

Contents

1	Algebra	3
2	Logarithmic and exponential functions	51
3	Trigonometry	91
4	Differentiation	123
5	Integration	183
6	Numerical solution of equations	245
A	Answers	287

Chapter 1

Algebra

2. 9709_s20_qp_21 Q: 4

- (a) Sketch, on the same diagram, the graphs of $y = |3x + 2a|$ and $y = |3x - 4a|$, where a is a positive constant.

Give the coordinates of the points where each graph meets the axes. [3]

- (b) Find the coordinates of the point of intersection of the two graphs. [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

- (c) Deduce the solution of the inequality $|3x + 2a| < |3x - 4a|$. [1]

.....

.....

.....

.....

3. 9709_s20_qp_22 Q: 5

(a) Sketch, on the same diagram, the graphs of $y = |2x - 3|$ and $y = 3x + 5$. [2]

(b) Solve the inequality $3x + 5 < |2x - 3|$. [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

6. 9709_w20_qp_21 Q: 4

(a) Solve the equation $|2x - 5| = |x + 6|$. [3]

.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....

(b) Hence find the value of y such that $|2^{1-y} - 5| = |2^{-y} + 6|$. Give your answer correct to 3 significant figures. [2]

.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....

7. 9709_w20_qp_22 Q: 3

- (a) Sketch, on a single diagram, the graphs of $y = \left| \frac{1}{2}x - a \right|$ and $y = \frac{3}{2}x - \frac{1}{2}a$, where a is a positive constant. [2]

- (b) Find the coordinates of the point of intersection of the two graphs. [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

- (c) Deduce the solution of the inequality $\left| \frac{1}{2}x - a \right| > \frac{3}{2}x - \frac{1}{2}a$. [1]

.....

.....

.....

.....

.....

9. 9709_m19_qp_22 Q: 4

- (i) Find the quotient when $4x^3 + 8x^2 + 11x + 9$ is divided by $(2x + 1)$, and show that the remainder is 5. [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

- ii) Show that the equation $4x^3 + 8x^2 + 11x + 4 = 0$ has exactly one real root. [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

Appendix A

Answers

1. 9709_s20_MS_21 Q: 2

	Substitute $x = 2$ and equate to zero	M1
	Substitute $x = -\frac{1}{2}$ and equate to zero	M1
	Obtain $4a + b + 66 = 0$ and $\frac{1}{4}a + b - \frac{21}{4} = 0$ or equivalents	A1
	Solve a relevant pair of linear simultaneous equations (Dependent on at least one M mark)	DM1
	Obtain $a = -19, b = 10$	A1
		5

2. 9709_s20_MS_21 Q: 4

(a)	Draw two V-shaped graphs with one vertex on negative x -axis and one vertex on positive x -axis	M1
	Draw correct graphs related correctly to each other	A1
	State correct coordinates $-\frac{2}{3}a, 2a, \frac{4}{3}a, 4a$	A1
		3
(b)	Solve linear equation with signs of $3x$ different or solve non-modulus equation $(3x + 2a)^2 = (3x - 4a)^2$	M1
	Obtain $x = \frac{1}{3}a$	A1
	Obtain $y = 3a$	A1
		3
(c)	State $x < \frac{1}{3}a$ (FT from part (b))	B1FT
		1

3. 9709_s20_MS_22 Q: 5

(a)	Draw V-shaped graph with vertex on positive x -axis	B1
	Draw (more or less) correct graph of $y = 3x + 5$	B1
		2
(b)	State equation $3x + 5 = -(2x - 3)$ or corresponding inequality	B1
	Attempt solution of linear equation / inequality where signs of $3x$ and $2x$ are different	M1
	State answer $x < -\frac{2}{5}$	A1
	Alternative method for question 5(b)	
	Square both sides of equation / inequality and attempt solution of 3-term quadratic equation / inequality	M1
	Obtain (eventually) only $-\frac{2}{5}$	A1
	State answer $x < -\frac{2}{5}$	A1
		3

4. 9709_w20_MS_21 Q: 1

	Answer	Mark	Partial Marks
	Use correct logarithm property to simplify left-hand side	M1	Or equivalent method
	Use correct process to obtain equation without logarithms	M1	
	Obtain $\frac{2x+1}{x-3} = e^2$	A1	OE
	Obtain $x = \frac{3e^2+1}{e^2-2}$	A1	OE
		4	

