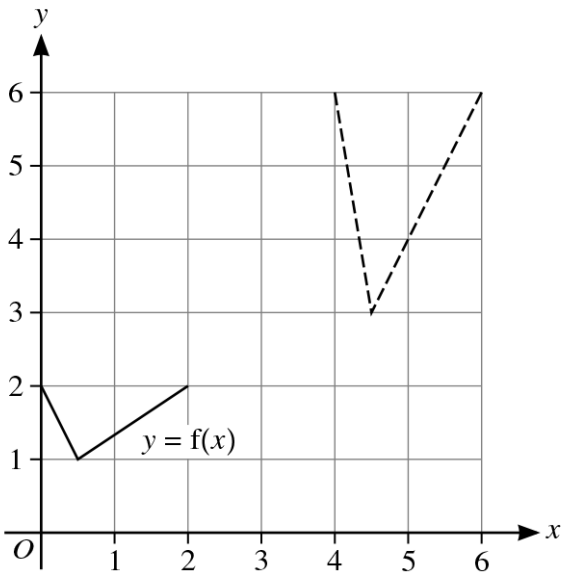
Functions f, g and h are defined as follows:

$$\text{f} : x \mapsto x - 4x^{\frac{1}{2}} + 1 \quad \text{for } x \geqslant 0,$$
$$\text{g} : x \mapsto mx^2 + n \quad \text{for } x \geqslant -2, \text{ where } m \text{ and } n \text{ are constants,}$$
$$\text{h} : x \mapsto x^{\frac{1}{2}} - 2 \quad \text{for } x \geqslant 0.$$

(a) Solve the equation $\text{f}(x) = 0$, giving your solutions in the form $x = a + b\sqrt{c}$, where a, b and c are integers. [4]

(b) Given that $f(x) \equiv gh(x)$, find the values of m and n. [4]

15. 9709_m21_qp_12 Q: 5



In the diagram, the graph of $y = f(x)$ is shown with solid lines. The graph shown with broken lines is a transformation of $y = f(x)$.

(a) Describe fully the two single transformations of $y = f(x)$ that have been combined to give the resulting transformation. [4]

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(b) State in terms of y, f and x, the equation of the graph shown with broken lines. [2]

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16. 9709_m21_qp_12 Q: 7

Functions f and g are defined as follows:

$$\text{f} : x \mapsto x^2 + 2x + 3 \text{ for } x \leqslant -1,$$
$$\text{g} : x \mapsto 2x + 1 \text{ for } x \geqslant -1.$$

(a) Express $\text{f}(x)$ in the form $(x + a)^2 + b$ and state the range of f. [3]

(b) Find an expression for $f^{-1}(x)$. [2]

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(c) Solve the equation $gf(x) = 13$. [3]

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17. 9709_s21_qp_11 Q: 9

Functions f and g are defined as follows:

$$\mathrm{f}(x) = (x-2)^2 - 4 \text{ for } x \geqslant 2,$$
$$\mathrm{g}(x) = ax + 2 \text{ for } x \in \mathbb{R},$$

where a is a constant.

(a) State the range of f. [1]

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(b) Find $\mathrm{f}^{-1}(x)$. [2]

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(c) Given that $a = -\frac{5}{3}$, solve the equation $\mathrm{f}(x) = \mathrm{g}(x)$. [3]

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(d) Given instead that $\text{ggf}^{-1}(12) = 62$, find the possible values of a. [5]

18. 9709_s21_qp_12 Q: 2

(a) The graph of $y = \mathrm{f}(x)$ is transformed to the graph of $y = 2\mathrm{f}(x - 1)$.

Describe fully the two single transformations which have been combined to give the resulting transformation. [3]

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(b) The curve $y = \sin 2x - 5x$ is reflected in the y-axis and then stretched by scale factor $\frac{1}{3}$ in the x-direction.

