Contents

Chapter 1

Algebra

1. 9709_s20_qp_31 Q: 2

(a) Expand $(2 - 3x)^{-2}$ in ascending powers of x, up to and including the term in x^2, simplifying the coefficients.   [4]

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(b) State the set of values of x for which the expansion is valid.   [1]

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2. 9709_s20_qp_32 Q: 1

Find the quotient and remainder when $6x^4 + x^3 - x^2 + 5x - 6$ is divided by $2x^2 - x + 1$. [3]

3. 9709_s20_qp_33 Q: 1

Solve the inequality $|2x - 1| > 3|x + 2|$.   [4]

4. 9709_w20_qp_31 Q: 1

Solve the inequality $2 - 5x > 2|x - 3|$. [4]

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5. 9709_w20_qp_31   Q: 9

Let $f(x) = \dfrac{8 + 5x + 12x^2}{(1 - x)(2 + 3x)^2}$.

(a) Express $f(x)$ in partial fractions.   [5]

(b) Hence obtain the expansion of f(x) in ascending powers of x, up to and including the term in x^2.

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6. 9709_w20_qp_32 Q: 2

(a) Expand $\sqrt[3]{1 + 6x}$ in ascending powers of x, up to and including the term in x^3, simplifying the coefficients. [4]

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(b) State the set of values of x for which the expansion is valid. [1]

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7. 9709_m19_qp_32 Q: 8

Let $f(x) = \dfrac{12 + 12x - 4x^2}{(2 + x)(3 - 2x)}$.

(i) Express $f(x)$ in partial fractions. [5]

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(ii) Hence obtain the expansion of $f(x)$ in ascending powers of x, up to and including the term in x^2.

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8. 9709_s19_qp_31 Q: 8

Let $f(x) = \dfrac{16 - 17x}{(2 + x)(3 - x)^2}$.

(i) Express $f(x)$ in partial fractions. [5]

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Eminence in exam preparation

Appendix A

Answers

1. 9709_s20_MS_31 Q: 2

(a)	State a correct unsimplified version of the x or x^2 term of the expansion of $(2-3x)^{-2}$ or $\left(1-\dfrac{3}{2}x\right)^{-2}$	M1
	State correct first term $\dfrac{1}{4}$	B1
	Obtain the next two terms $\dfrac{3}{4}x + \dfrac{27}{16}x^2$	A1 + A1
		4
(b)	State answer $|x| < \dfrac{2}{3}$, or equivalent	B1
		1

2. 9709_s20_MS_32 Q: 1

	Commence division and reach partial quotient $3x^2 + kx$	M1
	Obtain quotient $3x^2 + 2x - 1$	A1
	Obtain remainder $2x - 5$	A1
		4

3. 9709_s20_MS_33 Q: 1

	State or imply non-modular inequality $(2x-1)^2 > 3^2(x+2)^2$, or corresponding quadratic equation, or pair of linear equations	B1
	Make reasonable attempt at solving a 3-term quadratic, or solve two linear equations for x	M1
	Obtain critical values $x = -7$ and $x = -1$	A1
	State final answer $-7 < x < -1$	A1
	Alternative method for question 1	
	Obtain critical value $x = -1$ from a graphical method, or by solving a linear equation or linear inequality	B1
	Obtain critical value $x = -7$ similarly	B2
	State final answer $-7 < x < -1$ [Do not condone $\leqslant$ for $<$ in the final answer.]	B1
		4

4. 9709 _w20_MS _31 Q: 1

	Answer	Mark	Partial Marks
	Make a recognisable sketch graph of $y = 2|x - 3|$ and the line $y = 2 - 5x$	B1	Need to see correct V at $x = 3$, roughly symmetrical, $x = 3$ stated, domain at least $(-2, 5)$.
	Find x-coordinate of intersection with $y = 2 - 5x$	M1	Find point of intersection with $y = 2|x - 3|$ or solve $2 - 5x$ with $2(x - 3)$ or $-2(x - 3)$
	Obtain $x = -\dfrac{4}{3}$	A1	
	State final answer $x < -\dfrac{4}{3}$	A1	Do not accept $x < -1.33$ [Do not condone $\leqslant$ for $<$ in the final answer.]
Alternative method for question 1			
	State or imply non-modular inequality/equality $(2 - 5x)^2 >, \geqslant, =, 2^2(x - 3)^2$, or corresponding quadratic equation, or pair of linear equations $(2 - 5x) >, \geqslant, =, \pm 2(x - 3)$	B1	Two correct linear equations only
	Make reasonable attempt at solving a 3-term quadratic, or solve one linear equation, or linear inequality for x	M1	$21x^2 + 4x - 32 = (3x + 4)(7x - 8) = 0$ $2 - 5x$ or $-(2 - 5x)$ with $2(x - 3)$ or $-2(x - 3)$
	Obtain critical value $x = -\dfrac{4}{3}$	A1	
	State final answer $x < -\dfrac{4}{3}$	A1	Do not accept $x < -1.33$ [Do not condone $\leqslant$ for $<$ in the final answer.]
		4	

