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Chapter 1

Quadratics

1. 9709_s20_qp_11 Q: 5

The equation of a line is $y = mx + c$, where m and c are constants, and the equation of a curve is $xy = 16$.

(a) Given that the line is a tangent to the curve, express m in terms of c . [3]

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(b) Given instead that $m = -4$, find the set of values of c for which the line intersects the curve at two distinct points. [3]

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6. 9709_s16_qp_11 Q: 6

- (a) Find the values of the constant m for which the line $y = mx$ is a tangent to the curve $y = 2x^2 - 4x + 8$. [3]
- (b) The function f is defined for $x \in \mathbb{R}$ by $f(x) = x^2 + ax + b$, where a and b are constants. The solutions of the equation $f(x) = 0$ are $x = 1$ and $x = 9$. Find
- (i) the values of a and b , [2]
- (ii) the coordinates of the vertex of the curve $y = f(x)$. [2]
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7. 9709_w16_qp_11 Q: 1

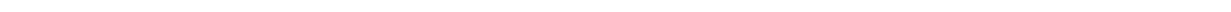
(i) Express $x^2 + 6x + 2$ in the form $(x + a)^2 + b$, where a and b are constants. [2]

(ii) Hence, or otherwise, find the set of values of x for which $x^2 + 6x + 2 > 9$. [2]

8. 9709_s15_qp_13 Q: 1

Express $2x^2 - 12x + 7$ in the form $a(x + b)^2 + c$, where a , b and c are constants.

[3]



9. 9709_w15_qp_13 Q: 3

- (i) Express $3x^2 - 6x + 2$ in the form $a(x + b)^2 + c$, where a , b and c are constants. [3]
- (ii) The function f , where $f(x) = x^3 - 3x^2 + 7x - 8$, is defined for $x \in \mathbb{R}$. Find $f'(x)$ and state, with a reason, whether f is an increasing function, a decreasing function or neither. [3]
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Chapter 2

Functions

Appendix A

Answers

1. 9709_s20_MS_11 Q: 5

(a)	$x(mx+c) = 16 \rightarrow mx^2 + cx - 16 = 0$	B1
	Use of $b^2 - 4ac = c^2 + 64m$	M1
	Sets to 0 $\rightarrow m = \frac{-c^2}{64}$	A1
		3
(b)	$x(-4x+c) = 16$ Use of $b^2 - 4ac \rightarrow c^2 - 256$	M1
	$c > 16$ and $c < -16$	A1 A1
		3

2. 9709_s19_MS_13 Q: 1

	Answer	Mark	Partial Marks
(i)	$[(x-2)^2]$ [+4]	B1 DB1	2nd B1 dependent on 2 inside bracket
		2	
(ii)	$(x-2)^2 < 5 \rightarrow -\sqrt{5} < x-2$ and/or $x-2 < \sqrt{5}$	M1	Allow e.g. $x-2 < \pm\sqrt{5}$, $x-2 = \pm\sqrt{5}$ and decimal equivalents for $\sqrt{5}$ For M1, ft from <i>their</i> (i). Also allow $\sqrt{13}$ instead of $\sqrt{5}$ for clear slip
	$2 - \sqrt{5} < x < 2 + \sqrt{5}$	A1A1	A1 for each inequality – allow two separate statements but there must be 2 inequalities for x . Non-hence methods, if completely correct, score SC 1/3. Condone \leq
		[3]	

3. 9709_s18_MS_13 Q: 1

	Answer	Mark	Partial Marks
	$[3] [(x-2)^2]$ [-5]	B1B1B1	OR $a = 3$, $b = -2$, $c = -5$. 1st mark is dependent on the form $(x+a)^2$ following 3
		3	

4. 9709_w18_MS_11 Q: 1

	Answer	Mark	Partial Marks
	$(4x^{3/2} - 3)(x^{3/2} - 2)$ oe soi Alt: $4x + 6 = 11\sqrt{x} \Rightarrow 16x^2 - 73x + 36$	M1	Attempt solution for $x^{3/2}$ or sub $u = x^{3/2}$
	$x^{3/2} = 3/4$ or 2 $(16x - 9)(x - 4)$	A1	Reasonable solutions for $x^{3/2}$ implies M1 ($x = 2, 3/4, \text{M1A0}$)
	$x = 9/16$ oe or 4	A1	Little or no working shown scores SCB3, spotting one solution, B0
		3	

5. 9709_m17_MS_12 Q: 1

	Answer	Mark	Partial Marks
	$(3k)^2 - 4 \times 2 \times k$	M1	Attempt $b^2 - 4ac$
	$9k^2 - 8k > 0$ soi Allow $9k^2 - 8k \geq 0$	A1	Must involve correct inequality. Can be implied by correct answers
	0, 8/9 soi	A1	
	$k < 0, k > 8/9$ (or 0.889)	A1	Allow $(-\infty, 0), (8/9, \infty)$
	Total:	4	

6. 9709_s16_MS_11 Q: 6

	Answer	Mark	Partial Marks
(a)	$y = 2x^2 - 4x + 8$ Equates with $y = mx$ and selects a, b, c Uses $b^2 = 4ac$ $\rightarrow m = 4$ or -12 .	M1 M1 A1 [3]	Equate + solution or use of dy/dx Use of discriminant for both.
(b) (i)	$f(x) = x^2 + ax + b$ Eqn of form $(x - 1)(x - 9)$ $\rightarrow a = -10, b = 9$ (or using 2 sim eqns M1 A1)	M1 A1 [2]	Any valid method allow $(x + 1)(x + 9)$ for M1 must be stated
(ii)	Calculus or $x = \frac{1}{2}(1 + 9)$ by symmetry $\rightarrow (5, -16)$	M1 A1 [2]	Any valid method

7. 9709_w16_MS_11 Q: 1

	Answer	Mark	Partial Marks
(i)	$(x + 3)^2 - 7$	B1B1	[2] For $a = 3, b = -7$
(ii)	1, -7 seen $x > 1, x < -7$ oe	B1 B1	[2] $x > 1$ or $x < -7$ Allow $x \leq -7, x \geq 1$ oe

8. 9709_s15_MS_13 Q: 1

	Answer	Mark	Partial Marks
	$2(x-3)^2 - 11$	B1B1B1 [3]	For 2, $(x-3)^2$, -11. Or $a=2, b= -3,$ $c= 11$

9. 9709_w15_MS_13 Q: 3

	Answer	Mark	Partial Marks
(i)	$[3] [(x-1)^2] [-1]$	B1B1B1 [3]	
(ii)	$f'(x) = 3x^2 - 6x + 7$ $= 3(x-1)^2 + 4$ > 0 hence increasing	B1 B1 [✓] DB1 [3]	Ft <i>their (i)</i> + 5 Dep B1 [✓] unless other valid reason

10. 9709_m20_MS_12 Q: 2

	Answer	Mark	Partial Marks
	[Stretch] [factor 2, x direction (or y-axis invariant)]	*B1 DB1	
	[Translation or Shift] [1 unit in y direction] or [Translation/Shift] $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$	B1B1	Accept transformations in either order. Allow (0, 1) for the vector
		4	