Write down the equation of the transformed curve. [2]

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Appendix A

Answers

1. 9709_m21_ms_12 Q: 2

Question	Answer	Marks	Guidance
	$u=2x-3$ leading to $u^4-3u^2-4\ [=0]$	**M1**	Or $u=(2x-3)^2$ leading to $u^2-3u-4\ [=0]$
	$(u^2-4)(u^2+1)\ [=0]$	**M1**	Or $(u-4)(u+1)\ [=0]$
	$2x-3=[\pm]2$	**A1**	
	$x=\frac{1}{2}$, $\frac{5}{2}$ **only**	**A1**	
		4	

2. 9709_s21_ms_11 Q: 6

Question	Answer	Marks	Guidance
	$(2k-3)x^2-kx-(k-2)=3x-4$	***M1**	Equating curve and line
	$(2k-3)x^2-(k+3)x-(k-6)[=0]$	**DM1**	Forming a 3-term quadratic
	$(k+3)^2+4(2k-3)(k-6)[=0]$	**DM1**	Use of discriminant (dependent on **both** previous M marks)
	$9k^2-54k+81[=0]$ [leading to $k^2-6k+9=0$]	**M1**	Simplifying and solving *their* 3-term quadratic in k
	$k=3$	**A1**	
	Alternative method for Question 6		
	$(2k-3)x^2-kx-(k-2)=3x-4$	***M1**	Equating curve and line
	$2(2k-3)x-k=3 \Rightarrow x=\frac{k+3}{4k-6}$ or $k=\frac{3+6x}{4x-1}$	**DM1**	Differentiating and solving for x or k
	Either $(2k-3)\left(\frac{k+3}{4k-6}\right)^2-k\left(\frac{k+3}{4k-6}\right)-(k-2)=3\left(\frac{k+3}{4k-6}\right)-4$ **Or** $4x\left(\frac{3x^2+3x-6}{2x^2-x-1}\right)-6x-\left(\frac{3x^2+3x-6}{2x^2-x-1}\right)=3$	**DM1**	Substituting *their* x into equation or *their* $k=\frac{3x^2+3x-6}{2x^2-x-1}$ or $k=\frac{3x+6}{2x+1}$ into derivative equation (dependent on **both** previous M marks)
	$9k^2-54k+81[=0]$ [leading to $k^2-6k+9=0$]	**M1**	Simplifying and solving *their* 3-term quadratic in k (or solving for x)
	$k=3$	**A1**	
			SC If M0, B1 for differentiating, equating to 3 and solving for x or k
		5	

3. 9709_s21_ms_12 Q: 1

Question	Answer	Marks	Guidance
(a)	$(4x-3)^2$ or $(4x+(-3))^2$ or $a=-3$	B1	$k(4x-3)^2$ where $k \neq 1$ scores B0 but mark final answer, allow recovery.
	$+1$ or $b=1$	B1	
		2	
(b)	[For one root] $k=1$ or *'their b'*	B1 FT	Either by inspection or solving or from $24^2-4\times16\times(10-k)=0$ WWW
	[Root or $x=$] $\frac{3}{4}$ or 0.75	B1	**SC B2** for correct final answer WWW.
		2	

4. 9709_s20_ms_11 Q: 5

(a)	$x(mx+c)=16 \rightarrow mx^2+cx-16=0$	**B1**
	Use of $b^2-4ac=c^2+64m$	**M1**
	Sets to 0 $\rightarrow m=\frac{-c^2}{64}$	**A1**
		3
(b)	$x(-4x+c)=16$ Use of $b^2-4ac \rightarrow c^2-256$	**M1**
	$c>16$ and $c<-16$	**A1 A1**
		3

5. 9709_s19_ms_13 Q: 1

	Answer	Mark	Partial Marks
(i)	$\left[(x-2)^2\right]$ [+4]	**B1 DB1**	2nd B1 dependent on 2 inside bracket
		2	
(ii)	$(x-2)^2<5 \rightarrow -\sqrt{5}<x-2$ and/or $x-2<\sqrt{5}$	**M1**	Allow e.g. $x-2<\pm\sqrt{5}$, $x-2=\pm\sqrt{5}$ and decimal equivalents for √5 For M1, ft from *their*(i). Also allow √13 instead of √5 for clear slip
	$2-\sqrt{5}<x<2+\sqrt{5}$	**A1A1**	A1 for each inequality – allow two separate statements but there must be 2 inequalities for x. Non-hence methods, if completely correct, score SC 1/3. Condone $\leqslant$
		[3]	

6. 9709_s18_ms_13 Q: 1

	Answer	Mark	Partial Marks
	[3] $\left[(x-2)^2\right]$ [–5]	**B1B1B1**	OR $a=3$, $b=-2$, c = –5. 1st mark is dependent on the form $(x+a)^2$ following 3
		3	