5. 9709_w20_MS_31 Q: 9

	Answer	Mark	Partial Marks
(a)	State or imply the form $\dfrac{A}{1-x}+\dfrac{B}{2+3x}+\dfrac{C}{(2+3x)^2}$	B1	
	Use a correct method for finding a coefficient	M1	
	Obtain one of $A=1$, $B=-1$, $C=6$	A1	
	Obtain a second value	A1	
	Obtain the third value	A1	In the form $\dfrac{A}{1-x}+\dfrac{Dx+E}{(2+3x)^2}$, where $A=1$, $D=-3$ and $E=4$ can score B1 M1 A1 A1 A1 as above.
		5	
(b)	Use a correct method to find the first two terms of the expansion of $(1-x)^{-1}$, $(2+3x)^{-1}$, $\left(1+\dfrac{3}{2}x\right)^{-1}$, $(2+3x)^{-2}$ or $\left(1+\dfrac{3}{2}x\right)^{-2}$	M1	Symbolic coefficients are not sufficient for the M1 $A\left[\dfrac{1+(-1)(-x)+\dfrac{(-1)(-2)(-x)^2}{2}\dots}{}\right]$ $A=1$ $\dfrac{B}{2}\left[\dfrac{1+(-1)\left(\dfrac{3x}{2}\right)+\dfrac{(-1)(-2)\left(\dfrac{3x}{2}\right)^2}{2}\dots}{}\right]$ $B=1$ $\dfrac{C}{4}\left[\dfrac{1+(-2)\left(\dfrac{3x}{2}\right)+\dfrac{(-2)(-3)\left(\dfrac{3x}{2}\right)^2}{2}\dots}{}\right]$ $C=6$
	Obtain correct un-simplified expansions up to the term in of each partial fraction	A1 FT +	$(1+x+x^2)+\left(-\dfrac{1}{2}+\left(\dfrac{3}{4}\right)x-\left(\dfrac{9}{8}\right)x^2\right)$
		A1 FT +	$+\left(\dfrac{6}{4}-\left(\dfrac{18}{4}\right)x+\left(\dfrac{81}{8}\right)x^2\right)$ [The FT is on A, B, C]
		A1 FT	$\left(1-\dfrac{1}{2}+\dfrac{6}{4}\right)+\left(1+\dfrac{3}{4}-\dfrac{18}{4}\right)x+\left(1-\dfrac{9}{8}+\dfrac{81}{8}\right)x^2$
	Obtain final answer $2-\dfrac{11}{4}x+10x^2$, or equivalent	A1	Allow unsimplified fractions $\dfrac{(Dx+E)}{4}\left[\dfrac{1+(-2)\left(\dfrac{3x}{2}\right)+\dfrac{(-2)(-3)\left(\dfrac{3x}{2}\right)^2}{2}\dots}{}\right]$ $D=-3$, $E=4$ The FT is on A, D, E.
		5	

6. 9709_w20_MS_32 Q: 2

	Answer	Mark	Partial Marks
(a)	State a correct unsimplified version of the x or x^2 or x^3 term	M1	For the given expression
	State correct first two terms $1+2x$	A1	
	Obtain the next two terms $-4x^2+\dfrac{40}{3}x^3$	A1 + A1	One mark for each correct term. ISW Accept $13\frac{1}{3}$ The question asks for simplified coefficients, so candidates should cancel fractions.
		4	
(b)	State answer $|x|<\dfrac{1}{6}$	B1	OE. Strict inequality
		1	

7. 9709_m19_MS_32 Q: 8

	Answer	Mark	Partial Marks
(i)	State or imply the form $A + \dfrac{B}{2+x} + \dfrac{C}{3-2x}$	**B1**	
	Use a correct method for finding a constant	**M1**	
	Obtain one of $A = 2$, $B = -4$ and $C = 6$	**A1**	
	Obtain a second value	**A1**	
	Obtain the third value	**A1**	
		5	
(ii)	Use correct method to find the first two terms of the expansion of $(2+x)^{-1}$ or $(3-2x)^{-1}$, or equivalent	**M1**	
	Obtain correct unsimplified expansions up to the term in x^2 of each partial fraction	**A1ft +A1ft**	The ft is on B and C
	Add the value of A to the sum of the expansions	**M1**	
	Obtain final answer $2 + \dfrac{7}{3}x + \dfrac{7}{18}x^2$	**A1**	
		5	

8. 9709_s19_MS_31 Q: 8

	Answer	Mark	Partial Marks
(i)	State or imply the form $\dfrac{A}{2+x} + \dfrac{B}{3-x} + \dfrac{C}{(3-x)^2}$	**B1**	
	Use a correct method to obtain a constant	**M1**	
	Obtain one of $A = 2$, $B = 2$, $C = -7$	**A1**	
	Obtain a second value	**A1**	
	Obtain the third value	**A1**	[Mark the form $\dfrac{A}{2+x} + \dfrac{Dx+E}{(3-x)^2}$, where $A = 2$, $D = -2$ and $E = -1$, B1M1A1A1A1.]
		5	
(ii)	Use a correct method to find the first two terms of the expansion of $(2+x)^{-1}$, $(3-x)^{-1}$ or $(3-x)^{-2}$, or equivalent, e.g. $\left(1+\dfrac{1}{2}x\right)^{-1}$	**M1**	
	Obtain correct unsimplified expansions up to the term in x^2 of each partial fraction	**A1** **A1** **A1**	**FT** on A, B and C $1 - \dfrac{x}{2} + \dfrac{x^2}{4} \quad \dfrac{2}{3}\left(1 + \dfrac{x}{3} + \dfrac{x^2}{9}\right) \quad -\dfrac{7}{9}\left(1 + \dfrac{2x}{3} + \dfrac{3x^2}{9}\right)$
	Obtain final answer $\dfrac{8}{9} - \dfrac{43}{54}x + \dfrac{7}{108}x^2$	**A1**	
			For the A, D, E form of fractions give M1A1**ft**A1**ft** for the expanded partial fractions, then, if $D \neq 0$, M1 for multiplying out fully, and A1 for the final answer.
		5	