5. 9709_w20_MS_21 Q: 2

	Answer	Mark	Partial Marks
	Substitute $x = -2$ and equate to zero	*M1	
	Substitute $x = 2$ and equate to 72	*M1	
	Obtain $4a - 2b + 8 = 0$ and $4a + 2b - 48 = 0$ or equivalents	A1	
	Solve a pair of relevant linear simultaneous equations	DM1	Dependent at least one M mark
	Obtain $a = 5, b = 14$	A1	
		5	

6. 9709_w20_MS_21 Q: 4

	Answer	Mark	Partial Marks
(a)	State or imply non-modulus equation $(2x-5)^2 = (x+6)^2$ or pair of linear equations	B1	
	Attempt solution of 3-term quadratic equation or of pair of linear equations	M1	
	Obtain $-\frac{1}{3}$ and 11	A1	
		3	
(b)	Apply logarithms and use power law for $2^{-y} = k$ where $k > 0$ from (a)	M1	
	Obtain -3.46	A1	AWRT
		2	

7. 9709_w20_MS_22 Q: 3

	Answer	Mark	Partial Marks
(a)	Draw V-shaped graph with vertex on positive x-axis	B1	
	Draw straight line graph correctly positioned with greater gradient	B1	
		2	
(b)	Solve linear equation with signs of $\frac{1}{2}x$ and $\frac{3}{2}x$ different or solve non-modulus equation $\left(\frac{1}{2}x-a\right)^2 = \left(\frac{3}{2}x-\frac{1}{2}a\right)^2$ to obtain $x =$	M1	
	Obtain $x = \frac{3}{4}a$	A1	
	Obtain $y = \frac{5}{8}a$	A1	And no other point
		3	
(c)	State $x < \frac{3}{4}a$	B1 FT	Following <i>their</i> (single) x-coordinate from part (b)
		1	

8. 9709_m19_MS_22 Q: 2

	Answer	Mark	Partial Marks
	Solve non-modular equation $(2x+3)^2 = (2x-1)^2$ or linear equation with signs of $2x$ different	M1	
	Obtain $x = -\frac{1}{2}$	A1	
	Substitute negative value into expression and show correct evaluation of modulus at least once	M1	
	Obtain $5-3=2$ with no errors seen	A1	
		4	

9. 9709_m19_MS_22 Q: 4

	Answer	Mark	Partial Marks
(i)	Carry out division at least as far as $2x^2 + kx$	M1	
	Obtain quotient $2x^2 + 3x + 4$	A1	
	Confirm remainder is 5	A1	Answer given; necessary detail needed
		3	
(ii)	State or imply equation is $(2x+1)(2x^2 + 3x + 4) = 0$	B1	FT their quotient from part (i)
	Calculate discriminant of 3-term quadratic expression or equivalent	M1	
	Obtain -23 or equiv and conclude appropriately	A1	
		3	

10. 9709_s19_MS_21 Q: 2

	Answer	Mark	Partial Marks
(i)	State or imply non-modular inequality $(3x-5)^2 < (x+3)^2$ or corresponding equation or pair of different linear equations/inequalities	B1	SC: Allow B1 for $x < 4$ from only one linear inequality
	Attempt solution of 3-term quadratic equation/inequality or of two different linear equations/inequalities	M1	For M1, must get as far as 2 critical values
	Obtain critical values $\frac{1}{2}$ and 4	A1	
	State answer $\frac{1}{2} < x < 4$ or equivalent	A1	If given as 2 separate statements, condone omission of 'and' or \cap but penalise inclusion of 'or' or \cup
		4	
(ii)	Attempt to find n (not necessarily an integer so far) from $3^{0.1n} =$ or $<$ their positive upper value from part (i) or $3^{0.1n+1} =$ or $<$ $3 \times$ their positive upper value from part (i)	M1	0/2 for trial and improvement
	Conclude 12	A1	
		2	

11. 9709_s19_MS_21 Q: 5

	Answer	Mark	Partial Marks
(i)	Substitute $x = 2$ and equate to zero	M1	Allow synthetic division for each– must result in an equation from each division
	Substitute $x = -1$ and equate to 27	M1	Allow unsimplified
	Obtain $4a + 2b = -24$ and $a - b = 48$ or equivalents	A1	Allow one error in each equation
	Solve a relevant pair of simultaneous linear equations	M1	Dependent at least one M mark
	Obtain $a = 12, b = -36$	A1	
		5	
(ii)	Divide by $x - 2$ at least as far as the x term to obtain $5x^2 + (\text{their } a + 10)x \dots$	M1	For synthetic division need to see 5 and their $a + 10$ in the bottom line
	Obtain $5x^2 + 22x + 8$	A1	
	Obtain $(x - 2)(5x + 2)(x + 4)$	A1	If solved using a calculator and then forming factors, must be correct for full marks
		3	