7. 9709_w18_ms_11 Q: 1

	Answer	Mark	Partial Marks
	$\left(4x^{½}-3\right)\left(x^{½}-2\right)$ oe soi Alt: $4x+6=11\sqrt{x} \Rightarrow 16x^2-73x+36$	**M1**	Attempt solution for $x^{½}$ or sub $u=x^{½}$
	$x^{½}=3/4$ or 2 $(16x-9)(x-4)$	**A1**	Reasonable solutions for $x^{½}$ implies M1 ($x = 2, 3/4$, M1A0)
	$x=9/16$ oe or 4	**A1**	Little or no working shown scores SCB3, spotting one solution, B0
		3	

8. 9709_m17_ms_12 Q: 1

	Answer	Mark	Partial Marks
	$(3k)^2-4\times2\times k$	**M1**	Attempt b^2-4ac
	$9k^2-8k>0$ soi Allow $9k^2-8k\geqslant 0$	**A1**	Must involve correct inequality. Can be implied by correct answers
	0, 8/9 soi	**A1**	
	$k<0$, $k>8/9$ (or 0.889)	**A1**	Allow $(-\infty, 0)$, $(8/9, \infty)$
	Total:	**4**	

9. 9709_s16_ms_11 Q: 6

	Answer	Mark	Partial Marks
(a)	$y = 2x^2 - 4x + 8$ Equates with $y = mx$ and selects a, b, c Uses $b^2 = 4ac$ $\rightarrow m = 4$ or -12.	**M1** **M1** **A1** [3]	Equate + solution or use of dy/dx Use of discriminant for both.
(b) (i)	$f(x) = x^2 + ax + b$ Eqn of form $(x-1)(x-9)$ $\rightarrow a = -10$, $b = 9$ (or using 2 sim eqns **M1 A1**)	**M1** **A1** [2]	Any valid method allow $(x+1)(x+9)$ for **M1** must be stated
(ii)	Calculus or $x = \frac{1}{2}(1 + 9)$ by symmetry $\rightarrow (5, -16)$	**M1** **A1** [2]	Any valid method

10. 9709_w16_ms_11 Q: 1

	Answer	Mark	Partial Marks
(i)	$(x+3)^2-7$	**B1B1** [2]	For $a = 3$, $b = -7$
(ii)	1,−7 seen $x>1$, $x<-7$ oe	**B1** **B1** [2]	$x>1$ or $x<-7$ Allow $x\leqslant -7, x\geqslant 1$ oe

11. 9709_s15_ms_13 Q: 1

	Answer	Mark	Partial Marks
	$2(x-3)^2-11$	**B1B1B1** **[3]**	For 2, $(x-3)^2$, −11. Or a=2, b= 3, c= 11

12. 9709_w15_ms_13 Q: 3

	Answer	Mark	Partial Marks
(i)	$[3]\left[(x-1)^2\right][-1]$	**B1B1B1** [3]	
(ii)	$f'(x) = 3x^2 - 6x + 7$	**B1**	Ft *their* **(i)** + 5
	$= 3(x-1)^2 + 4$	**B1√**	
	> 0 hence increasing	**DB1** [3]	Dep B1√ unless other valid reason

13. 9709_m22_ms_12 Q: 5

Question	Answer	Marks	Guidance
(a)	$2[\{(x-2)^2\}\ \{+3\}]$	**B1 B1**	B1 for $a = 2$, B1 for $b = 3$. $2(x-2)^2+6$ gains B1B0
		2	
(b)	{Translation} $\begin{pmatrix}\{2\}\\\{3\}\end{pmatrix}$ OR {Stretch} {y direction} {factor 2}	**B2,1,0**	B2 for fully correct, B1 with two elements correct. {} indicates different elements.
	{Stretch} {y direction} {factor 2} OR {Translation} $\begin{pmatrix}\{2\}\\\{6\}\end{pmatrix}$	**B2,1,0**	B2 for fully correct, B1 with two elements correct. {} indicates different elements.
		4	

14. 9709_m22_ms_12 Q: 9

Question	Answer	Marks	Guidance
(a)	$\left[x^{\frac{1}{2}} = \right]\dfrac{4 \pm \sqrt{16-4}}{2} = 2 \pm \sqrt{3}$	**M1 A1**	OE. Answer must come from formula or completing square. If M0A0 scored then **SC B1** for $2 \pm \sqrt{3}$ only.
	$[x =]\left(2 \pm \sqrt{3}\right)^2$	**M1**	Attempt to square *their* $2 \pm \sqrt{3}$
	$7 + 4\sqrt{3}\ ,\ 7 - 4\sqrt{3}$	**A1**	Accept $7 \pm 4\sqrt{3}$ or $a = 7, b = \pm 4, c = 3$ **SC B1** instead of second M1A1 for correct final answer only.
	Alternative method for question 9(a)		
	$-4x^{\frac{1}{2}} + 1 = 0$ leading to $(x+1)^2 = 16x$ leading to $x^2 - 14x + 1 = 0$	***M1 A1**	OE
	$x = \dfrac{14 \pm \sqrt{196-4}}{2}$	**DM1**	Attempt to solve for x
	$7 + 4\sqrt{3}\ ,\ 7 - 4\sqrt{3}$	**A1**	**SC B1** instead of second M1A1 for correct final answer only.
		4	
(b)	$[\mathrm{gh}(x) =]\ m\left(x^{\frac{1}{2}} - 2\right)^2 + n$	**M1**	SOI
	$\left[\mathrm{gh}(x) =\right]\ m\left(x - 4x^{\frac{1}{2}} + 4\right) + n \equiv x - 4x^{\frac{1}{2}} + 1$	**A1**	SOI
	$m = 1,\ n = -3$	**A1 A1**	WWW
		4	

15. 9709_m21_ms_12 Q: 5

Question	Answer	Marks	Guidance
(a)	(Stretch) (factor 3 in y direction **or** parallel to the y-axis)	**B1 B1**	
	(Translation) $\begin{pmatrix}4\\0\end{pmatrix}$	**B1 B1**	Allow Translation 4 (units) in x direction. N.B. Transformations can be given in either order.
		4	
(b)	[y =] 3f($x-4$)	**B1 B1**	B1 for 3 , B1 for $(x-4)$ with no extra terms.
		2	

16. 9709_m21_ms_12 Q: 7

Question	Answer	Marks	Guidance
(a)	$\left[\text{f}\left(x\right)=\right]\left(x+1\right)^2+2$	**B1 B1**	Accept $a=1, b=2$.
	Range [of f is $\left(y\right)$] $\geqslant 2$	**B1FT**	OE. Do not allow $x \geqslant 2$, FT on *their b*.
		3	
(b)	$y=\left(x+1\right)^2+2$ leading to $x=\left[\pm\right]\sqrt{y-2}-1$	**M1**	Or by using the formula. Allow one sign error.
	$\text{f}^{-1}\left(x\right)=-\sqrt{x-2}-1$	**A1**	
		2	

Question	Answer	Marks	Guidance
(c)	$2\left(x^2+2x+3\right)+1=13$	**B1**	Or using a correct completed square form of f(x).
	$2x^2+4x-6\left[=0\right]$ leading to $\left(2\right)\left(x-1\right)\left(x+3\right)\left[=0\right]$	**B1**	Or $x=1, x=-3$ using formula or completing square. Must reach 2 solutions.
	$x=-3$ **only**	**B1**	
		3	

17. 9709_s21_ms_11 Q: 9

Question	Answer	Marks	Guidance
(a)	Range of f is f(x) $\geqslant -4$	**B1**	Allow y, f or 'range' or $[-4,\infty)$
		1	
(b)	$y=(x-2)^2-4 \Rightarrow (x-2)^2=y+4 \Rightarrow x-2=+\sqrt{(y+4)}$ or $\pm\sqrt{(y+4)}$	**M1**	May swap variables here
	$\left[\text{f}^{-1}\left(x\right)\right]=\sqrt{(x+4)}+2$	**A1**	
		2	
(c)	$(x-2)^2-4=-\frac{5}{3}x+2 \Rightarrow x^2-4x+4-4=-\frac{5}{3}x+2$ [$\Rightarrow x^2-\frac{7}{3}x-2=0$]	**M1**	Equating and simplifying to a 3-term quadratic
	$(3x+2)(x-3)\left[=0\right]$ or $\frac{7\pm\sqrt{7^2-4(3)(-6)}}{6}$ OE	**M1**	Solving quadratic
	$x=3$ only	**A1**	
		3	

Question	Answer	Marks	Guidance
(d)	$f^{-1}(12) = 6$	**M1**	Substitute 12 into *their* $f^{-1}(x)$ and evaluate
	$g(f^{-1}(12)) = 6a + 2$	**M1**	Substitute *their* '6' into $g(x)$
	$g(g(f^{-1}(12))) = a(6a + 2) + 2 = 62$	**M1**	Substitute the result into $g(x)$ and = 62
	$6a^2 + 2a - 60$ [= 0]	**M1**	Forming and solving a 3-term quadratic
	$a = -\frac{10}{3}$ or 3	**A1**	
	Alternative method for Question 9(d)		
	$g(f^{-1}(x)) = a\left(\sqrt{x+4}+2\right)+2$ or $gg(x) = a(ax+2)+2$	**M1**	Substitute *their* $f^{-1}(x)$ or $g(x)$ into $g(x)$
	$g(g(f^{-1}(x))) = a\left(a\left(\sqrt{x+4}+2\right)+2\right)+2$	**M1**	Substitute the result into $g(x)$
	$g(g(f^{-1}(12))) = a(6a + 2) + 2 = 62$	**M1**	Substitute 12 and = 62
	$6a^2 + 2a - 60$ [= 0]	**M1**	Forming and solving a 3-term quadratic
	$a = -\frac{10}{3}$ or 3	**A1**	
		5	

18. 9709_s21_ms_12 Q: 2

Question	Answer	Marks	Guidance
(a)	Translation $\begin{pmatrix}1\\0\end{pmatrix}$	B1	Allow shift and allow by 1 in x-direction or [parallel to/on/in/ along/against] the x-axis or horizontally. 'Translation by 1 to the right' only, scores B0
	Stretch	B1	Stretch. **SC B2** for amplitude doubled.
	Factor 2 in y-direction	B1	With/by **factor** 2 in y-direction or [parallel to/on/in/along/against] the y-axis or vertically or with x axis invariant 'With/by factor 2 upwards' only, scores B0. Accept SF as an abbreviation for scale factor.
		3	**Note:** Transformations can be in either order
(b)	$[-\sin 6x][+ 15x]$ or $[\sin(-6x)][+ 15x]$ OE	B1 B1	Accept an unsimplified version. ISW. B1 for each correct component – square brackets indicate each required component.
			If B0, **SC B1** for either $\sin(-2x) + 5x$ or $-\sin(2x) + 5x$ or $\sin 6x - 15x$ or $\sin\left(-\frac{2}{3}x\right)+\frac{5}{3}x$
		2	

19. 9709_s21_ms_12 Q: 5

Question	Answer	Marks	Guidance
(a)	$\text{ff}(x) = 2(2x^2 + 3)^2 + 3$	M1	Condone = 0.
	$8x^4 + 24x^2 + 21$	A1	ISW if correct answer seen. Condone = 0.
		2	
(b)	$8x^4 + 24x^2 + 21 = 34x^2 + 19 \Rightarrow 8x^4 + 24x^2 - 34x^2 + 21 - 19$ [= 0]	M1	Equating $34x^3 + 19$ to *their* 3-term ff(x) and collect all terms on one side condone ± sign errors.
	$8x^4 - 10x^2 + 2$[= 0]	A1	
	$[2](x^2 - 1)(4x^2 - 1)$	M1	Attempt to solve 3-term quartic or 3-term quadratic by factorisation, formula or completing the square or factor theorem.
	$\left[x^2 = 1 \text{ or } \frac{1}{4} \text{ leading to}\right] x = 1 \text{ or } x = \frac{1}{2}$	A1	If factorising, factors must expand to give $8x^4$ or $4x^4$ 4 or *their* ax^4 otherwise M0A0 due to calculator use. Condone ± 1, $\pm\frac{1}{2}$ but not $\sqrt{\frac{1}{4}}$ or $\sqrt{1}$.
		4	

20. 9709_s21_ms_13 Q: 6

Question	Answer	Marks	Guidance
(a)	$f(x) = (x-1)^2 + 4$	**B1**	
	$g(x) = (x+2)^2 + 9$	**B1**	
	$g(x) = f(x+3) + 5$	**B1 B1**	B1 for each correct element. Accept $p = 3, q = 5$
		4	

Question	Answer	Marks	Guidance
(b)	Translation or Shift	**B1**	
	$\begin{pmatrix}-3\\5\end{pmatrix}$ or acceptable explanation	**B1 FT**	If given as 2 single translations both must be described correctly e.g. $\begin{pmatrix}-3\\0\end{pmatrix}$ & $\begin{pmatrix}0\\5\end{pmatrix}$ FT from *their* $f(x+p)+q$ *or their* $f(x) \to g(x)$ Do not accept $\begin{pmatrix}1\\4\end{pmatrix}$ or $\begin{pmatrix}-2\\9\end{pmatrix}$
		2	

21. 9709_s21_ms_13 Q: 8

Question	Answer	Marks	Guidance
(a)	$[\text{fg}(x) =] 1/(2x+1)^2 - 1$	**B1**	SOI
	$1/(2x+1)^2 - 1 = 3$ leading to $4(2x+1)^2 = 1$ or $\frac{1}{(2x+1)} = [\pm]2$ or $16x^2 + 16x + 3 = 0$	**M1**	Setting $\text{fg}(x) = 3$ and reaching a stage before $2x+1 = \pm½$ or reaching a 3 term quadratic in x
	$2x+1 = \pm½$ or $2x+1 = -½$ or $(4x+1)(4x+3)[=0]$	**A1**	Or formula or completing square on quadratic
	$x = -\frac{3}{4}$ only	**A1**	
	Alternative method for Question 8(a)		
	$x^2 - 1 = 3$	**M1**	
	$\text{g}(x) = -2$	**A1**	
	$\frac{1}{(2x+1)} = -2$	**M1**	
	$x = -\frac{3}{4}$ only	**A1**	
		4	

Question	Answer	Marks	Guidance
(b)	$y = \frac{1}{(2x+1)^2} - 1$ leading to $(2x+1)^2 = \frac{1}{y+1}$ leading to $2x+1 = [\pm]\frac{1}{\sqrt{y+1}}$	***M1**	Obtain $2x+1$ or $2y+1$ as the subject
	$x = [\pm]\frac{1}{2\sqrt{y+1}} - \frac{1}{2}$	**DM1**	Make x (or y) the subject
	$-\frac{1}{2\sqrt{x+1}} - \frac{1}{2}$	**A1**	OE e.g. $-\frac{\sqrt{x+1}}{2x+2} - \frac{1}{2}$, $-\left(\sqrt{\frac{-x}{4x+4}} + \frac{1}{4} + \frac{1}{2}\right)$
		3	

22. 9709_w21_ms_11 Q: 8

Question	Answer	Marks	Guidance
(a)	$\{-3(x-2)^2\}$ $\{+14\}$	**B1 B1**	B1 for each correct term; condone $a = 2$, $b = 14$.
		2	
(b)	$[k =]$ 2	**B1**	Allow $[x] \leqslant 2$.
		1	

Question	Answer	Marks	Guidance
(c)	[Range is] $[y] \leqslant -13$	**B1**	Allow [f(x)] $\leqslant -13$, [f] $\leqslant -13$ but NOT $x \leqslant -13$.
		1	
(d)	$y = -3(x-2)^2 + 14$ leading to $(x-2)^2 = \frac{14-y}{3}$	**M1**	Allow $\frac{y-14}{-3}$. Allow 1 error in rearrangement if x, y on opposite sides.
	$x = 2(\pm)\sqrt{\frac{14-y}{3}}$	**A1**	Allow $\frac{y-14}{-3}$.
	$[f^{-1}(x)] = 2 - \sqrt{\frac{14-x}{3}}$	**A1**	OE. Allow $\frac{x-14}{-3}$. Must be x on RHS; must be negative square root only.
	Alternative method for question 8(d)		
	$x = -3(y-2)^2 + 14$ leading to $(y-2)^2 = \frac{14-x}{3}$	**M1**	Allow $\frac{x-14}{-3}$. Allow 1 error in rearrangement if x, y on opposite sides.
	$= 2(\pm)\sqrt{\frac{14-x}{3}}$	**A1**	Allow $\frac{x-14}{-3}$.
	$[f^{-1}(x)] = 2 - \sqrt{\frac{14-x}{3}}$	**A1**	OE. Allow $\frac{x-14}{-3}$. Must be x on RHS; must be negative square root only.
		3	

Question	Answer	Marks	Guidance
(e)	[g(x) =] $\{-3(x+3-2)^2\} + \{14+1\}$	**B2, 1, 0**	OR $\{-3(x+3)^2\} + \{12(x+3)\} + \{3\}$
	$g(x) = -3x^2 - 6x + 12$	**B1**	
		3	

23. 9709_w21_ms_12 Q: 2

Question	Answer	Marks	Guidance
(a)	Stretch with [scale factor] either ± 2 or $\pm\frac{1}{2}$	**B1**	
	Scale factor $\frac{1}{2}$ in the x-direction	**B1**	
	Translation $\begin{pmatrix} 0 \\ -3 \end{pmatrix}$ or translation of 3 units in negative y-direction	**B1**	
		3	
(b)	(10 , 9)	**B1 B1**	B1 for each correct co-ordinate.
		2	

24. 9709_w21_ms_12 Q: 3

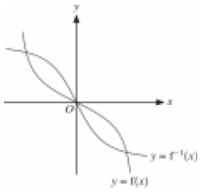
Question	Answer	Marks	Guidance
(a)	$f(5) = [2]$ and $f(\textit{their}\,2) = [5]$ OR $ff(5) = \left[\frac{2+3}{2-1}\right]$ OR $\frac{\frac{x+3}{x-1}+3}{\frac{x+3}{x-1}-1}$ and an attempt to substitute x =5.	**M1**	Clear evidence of applying f twice with x = 5.
	5	**A1**	
		2	

Question	Answer	Marks	Guidance
(b)	$\frac{x+3}{x-1}=y\Rightarrow x+3=xy-y$ OR $\frac{y+3}{y-1}=x\Rightarrow y+3=xy-x$	*M1	Setting f(*x*) = *y* or swapping *x* and *y*, clearing of fractions and expanding brackets. Allow ± sign errors.
	$xy-x=y+3\Rightarrow x=\frac{y+3}{y-1}$ OE OR $y+3=xy-x\Rightarrow y=\left[\frac{x+3}{x-1}\right]$ OE	DM1	Finding *x* or *y* = . Allow ± sign errors.
	$[f^{-1}(x)\text{or } y]=\frac{x+3}{x-1}$	A1	OE e.g. $1+\frac{4}{x-1}$ etc. Must be a function of *x*, cannot be *x* = .
		3	

25. 9709_w21_ms_13 Q: 1

Question	Answer	Marks	Guidance
	{Reflection} {[in the] *x*-axis} or {Stretch of scale factor -1} {parallel to y-axis}	*B1 DB1	{} indicate how the B1 marks should be awarded throughout.
	Then {Translation} $\left\{\begin{pmatrix}0\\3\end{pmatrix}\right\}$	B1 B1	Or Translation 3 units in the positive *y*-direction. **N.B.** If order reversed a maximum of 3 out of 4 marks awarded.
	Alternative method for question 1		
	{Translation} $\left\{\begin{pmatrix}0\\-3\end{pmatrix}\right\}$	B1 B1	Or Translation 3 units in the negative *y*-direction.
	Then {Reflection} {in the *x*-axis} or {Stretch of scale factor -1} {parallel to *y*-axis}	*B1 DB1	**N.B.** If order reversed a maximum of 3 out of 4 marks awarded.
		4	

26. 9709_w21_ms_13 Q: 6

Question	Answer	Marks	Guidance
(a)		B1	A reflection of the given curve in *y* = *x* (the line *y* = *x* can be implied by position of curve).
		1	

Question	Answer	Marks	Guidance
(b)	$y=\frac{-x}{\sqrt{4-x^2}}$ leading to $x^2=y^2\left(4-x^2\right)$	*M1	Squaring and clearing the fraction. Condone one error in squaring $-x$ or y
	$x^2\left(1+y^2\right)=4y^2$	DM1	OE. Factorisation of the new subject with order of operations correct. Condone sign errors.
	$x=(\pm)\frac{2y}{\sqrt{1+y^2}}$	DM1	$x=(\pm)\sqrt{\left(\frac{4y^2}{(1+y^2}\right)}$ OE is acceptable for this mark. Isolating the new subject. Order of operations correct. Condone sign errors.
	$\mathrm{f}^{-1}(x)=\frac{-2x}{\sqrt{1+x^2}}$	A1	Selecting the correct square root. Must not have fractions in numerator or denominator.
		4	
(c)	1 or $a=1$	B1	Do not allow $x=1$ or $-1<x<1$
		1	
(d)	$[\mathrm{fg}(x)=\mathrm{f}(2x)=]\ \frac{-2x}{\sqrt{4-4x^2}}$	B1	Allow $\frac{-2x}{\sqrt{4-(2x)^2}}$ or any correct unsimplified form.
	$\mathrm{fg}(x)=\frac{-x}{\sqrt{1-x^2}}$ or $\frac{-x}{1-x^2}\sqrt{1-x^2}$ or $\frac{x}{x^2-1}\sqrt{1-x^2}$	B1	Result of cancelling 2 in numerator and denominator.
		2	

27. 9709_m20_ms_12 Q: 2

	Answer	Mark	Partial Marks
	[Stretch] [factor 2, x direction (or y-axis invariant)]	*B1 DB1	
	[Translation or Shift] [1 unit in y direction] **or** [Translation/Shift] $\left[\begin{pmatrix}0\\1\end{pmatrix}\right]$	B1B1	Accept transformations in either order. Allow (0, 1) for the vector
		4	

28. 9709_m20_ms_12 Q: 9

	Answer	Mark	Partial Marks
(a)	$\left[2(x+3)^2\right][-7]$	**B1B1**	Stating $a=3, b=-7$ gets B1B1
		2	
(b)	$y=2(x+3)^2-7 \rightarrow 2(x+3)^2=y+7 \rightarrow (x+3)^2=\frac{y+7}{2}$	**M1**	First 2 operations correct. Condone sign error or with x/y interchange
	$x+3=(\pm)\sqrt{\frac{y+7}{2}} \rightarrow x=(\pm)\sqrt{\frac{y+7}{2}}-3 \rightarrow \ \mathrm{f}^{-1}(x)=-\sqrt{\frac{x+7}{2}}-3$	**A1FT**	FT on *their* a and b. Allow y = ...
	Domain: $x \geqslant -5$ **or** $\geqslant$ -5 **or** $[-5, \infty)$	**B1**	Do not accept $y=\ldots,\ f(x)=\ldots, f^{-1}(x)=\ldots$
		3	
(c)	$\mathrm{fg}(x) = 8x^2-7$	**B1FT**	SOI. FT on *their* -7 from part **(a)**
	$8x^2-7=193 \rightarrow x^2=25 \rightarrow x=-5$ only	**B1**	
	Alternative method for question 9(c)		
	$\mathrm{g}(x) = \mathrm{f}^{-1}(193) \rightarrow\ 2x-3=-\sqrt{100}-3$	**M1**	FT on *their* $\mathrm{f}^{-1}(x)$
	$x=-5$ only	**A1**	
		2	
(d)	(Largest k is) $-\frac{1}{2}$	**B1**	Accept $-\frac{1}{2}$ or $k \leqslant -\frac{1}{2}$
		1	

29. 9709_s20_ms_11 Q: 6

(a)	$3(3x+b)+b=9x+4b \rightarrow 10=18+4b$	**M1**
	$b=-2$	**A1**
	Either $\mathrm{f}(14)=2$ **or** $\mathrm{f}^{-1}(x)=2(x+a)$ etc.	**M1**
	$a=5$	**A1**
		4
(b)	$\mathrm{gf}(x) = 3\left(\frac{1}{2}x-5\right)-2$	**M1**
	$\mathrm{gf}(x) = \frac{3}{2}x-17$	**A1**
		2

30. 9709_s20_ms_12 Q: 5

(a)	$\mathrm{ff}(x) = a-2(a-2x)$	**M1**
	$\mathrm{ff}(x) = 4x-a$	**A1**
	$\mathrm{f}^{-1}(x) = \frac{a-x}{2}$	**M1 A1**
		4
(b)	$4x-a=\frac{a-x}{2} \rightarrow 9x=3a$	**M1**
	$x=\frac{a}{3}$	**A1**
		2